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A TEXT BOOK

ON

GRAPHIC STATICS

BY

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PREFACE.

This text was prepared for the author's students in elementary Graphic Statics and Stresses. It has not been the object of the writer to discover new principles, but rather to present the subject clearly and logically. Many texts on Graphic Statics are open to the criticism that they have a tendency to state principles and give constructions without proofs, and the student is therefore compelled to memorize propositions and constructions without being taught the underlying principles. It has been the aim of the writer to give the proofs, together with full explanations of the constructions. No attempt has been made to give elaborate solutions which have little or no practical applications. Particular attention has been given to the order of presentation; and the text has been divided into chapters, articles, and numbered sections to facilitate easy reference.

Most of the material in Part I and Part II has been used in printed form by the author's students for several years. It is hoped that the material in Part II, Part III, and Part IV will be found of assistance to the practicing engineer.

The author gratefully acknowledges his indebtedness to Ira O. Baker, Professor of Civil Engineering, University of Illinois, for many valuable suggestions and criticisms.

Urbana, Illinois,
August, 15, 1909.

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GRAPHIC STATICS.

INTRODUCTION.

This text will treat of the general principles of Graphic Statics, and of the application of these principles to the solution of some of the problems of especial interest to the Civil and Structural Engineer. For convenience, the subject will be divided into four parts as follows:

Part I. General Principles.
Part II. Framed Structures—Roof Trusses.
Part III. Beams.
Part IV. Bridges.

PART I.

GENERAL PRINCIPLES.

CHAPTER I.

DEFINITIONS.

1. **Dynamics** is the science that treats of the action of forces upon a body at rest or in motion. Its two main branches are kinetics and statics.

Kinetics treats of the motion of bodies and of the laws governing the production of motion by forces.

Statics treats of the action of forces under such conditions that no change of motion is produced in the bodies acted upon. It is therefore the science of equilibrium—the science by the aid of which are determined the forces necessary to maintain a body in its state of rest or motion, notwithstanding disturbing tendencies.

2. **Graphic Statics** has for its object the deduction of the principles of statics and the solution of statical problems by means of geometrical constructions.

3. **Rigid Body.** A rigid body is a body which is incapable of change in shape or size when acted upon by forces. Such a body is only imaginary, in that all solids possess elasticity to a greater or lesser degree. Many solids, however, closely approximate a condition of rigidity and for practical purposes may be considered rigid if the forces acting upon them are not great enough to cause rupture. A well designed and properly

constructed truss approximates a rigid body, in that it acts as a whole to resist external forces.

Particle or Point. A particle or point is the smallest conceivable rigid body.

Every problem in statics presupposes the existence of a rigid body or particle upon which the forces act.

4. Rest and Motion. *Rest* is the relation existing between two particles when a line joining them does not change either in length or in direction. *Motion* is the relation existing between two particles when the line connecting them changes either in length or in direction.

There are two kinds of motion, viz.: translation and rotation.

Translation. Translation is the motion corresponding to the change in length of the line connecting two particles. The motion of a body is translation when every point in the body travels in a straight line.

Rotation. Rotation is the motion of a body when all points in the body, that change their positions at all, describe concentric circles in parallel planes. The common normal to these planes, which contains the centers of all the circles, is called the axis of rotation. If the axis of rotation is fixed in position, the motion is pure rotation.

There may be compound translation, as that of a body sliding across a moving car; or compound rotation, as that of the earth revolving around its own axis and also about the sun; or combined translation and rotation, as that of a ball rolling along a straight path.

Two bodies may also be at rest with respect to each other and in motion with respect to a third body. Rest, then, means motionlessness with respect to a definite body of reference, which in this work is understood to be the earth.

5. Force. Force is an action exerted upon a body tending to change its state of rest or motion.

6. Elements of a Force. In order that a force may be completely known, four characteristics of it, called elements, are essential, viz.:

1. *Magnitude.*
2. *Direction.*
3. *Line of Action.*
4. *Point of Application.*

7. **Magnitude.** The magnitude of a force is given by stating, numerically, the ratio of its effectiveness in producing motion to that of the unit force. The magnitude of a force may be expressed, graphically, by the length of a line, the magnitude of the force being the ratio of the given length to the unit length.

8. **Direction.** Direction is the specification as to which of the two ways along the line the force tends to produce motion. Direction is expressed, graphically, by an arrow placed on the line representing the force, indicating which way the force tends to produce motion.

9. **Line of Action.** The line of action of a force is the path along which the force tends to produce motion. The line of action is expressed, graphically, by the position of the line representing the force.

10. **Point of Application.** The point of application is the place (considered as a point) where the force is brought to bear upon the body. The point of application is on the line of action of the force, and together with its position locates the line.

11. **Concurrent and Non-concurrent Forces.** *Concurrent* forces are those whose lines of action meet in a point. *Non-concurrent* forces are those whose lines of action do not meet in a point. Whether or not a given system of forces is concurrent or non-concurrent affects rotation only, since translation is entirely independent of the position of the point of application of the forces.

12. **Coplanar and Non-coplanar Forces.** *Coplanar* forces are those whose lines of action lie in the same plane. *Non-coplanar* forces are those whose lines of action do not lie in the same plane. Except where statements are of a general character, coplanar forces only will be treated in this work.

13. **Equilibrium.** A system of two or more forces is in

equilibrium when the combined effect of the forces produces no change in the body with respect to its state of rest or motion.

14. **Equivalence.** Two forces or systems of forces are equivalent when they have identical effects upon the body acted upon, with respect to its state of rest or motion.

15. **Resultant.** The resultant of a system of forces is the simplest system which is equivalent to the given system. Usually the given system is equivalent to a single force, although this is not always the case.

16. **Equilibrant.** A single force, or the simplest system, which will exactly neutralize the effect of a system of forces, is called the equilibrant of the system. The equilibrant is numerically equal to the resultant, but acts in an opposite direction.

17. **Components.** Any one of a system of forces having a given force for its resultant is called a component of that force. It is evident that a force may have any number of components.

18. **Composition of Forces.** Composition of forces is the process of finding, for a given system of forces, an equivalent system having a smaller number of forces than the given system. The process of finding a single force to replace the given system is the most important case of composition.

19. **Resolution of Forces.** Resolution of forces is the process of finding, for a given system of forces, an equivalent system having a greater number of forces than the given system. The process of finding two or more forces which are equivalent to a given force is the most important case of resolution.

20. **Couple.** A couple consists of two equal, parallel forces, opposite in direction, having different lines of action. The arm of the couple is the perpendicular distance between the lines of action of the two forces.

21. **Force and Space Diagrams.** In the solution of problems in graphic statics, it is usually most convenient to draw two separate figures, one of which shows the forces in magni-

tude and direction and the other in line of action. The former is called the *force diagram,* and the latter the *space diagram.*

22. Notation. In solving problems, graphically, it is often convenient to draw both the force and the space diagrams, and these diagrams are so related that for every line in one there is a corresponding line in the other. The solution is greatly facilitated if a convenient system of notation is adopted for the diagrams.

In the force diagram, each line represents a force in magnitude and direction, and this line will be designated by placing a capital letter at each extremity of the line. In the space diagram, the corresponding line represents the line of action of the force, and this line of action will be marked by the corresponding small letters, one on each side of the line. An arrow placed on the line indicates the direction of the force. The sequence of the letters representing the force also indicates the direction of the force; thus, the force represented by AB acts in a direction from A towards B. This system of notation is illustrated in Fig. 1, AB representing the force in magnitude and direction, while its line of action is marked by the letters a b placed as shown.

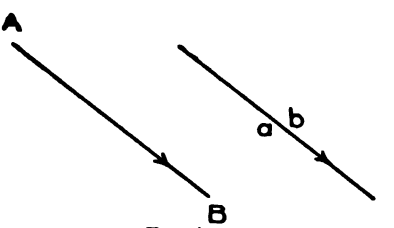


FIG. 1.

CHAPTER II.

CONCURRENT FORCES.

The subject matter given in this chapter will be divided into three articles, as follows: Art. 1, Composition of Concurrent Forces; Art. 2, Resolution of Concurrent Forces; Art. 3, Equilibrium of Concurrent Forces.

ART. I. COMPOSITION OF CONCURRENT FORCES.

23. Resultant of Two Concurrent Forces. Given the two concurrent forces represented in magnitude and direction by the lines BA and BC. It is required to find their resultant.

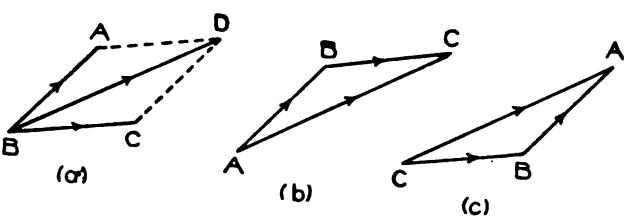


FIG. 2.

Let BA and BC (Fig. 2, a) represent two concurrent forces in magnitude and direction, acting at B. It is required to find their resultant BD. Complete the parallelogram of forces (Fig. 2, a) by drawing the line CD parallel to BA, and AD parallel to BC; then the line BD, connecting the points B and D, will represent the resultant of the two forces in magnitude. The direction of the resultant will be as indicated by the arrow placed on the line BD. (The proof of the above will not be

given here as it may be found in any elementary treatise on mechanics.)

It is unnecessary to construct the entire force parallelogram, for, draw AB (Fig. 2, b) equal and parallel to BA (Fig. 2, a), and BC equal and parallel to BC. Then AC will be equal and parallel to BD; for, by construction, the triangle ABC is equal to the triangle ABD and has its corresponding sides parallel; hence, AC is equal and parallel to BD the required resultant. Likewise, CA (Fig. 2, c) is the resultant of the two concurrent forces BA and BC. It should be noted that the two forces act around the triangle in the same direction, and that the resultant acts in a direction opposite to them.

It is thus seen that any two concurrent forces may be combined into a single resultant force, and further that it is immaterial in which order the forces are taken so long as they act in the same direction around the triangle.

24. Force Triangle and Force Polygon. The triangle shown in either Fig. 2, b, or Fig. 2, c, is called a *force triangle,* and its principles form the basis of the science of graphic statics. If the figure has more than three sides it is called a *force polygon.*

25. Resultant of Any Number of Concurrent Forces. The resultant of any number of concurrent forces may be found (1) by the method of the force triangle, or (2) by the method of the force polygon.

(1) *Solution by Force Triangle.* Let AB, AC, AD, AE, and AF (Fig. 3) represent in magnitude and direction a system of forces meeting at A. It is required to find their resultant R.

The method explained in § 23 may be applied to any number of forces. Commencing at B (Fig. 3), the extremity of the force AB, draw the line BG equal and parallel to the force AC; then AG, acting in the direction shown, is the resultant of the forces AB and AC. In like manner, from the point G, draw the line GH equal and

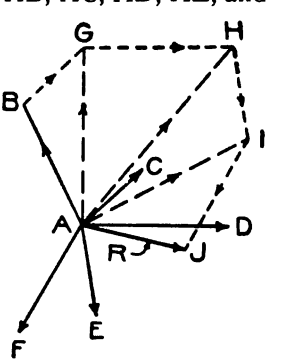


FIG 3.

parallel to the force AD; then AH, acting in the direction shown, is the resultant of the forces AG and AD, or, since AG is the resultant of AB and AC, then AH is also the resultant of the forces AB, AC, and AD. In like manner, AI is the resultant of the forces AB, AC, AD, and AE; and AJ, acting in the direction shown, is the required resultant R of the given system of forces AB, AC, AD, AE, and AF.

(2) *Solution by Force Polygon.* Let AB, BC, CD, DE, and EF (Fig. 4) represent in magnitude and direction a system of forces meeting at O. It is required to find their resultant R.

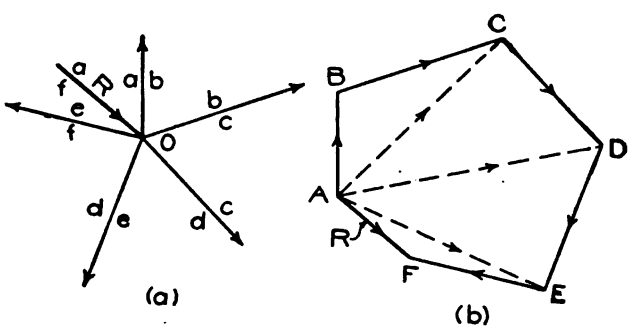


FIG. 4.

The given system of forces is represented in magnitude and direction in Fig. 4, b, and in line of action in Fig. 4, a. Commencing at any point A (Fig. 4, b), draw in succession the lines AB, BC, CD, DE, and EF, parallel respectively to the lines of action ab, bc, cd, de, and ef, representing the given forces in magnitude and direction, each force beginning at the end of the preceding one. Then AF, the line connecting the starting point of the force polygon with the end of the last force, represents in magnitude and direction the required resultant R. For, inserting the dotted lines AC, AD, and AE, which divide the polygon into triangles, it is evident that AC, acting in the direction shown, represents in magnitude and direction the resultant of the two forces AB and BC. Likewise, AD represents in magnitude and direction the resultant of the forces AC and CD, or in other words, AD is the result-

ant of the forces AB, BC, and CD. In like manner, AE is the resultant of the forces AB, BC, CD, and DE; and AF, acting in the direction shown, represents in magnitude and direction the required resultant R of the given system of forces. The line of action of R passes through O, and its direction is as indicated by the arrow.

It will be seen by referring to the force polygon that all the forces except the resultant act around the force polygon in the same direction, and that the resultant acts in a direction opposite to them. By taking the forces in a different order from that shown in Fig. 4, it will be found that the order in which the forces are taken is immaterial, so long as the given forces act around the force polygon in the same direction; since the positions of the initial and final points remain the same.

If the points A and F coincide, the force polygon is said to be *closed.*

ART. 2. RESOLUTION OF CONCURRENT FORCES.

26. It is readily seen from what has been given in the preceding sections that, to resolve a given force into any number of components, it is only necessary to draw a closed polygon, one side of which represents in magnitude the given force and is parallel to its line of action; then the other sides of the polygon will be parallel to, and will represent in magnitude, the components into which the given force is resolved. The given force will act around the force polygon in a direction opposite to that of the components.

27. To Resolve a Given Force Into Two Components. It is evident that this problem is indeterminate unless other conditions are imposed; as an infinite number of triangles may be drawn with one side of the given length.

There are four cases of the resolution of a force into two components, corresponding to the four cases of the solution of a plane triangle, which may be stated as follows:

(1) It is required to resolve a given force into two components which are known in line of action only.

(2) It is required to resolve a given force into two components which are known in magnitude only.

(3) It is required to resolve a given force into two components, one of which is known in line of action only, and the other in magnitude only (two solutions).

(4) It is required to resolve a given force into two components, one of which is completely known, while the other is completely unknown.

The solution of the first case is given below, the other three cases being left for the student to solve.

Case 1. Let AB (Fig. 5) represent in magnitude and direction the known force, and let ac and cb represent the lines of action of the two components meeting at O. It is required to find the unknown elements of the two components.

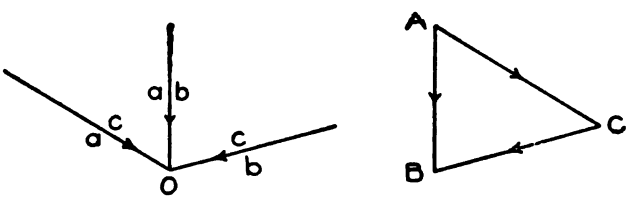


FIG 5.

From the extremities of the known force AB (Fig. 5), draw the lines AC and CB, parallel respectively to ac and cb, intersecting at the point C; then AC and CB represent in magnitude the two components. The directions of these components are shown by the arrows.

ART. 3. EQUILIBRIUM OF CONCURRENT FORCES.

28. Equilibrium of Concurrent Forces. A system of concurrent forces is in equilibrium if the resultant of the system is equal to zero; since this is the condition which must exist if no motion takes place. Referring to Fig. 4, it is seen that if

the resultant is zero, the force polygon is closed; hence the following proposition: If any system of concurrent forces is in equilibrium, the force polygon must close; and conversely, if the force polygon closes and the forces act in the same direction around the polygon, the system is in equilibrium. This is equivalent to the algebraic statements that the summation of the horizontal components of the forces is equal to zero, and that the summation of the vertical components is equal to zero.

If the system of forces is not in equilibrium, the resultant is represented in magnitude and direction by the closing line of the force polygon, this resultant acting around the force polygon in a direction opposite to the other forces. If a force having the same magnitude but acting in an opposite direction is substituted for the resultant, the system is then in equilibrium. It has been shown that the resultant acts around the force polygon in a direction opposite to the given forces; and therefore the force that will hold the given system in equilibrium, called the *equilibrant,* acts around the force polygon in the same direction as the given forces.

29. Solution of Problems in Equilibrium. The fact that the force polygon for a system of concurrent forces is closed if that system is in equilibrium, furnishes a method for solving problems in equilibrium when all the elements of the forces are not known.

The following problems illustrate the general principles employed in the solution of concurrent forces in equilibrium:

Problem 1. Given a system of five concurrent forces in equilibrium, three of which are completely known; of the remaining two, one is known in magnitude only, and the other in line of action only. It is required to find the unknown elements of the two forces. Give two solutions.

Problem 2. Given a system of five concurrent forces in equilibrium, three of which are completely known, the other two being known in line of action only. It is required to find the unknown elements of the two forces.

Problem 3. Given a system of five concurrent forces in equi-

librium, one of which is completely unknown. It is required to fully determine the unknown force.

Problem 4. Given a system of five concurrent forces in equilibrium, three of which are completely known, the other two being known in magnitude only. It is required to find the unknown elements of the two forces. Give two solutions.

The solution of the first problem is given below, the others being left for the students to solve.

Problem 1. Let the forces AB, BC, and CD (Fig. 6) be completely known, let DE be known in magnitude only, and EA in line of action only. It is required to find the unknown elements of the two forces.

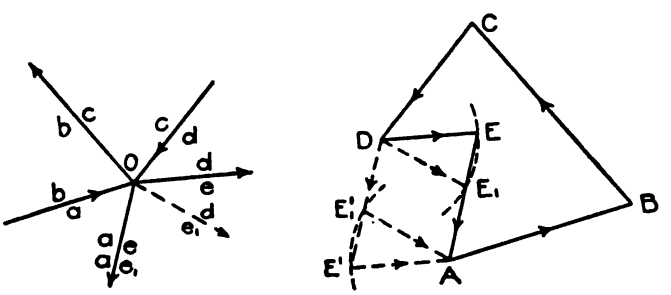


FIG. 6.

First construct the portion of the force polygon ABCD (Fig. 6), using as sides the known forces AB, BC, and CD, taking care that the forces act progressively around the force polygon. Complete the polygon by drawing the line EA through A, parallel to the known line of action of EA. Then, using D as a center and the known magnitude of DE as a radius, draw an arc of a circle intersecting EA at the point E. The lines DE and EA will then represent in magnitude the forces DE and EA, these forces acting around the force polygon in the same direction as the known forces AB, BC, and CD. The entire force polygon is ABCDE, a closed polygon; and since all the forces act around the force polygon in the same direction, they are in equilibrium. ABCDE′ is also a true form of the force polygon, and gives correct values

for the unknown forces; since all the conditions of the problem are fulfilled, i.e., the force polygon is closed, and the forces act continuously around the polygon. Another solution, which gives different values for the unknown forces, is indicated by the force polygon $ABCDE_1$.

CHAPTER III.

NON-CONCURRENT FORCES.

The subject matter given in this chapter will be divided into four articles, as follows: Art. 1, Composition of Non-concurrent Forces; Art. 2, Resolution of Non-concurrent Forces; Art. 3, Equilibrium of Non-concurrent Forces; and Art. 4, Special Constructions for Funicular Polygons.

ART. 1. COMPOSITION OF NON-CONCURRENT FORCES.

30. **Non-concurrent Forces.** In many engineering problems, the forces acting upon the body or structure do not meet at a point, but are applied at different points along the structure. Such forces are non-concurrent, and their lines of action may, or may not, be parallel.

31. **Resultant of Two Non-parallel, Non-concurrent Forces.** The determination of the resultant of two non-parallel, non-concurrent forces requires one of two somewhat different methods of solution, depending upon whether the given forces intersect inside or outside the limits of the drawing.

(1) *Forces Intersecting Within the Limits of the Drawing.* If the two forces are not parallel, their lines of action must intersect at a point, which may be taken as the point of application of each force. The two forces may therefore be treated as concurrent forces, and their resultant may be determined as in § 23.

(2) *Forces Intersecting Outside of the Limits of the Drawing.* Let AB and BC (Fig. 7) represent in magnitude and direc-

tion two non-parallel forces whose lines of action ab and bc intersect outside of the limits of the drawing. It is required to find their resultant.

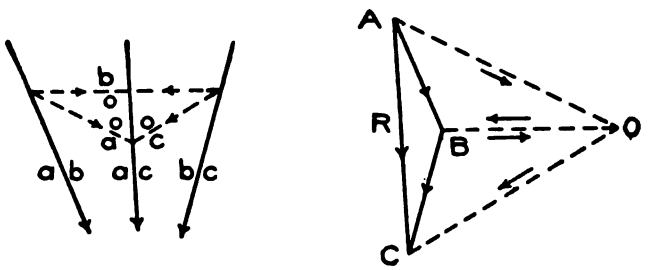


FIG 7

Draw the force polygon ABC (Fig. 7), and connect the points A and C by the closing line AC; then AC, acting as shown by the arrow, represents in magnitude and direction the resultant of the two forces AB and BC, its line of action being as yet unknown. To determine this line of action, an auxiliary construction is necessary. From any point O, draw the lines OA, OB, and OC connecting the point O with the points A, B, and C of the force polygon. These lines represent in magnitude and direction components into which the forces AB and BC may be resolved. Thus, AB is equivalent to the two forces represented in magnitude by AO and OB, acting in the directions shown by the arrows. Likewise, BC is equivalent to the two forces represented in magnitude and direction by BO and OC. To determine the lines of action of these components, start at any point on the line of action ab, and draw ao and ob parallel respectively to AO and OB. Prolong ob until it intersects the line of action bc; and from the intersection of ob and bc, draw the line oc parallel to OC to intersect the line ao. The lines ao, ob, bo, and oc are then the lines of action of the components AO, OB, BO, and OC, into which the original forces have been resolved. Now the forces represented by OB and BO are equal, have the same line of action, and act in opposite directions; hence they neutralize each other and may be omitted from the system,

thus leaving the two forces represented in magnitude and direction by AO and OC and in line of action by ao and oc, respectively. The resultant of these two forces, which have been shown equivalent to the given forces AB and BC, is the force represented in magnitude and direction by AC and in line of action by ac, drawn through the intersection of ao and oc, parallel to AC.

The method explained above is of great importance and should be thoroughly understood by the student; as the principles employed in it will be of great use in solving succeeding problems.

32. **Resultant of Any Number of Non-concurrent, Non-parallel Forces.** There are two methods in general use for finding the resultant of any number of non-concurrent forces. These methods embody the principles explained in § 31, and are merely applications of the method explained in that section. They depend upon whether the given forces have intersections (1) inside of the limits of the drawing, or (2) outside of the limits of the drawing. The first method to be described is limited in its application, but permits of a simpler construction for some problems. The second method, however, is of more general use; as it may be employed for finding the resultant of parallel, as well as non-parallel, forces.

(1) *Forces Having Intersections Within the Limits of the Drawing.* Let AB, BC, CD, and DE (Fig. 8) represent in magnitude and direction a system of four non-parallel, non-concurrent forces; and let ab, bc, cd, and de, respectively, represent their lines of action. It is required to find the resultant of the system of forces.

Applying the method explained in (1) § 31, the resultant of the forces represented by the lines AB and BC (Fig. 8) is represented in magnitude and direction by AC, acting as shown by the arrow, and in line of action by ac, drawn through the intersection of ab and bc, parallel to AC. This resultant may then be combined with the force represented by CD, giving as their resultant the force represented in magnitude and direction by AD and in line of action by ad, drawn through the

intersection of ac and cd, parallel to AD. In like manner, AD may be combined with the force DE, giving as their resultant the force represented in magnitude and direction by AE and in line of action by ae, drawn through the intersection of ad and de, parallel to AE. This last force AE represents in mag-

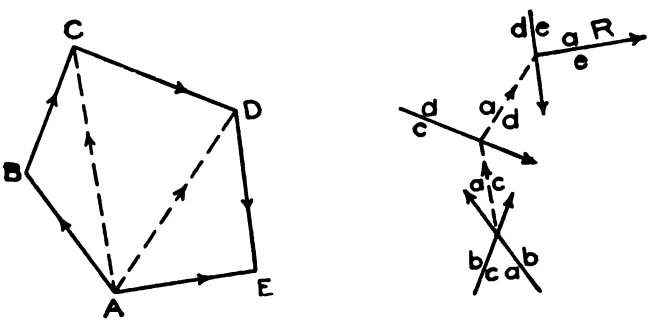


FIG. 8.

nitude and direction the resultant of the given system of forces AB, BC, CD, and DE, its line of action being ae.

It should be borne in mind that AE does not represent the magnitude and direction of any actual force. By the resultant AE is meant a force which, if applied, would produce the same effect upon the body as the given forces.

(2) *Forces Having Intersections Outside of the Limits of the Drawing.* Let AB, BC, CD, DE, and EF (Fig. 9) represent

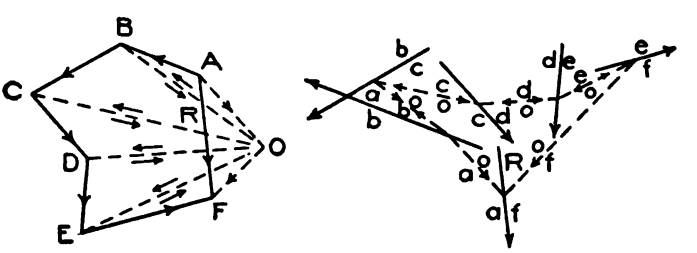


FIG. 9.

in magnitude and direction a system of non-concurrent forces, and let ab, bc, cd, de, and ef, respectively, represent their lines

of action. It is required to find the resultant of the given system of forces.

The resultant of the given system of forces may be found by applying the method explained in (2) § 31. Construct the force polygon ABCDEF, and draw the closing line AF. Then AF, acting in the direction shown by the arrow, represents in magnitude and direction the resultant of the given system of forces. To find its line of action, assume any point O, and draw the lines OA, OB, OC, OD, OE, and OF. Then construct the polygon whose sides are oa, ob, oc, od, oe, and of (see (2) § 31). Note that the two lines, which represent the components into which each force is resolved by the lines drawn from the point O to the extremities of that force, are respectively parallel to the two lines which meet on the line of action on that force. To find the line of action of the resultant, prolong the extreme lines oa and of until they intersect, and through the point of intersection draw af parallel to AF. This is the line of action of the required resultant; for, all the components, into which the given forces are resolved by the lines drawn from the point O to the extremities of the forces, are neutralized (as shown by the arrows) except the two forces represented in magnitude and direction by AO and OF and in line of action by ao and of, respectively. The resultant of these two forces, which are equivalent to the given system, is found in magnitude by completing the force triangle OAF and in direction by making the resultant act around the force triangle in a direction opposite to the other two forces. Since the line of action of the resultant must be on the line of action of each force, it must act through the common point of each line, viz: their point of intersection.

33. **Funicular Polygon.** The polygon whose sides are oa, ob, oc, od, oe, and of (Fig. 9) is called the funicular polygon (also called the equilibrium polygon).

Pole. The point O (Fig. 9) is called the pole of the force polygon.

Pole Distance. The perpendicular distance from the pole to

the line representing the force in the force polygon is called the pole distance.

Rays. The lines OA, OB, OC, OD, OE, and OF (Fig 9), drawn from the pole O to the extremities of the lines representing the forces in the force polygon, are called rays. The rays terminating at the extremities of any side of the force polygon, represent in magnitude the two components which may replace the force represented by that side.

Strings. The sides oa, ob, oc, od, oe, and of of the funicular polygon are called strings. The strings are parallel to the corresponding rays of the force polygon, and are the lines of action of the forces represented by the rays.

Referring to the diagram (Fig. 9), it is seen that for each ray in the force polygon there is a string parallel to it in the funicular polygon; and further, that the two rays drawn to the extremities of any force in the force polygon are respectively parallel to the two strings which intersect on the line of action of that force. Keeping in mind these facts will greatly facilitate the construction of the funicular polygon.

34. In both cases given in § 32, the resultant has been found to be a single force. This may not always be true, and the simplest system that will replace the given system may be a couple; as will be shown in the following section.

35. Resultant of Any Number of Non-parallel, Non-concurrent Forces.—Resultant a Couple. Upon determining the resultant of a given system of forces, it may be found that the first and the last sides of the funicular polygon are parallel. If this is the case, the constructions shown in § 32 do not determine the line of action of the resultant. Referring to Fig. 9, suppose the pole is taken on the line AF; then the first and the last strings of the funicular polygon are respectively parallel to AO and OF, and are therefore parallel to each other. In this case the difficulty is avoided by taking the pole at some point not on the closing line AF. There is, however, one particular case in which AO and OF will be parallel no matter where the pole is taken. Again referring to Fig. 9, it is seen that AO and OF will be parallel if the points A and F coincide.

These two rays will then represent equal and opposite forces, which cannot be combined into a simpler system unless their lines of action coincide. If their lines of action ao and of are coincident, the two forces, being equal and opposite in direction, neutralize each other, and their resultant is equal to zero. If their lines of action are not coincident, the system reduces to a couple. Even if the lines of action of the forces represented by AO and OF are coincident, the forces may still be considered as a couple with an arm equal to zero; hence the following proposition: If the force polygon for any system of forces closes, the resultant is a couple.

By changing the starting point of the funicular polygon, the lines of action of the forces represented by AO and OF will be changed; and by taking a new pole, either their magnitudes, or directions, or both their magnitudes and directions may be changed. Hence, it is seen that any number of couples may be found which are equivalent to each other; since they are equivalent to the same system of forces.

36. Resultant of Any Number of Parallel Forces. Let AB, BC, CD, DE, and EF (Fig. 10) represent in magnitude and direction a system of parallel forces; and let ab, bc, cd, de, and ef represent their lines of action, respectively. It is required to find the resultant of the given system.

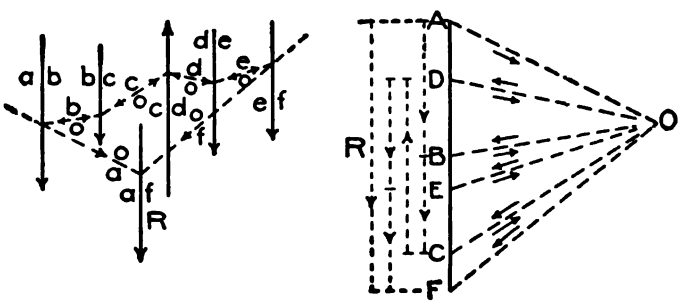


FIG. 10.

Construct the force polygon ABCDEF, which in this case is a straight line; since the forces are parallel. Then AF, acting in the direction shown by the arrow, represents in

magnitude and direction the resultant of the given system of forces. To determine its line of action, assume any pole O, and draw the rays OA, OB, OC, OD, OE, and OF. Then construct the funicular polygon whose sides are oa, ob, oc, od, oe, and of, and prolong the extreme strings oa and of until they intersect. Through this point of intersection, draw af parallel to AF, which gives the line of action of the resultant. For, the given system of forces may be considered to be replaced by the forces represented in magnitude and direction by AO and OF and in line of action by ao and of; since the other forces represented by the rays neutralize each other. The resultant of these two forces is given in magnitude and direction by the closing line AF of the force triangle OAF, and in line of action by af, acting through the intersection of ao and of.

37. **Closing of the Funicular Polygon.** The given system of forces represented in Fig. 9 has been shown to be equivalent to the two forces represented in magnitude and direction by AO and OF and in line of action by ao and of, the first and last strings of the funicular polygon. In general these lines of action are not parallel, but it may happen that they are parallel or that they coincide. In case they coincide, the funicular polygon is said to be *closed.*

ART. 2. RESOLUTION OF NON-CONCURRENT FORCES.

38. The problem of resolving a given force into two or more non-concurrent components is indeterminate unless additional data are given concerning the magnitudes and the lines of action of the required components. A given force may be resolved into three non-parallel, non-concurrent components, or into two parallel components, provided the lines of action of these two components are given. For a greater number of components the problem is indeterminate.

39. **Resolution of a Force Into Three Non-parallel, Non-concurrent Components Having Known Lines of Action.** Let

AB (Fig. 11) represent in magnitude and direction a given force, and let ab be its line of action. It is required to resolve this force into three components acting along the lines cb, dc, and ad.

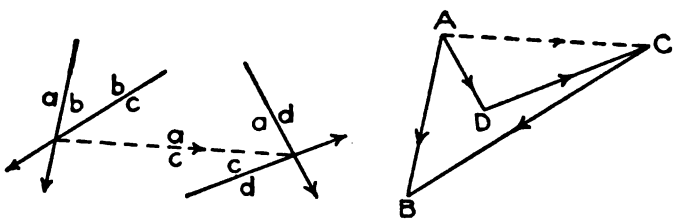


FIG. 11.

Since the given force AB may be assumed to act at any point in its line of action, let its point of application be taken at the intersection of ab and cb. Prolong the lines of action ad and dc until they intersect, and connect this point with the point of intersection of ab and cb. Resolve the given force AB into two components acting along ac and cb (§ 27); then AC and CB, acting as shown by the arrows, will represent in magnitude and direction two components of AB. In like manner, resolve the force represented by AC, whose line of action is ac, into two components acting along the lines ad and dc. These two components are given in magnitude and direction by the lines AD and DC, drawn parallel respectively to ad and dc. Hence, since the given force AB is equivalent to the two forces AC and CB, and AC is equivalent to the two forces AD and DC; therefore, the force AB is equivalent to the three forces represented in magnitude and direction by AD, DC, and CB.

If the line of action of the given force does not intersect any of the given lines of action within the limits of the drawing, the given force may be replaced by two components, each component then resolved by the above method, and the resulting forces combined.

40. Resolution of a Force Into Two Parallel Components Having Known Lines of Action. There are two special cases

of this problem depending upon whether the line of action of the given force is (1) between the lines of action of the two components, or (2) is outside of the lines of action of the two components.

(1) *Line of Action of Known Force Between the Lines of Action of the Two Components.* Let AB (Fig. 12) represent in magnitude and direction the given force, and let ab be its line of action; also let ac and cb be the lines of action of the two parallel components. It is required to find the magnitudes and directions of the two components.

The method explained in (2) § 31 may be used to find the resultant of two parallel forces; and conversely, it may be used to resolve a force into its two components.

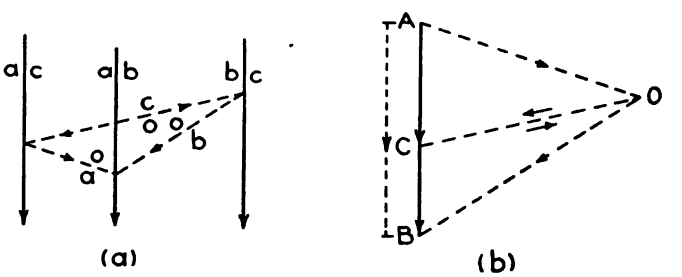


FIG. 12.

Assume any pole O (Fig. 12), and draw the rays OA and OB. At any point on ab, the line of action of the force AB, draw the string ao parallel to AO, and ob parallel to OB. Prolong the two strings until they intersect ac and cb; join these points of intersection by the string oc; and from the pole O, draw the ray OC parallel to the string oc, cutting the line AB at the point C. Then AC and CB, acting in the directions shown by the arrows, represent in magnitude and direction the two components whose lines of action are ac and cb, respectively. For, the diagram shown in Fig. 12, b is the force polygon for the two components AC and CB and their resultant AB; conversely, AB is resolved into its two components AC and CB.

A practical application of this case is the determination of

the magnitudes of the two reactions of a simple beam loaded at any point with a single concentrated load.

The line oc is called the *closing string* of the funicular polygon, and the ray OC is called the *dividing ray* of the force polygon; since it divides the given force into its two components.

(2) *Line of Action of the Known Force Outside of the Lines of Action of the Two Components.* The solution of this case will be left to the student.

Art. 3. Equilibrium of Non-concurrent Forces.

41. Conditions for Equilibrium of Non-Concurrent Forces. It has been shown that a system of concurrent forces is in equilibrium if the force polygon closes; as the resultant is then equal to zero. In order that a system of non-concurrent forces be in equilibrium, it is necessary that the force polygon close, but this single condition is not sufficient to insure equilibrium. For, referring to Fig. 9, it is seen that the given system of forces may be reduced to two forces represented in magnitude and direction by AO and OF and in lines of action by ao and of, respectively. For equilibrium, these two forces must be equal, must have the same line of action, and must act in opposite directions. It is readily seen that the two forces are equal and act in opposite directions if the points A and F coincide; or in other words, if the force polygon closes. In order that they have the same line of action, the first and last strings, ao and of, of the funicular polygon must coincide. From the above, it is seen that for equilibrium of a system of non-concurrent forces:

(1) *The force polygon must close.*

(2) *The funicular polygon must close.*

Therefore, a system of non-concurrent forces is in equilibrium if both the force and funicular polygons close; and conversely, if both the force and funicular polygons close, the system is in equilibrium. For, if the force polygon closes, the two forces AO and OF (Fig. 9) are equal and opposite in

direction, and if the funicular polygon closes, they have the same line of action and neutralize each other.

42. Use of Force and Funicular Polygons in Solving Problems in Equilibrium. It has been shown in Art. 2 that the force and the funicular polygon construction is especially adapted to the solution of problems involving non-concurrent forces. The two conditions necessary for equilibrium of non-concurrent forces, viz.: that the force polygon must close and that the funicular polygon must close, furnish a convenient graphical method for the solution of problems in equilibrium. In order that such problems may be solved, it is necessary that a sufficient number of forces be completely or partly known to permit of the construction of the complete force and funicular polygons; otherwise the problem is indeterminate.

The general method of procedure is as follows: First construct as much of the force and funicular polygons as is possible from the given data; then complete these polygons, keeping in mind the facts that both polygons must close, and that the forces must act continuously around the force polygon.

The application of the above principles to the solution of some of the most important cases arising in practice will now be given.

43. Problem 1. Parallel Forces. Given a system of parallel forces in equilibrium, all being completely known except two, these two being known in line of action only. It is required to find the unknown elements of the two forces.

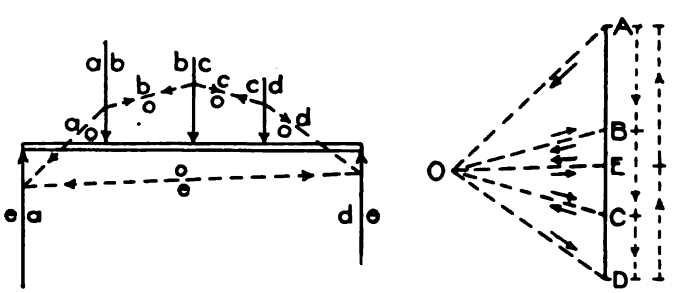


FIG. 13.

Let the known forces be represented by the three loads AB, BC, and CD (Fig. 13), applied to the beam along the lines ab, bc, and cd, and let the unknown forces be the supporting forces of the beam, EA and DE.

Draw the portion of the force polygon ABCD containing the three known forces laid off consecutively. From the condition that the force polygon must close, the combined magnitudes of the two supporting forces must be equal to DA and must act in the direction from D towards A. To determine the magnitude of each supporting force, take any pole O, and draw the rays OA, OB, OC, and OD. Then, commencing at any point on ea, the line of action of the left supporting force, construct the portion of the funicular polygon whose sides are oa, ob, oc, and od, drawn parallel respectively to the rays OA, OB, OC, and OD. Prolong od until it intersects de, and close the funicular polygon by drawing the line oe. Also, from the pole O, draw the ray OE parallel to the closing line oe, cutting DA at E. Then EA and DE, acting in an upward direction, represent in magnitude and direction the two supporting forces.

To thoroughly understand this solution, the student should follow through the constructions from the principle of the triangle of forces. Thus the force AB is resolved into two components represented in magnitude and direction by AO and OB and in line of action by ao and ob, respectively. In like manner, the force BC is resolved into the two components BO and OC, acting along the lines bo and oc; and the force CD is resolved into the two components CO and OD, acting along the lines co and od, respectively. The two forces BO and OB neutralize each other; since they are equal, act in opposite directions, and have the same line of action. For the same reasons, EO neutralizes OE, and CO neutralizes OC. The three given forces are therefore equivalent to the two forces AO and OD, acting along the lines ao and od, respectively. Now in order to have equilibrium, the resultant of AO and the left supporting force EA must neutralize the resultant of OD and the right supporting force DE. The lines of action

of these two resultants must coincide in eo in order that they may neutralize each other. Since the resultant EO and the two forces AO and EA all intersect at a point, they must form a force triangle AOE (§ 23), of which AO is the known side. Then EA, acting in a direction from E towards A, represents the left supporting force in magnitude and direction. In like manner, it may be shown that DE represents the right supporting force in magnitude and direction.

44. Problem 2. Non-parallel Forces. Given a system of non-parallel forces in equilibrium, all completely known except two. Of these two, the line of action of one and the point of application of the other are known. It is required to find the unknown elements of the two forces.

Let the known forces be the three wind loads AB, BC, and CD (Fig. 14) acting on the roof truss, and let the unknown forces be the two supporting forces of the truss. The line of action de of the right supporting force is given as vertical, and the point of application of the left supporting force ea is given at the left end of the truss.

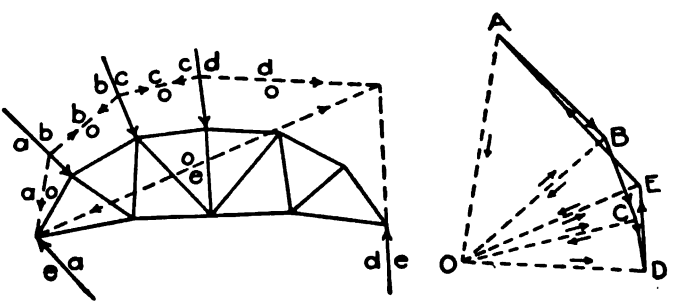


FIG. 14.

Construct the portion of the force polygon ABCD, containing the three known wind forces AB, BC, and CD laid off consecutively; and from any pole O, draw the rays OA, OB, OC, and OD. The side DE of the force polygon must be parallel to the known direction de; but as its magnitude is unknown, as is also the magnitude of the supporting force EA, the polygon cannot be closed. Since the only known

point in the line of action of the left supporting force is its point of application at the left end of the truss, let the funicular polygon be started at this point. Draw the strings oa, ob, oc, and od parallel respectively to the rays OA, OB, OC, and OD, keeping in mind the fact that the lines of action of the two components, into which each force is resolved by the rays, must intersect on the line of action of that force. Produce the string od until it intersects de, the line of action of the right supporting force, and close the funicular polygon by drawing the line oe. From the pole O, draw the ray OE parallel to oe, and from D draw the line DE parallel to the known direction de; their intersection then determines the point E of the force polygon. Close the force polygon by drawing the line EA; then DE and EA, acting in the directions shown by the arrows, represent in magnitude and direction the two supporting forces. The line of action of the left supporting force is found by drawing the line ea parallel to EA, through its known point of application at the left end of the truss.

45. Problem 3. Non-parallel Forces. Given a system of non-parallel forces in equilibrium, all completely known except three, these three being known in line of action only. It is required to determine the unknown elements of the three forces.

Let AB, BC, and CD (Fig. 15) represent the forces completely known, and let DE, EF, and FA represent the three forces which are known in line of action only.

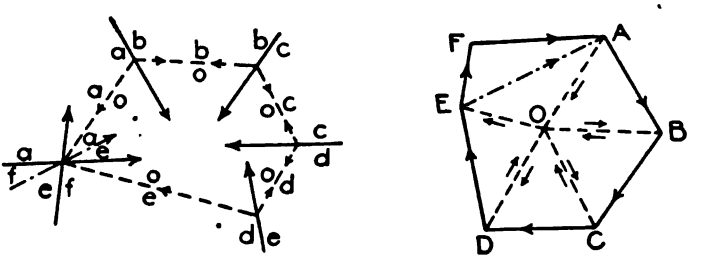


FIG. 15.

Since the resultant of any two forces must pass through

their point of intersection, let two of the unknown forces, whose lines of action are represented by ef and fa, be replaced by their resultant acting through their point of intersection. The problem then becomes identical with Problem 2, de being the line of action of one unknown force; and the intersection of ef and fa, the point of application of the other unknown force. The construction of Problem 2 having been made, and the resultant EA of the two forces found, the magnitudes of these forces, whose lines of action are represented by ef and fa, may be found by resolving this resultant into two components parallel to these known lines of action. In Fig. 15, EA represents this resultant in magnitude and direction, and EF and FA represent the two components into which it is resolved. Since the force DE is given by the construction of Problem 2, the three unknown forces are therefore represented in magnitude and direction by DE, EF, and FA.

In these problems and in those given in the preceding sections, it should particularly be noted that the forces should be used in such an order that those which are completely known are consecutive.

46. Inaccessible Points of Intersection. In Problem 3 and in other problems, it may happen that the point of intersection of the two forces falls outside of the limits of the drawing. When this is the case, and when it is required to draw a line from a given point through the point of intersection of two forces whose lines of action intersect outside of the limits of the drawing, the following geometrical construction may be employed:

Let XX and YY (Fig. 16) be two lines intersecting outside of the limits of the drawing. It is required to draw a line through their point of intersection from a given point P.

Draw any line PA through the point P to intersect the lines XX and YY at the points A and B,

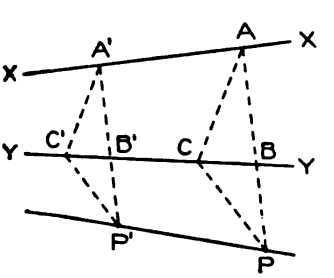


FIG. 16.

respectively. Also, draw the lines AC and PC intersecting on YY to form the triangle ACP. From any point A′ on XX, draw the lines A′P′ and A′C′, parallel respectively to AP and AC, and from C′ draw the line C′P′ parallel to CP to intersect A′P′ at P′. Then PP′, prolonged, will pass through the point of intersection of XX and YY. For, since the triangles ABC and BCP are similar to the triangles A′B′C′ and B′C′P′, respectively, therefore AB : A′B′ :: CB : C′B′, and BP : B′P′ :: CB : C′B′, which gives AB : A′B′ :: BP : B′P′, thus proving that the three lines XX, YY, and PP′ meet in a point.

47. Problems. (1). A rigid beam 16 feet in length rests horizontally upon supports at its ends and carries the following loads: its own weight of 200 lbs., acting at its center, and three loads of 150 lbs., 80 lbs., and 120 lbs., acting at points which are distant 4 ft., 10 ft., and 13 ft., respectively, from the left support. Find the upward pressures, or reactions, at the supports.

(2). Given the roof truss and wind loads as shown (Fig. 17). The truss is fixed to the wall at the right end, and is supported on rollers at the left. It is required to find the magnitude and line of action of the right reaction and the magnitude of the left. (The left reaction must be vertical; since the left end of the truss is on rollers and can have no horizontal component.)

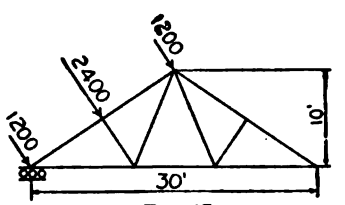


FIG. 17.

(3). A uniform bar 24 inches long, weighing 20 lbs., has one end resting against a smooth wall, and is supported on a smooth peg, which is 13 inches from the wall. The bar is held in equilibrium at an angle of 60 degrees with the vertical by a weight W suspended from the free end of the bar. It is required to find the weight W, and the pressures against the bar by the wall and the peg.

48. Special Method. If any system of forces in equilibrium is divided into two groups, the resultants of the two groups must be equal, must act in the opposite directions, and must have the same line of action. These facts suggest a special method for solving certain problems, which in some cases is simpler than the general method of constructing the force and funicular polygons.

Problem. Let AB (Fig. 18) represent a force which is completely known; and let BC, CD, and DA be known in line of action only, the lines of action of the four forces being ab, bc, cd, and da.

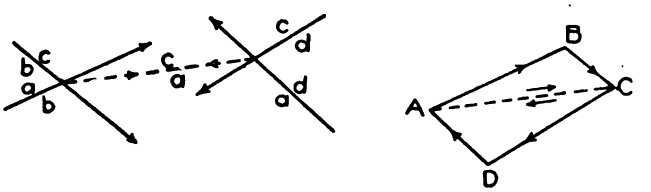


FIG. 18.

The resultant of the forces whose lines of action are ab and bc must pass through their point of intersection; also, the resultant of the forces whose lines of action are cd and da must pass through the intersection of cd and da. For equilibrium, these two resultants must be equal, must have opposite directions, and must have the same line of action; hence each must act through the line ac. Draw AB representing the magnitude and direction of the known force, and from A and B, the extremities of the line AB, draw lines parallel respectively to ac and bc. Then BC represents the magnitude and direction of the force whose line of action is bc; and AC, acting in the direction from A towards C, represents the magnitude and direction of the resultant of AB and BC. But for equilibrium, CA, acting in the opposite direction, i. e., from C towards A, must represent the resultant of the forces acting along the lines cd and da; hence these forces are represented in magnitude and direction by CD and DA, drawn from the points C and A, parallel respectively to cd and da.

Problem. A beam with rounded ends has one end resting against a smooth wall and the other end on a smooth floor. The weight of the beam is 60 lbs. acting through its center. Neglecting the friction of the beam against the wall and floor, what force applied horizontally at the lower end of the beam would support it at an angle of 60 degrees with the horizontal?

ART. 4. SPECIAL CONSTRUCTIONS FOR FUNICULAR POLYGONS.

49. Relation Between Different Funicular Polygons for the Same Forces. It will now be shown that if two funicular polygons are drawn, using the same forces and force polygons but different poles, the intersections of corresponding strings of these funicular polygons will meet on a straight line which is parallel to the line joining the two poles.

Let AB, BC, and CD (Fig. 19) represent in magnitude and direction a system of forces, and let ab, bc, and cd, respectively, be their lines of action.

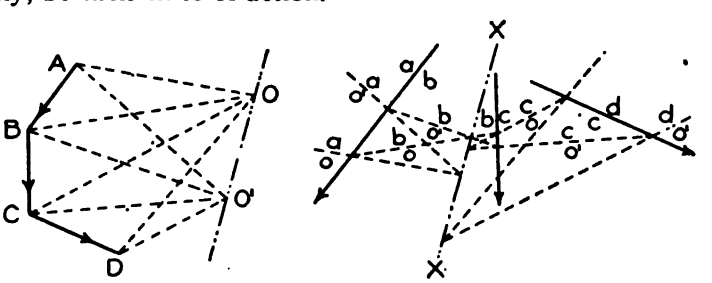


FIG. 19.

Draw the force polygon ABCD for the given system of forces, and with the pole O, draw the rays OA, OB, OC, and OD; also, with any other pole O', draw the rays O'A, O'B, O'C, and O'D. Construct the funicular polygons whose sides are oa, ob, oc, and od, having the strings respectively parallel to the rays OA, OB, OC, and OD; also, construct the funicular polygon whose sides are o'a, o'b, o'c, and o'd, having the

strings respectively parallel to the rays O'A, O'B, O'C, and O'D. Produce the strings oa and o'a until they intersect. In like manner, produce the other corresponding strings until each pair intersects. Draw the line XX through these points of intersection; then XX will be parallel to OO', the line joining the two poles. For, the quadrilaterals whose sides are AB, OA, OO', O'B; and ab, oa, XX, o'b have by construction three sides and two diagonals respectively parallel each to each; AB parallel to ab, OA parallel to oa, O'B parallel to o'b, O'A parallel to o'a, and OB parallel to ob: hence the fourth sides OO' and XX are also parallel. The same relation may be proved for the quadrilaterals whose sides are OB, BC, CO', OO'; and ob, bc, co', XX, and so on.

This relation between the two funicular polygons is employed for drawing the line of pressure of an arch.

50. To Draw a Funicular Polygon Through Two Given Points. Let AB, BC, and CD (Fig. 20) represent in magnitude and direction a system of forces acting on the beam; and let ab, bc, cd, respectively, be their lines of action. It is required to draw a funicular polygon passing through the two points M and N, which points are on the lines of action of the two reactions.

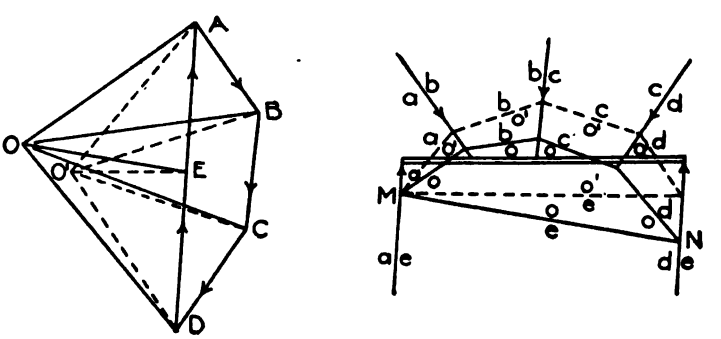


FIG. 20. FUNICULAR POLYGON THROUGH TWO POINTS.

Assume any pole O', and draw the rays O'A, O'B, O'C, and O'D. Construct the funicular polygon whose sides are o'a, o'b, o'c, and o'd, and close the polygon by drawing the

closing line o′e. From the pole O′, draw the dividing ray O′E, cutting AD at the point E. Then DE and EA represent the two reactions whose lines of action are de and ea, respectively. Now the point E will remain fixed, no matter where the pole is taken; since the two reactions must be the same for the given system of forces. From E draw the line EO parallel to the line connecting the two given points M and N. Then the new pole O, for the funicular polygon passing through the two given points, must be on this line; since OE is the dividing ray of the force polygon, drawn parallel to the closing string MN. With the new pole O (at any point on OE), commence at M, and draw the funicular polygon whose sides are oa, ob, oc, od, and oe, which must pass through the given points M and N.

51. **To Draw a Funicular Polygon Through Three Given Points.** Let AB, BC, CD, and DE (Fig. 21) represent in magnitude and direction a system of forces acting on the beam; and let ab, bc, cd, and de, respectively, be their lines of action. It is required to draw a funicular polygon passing through the three given points a, b, and c.

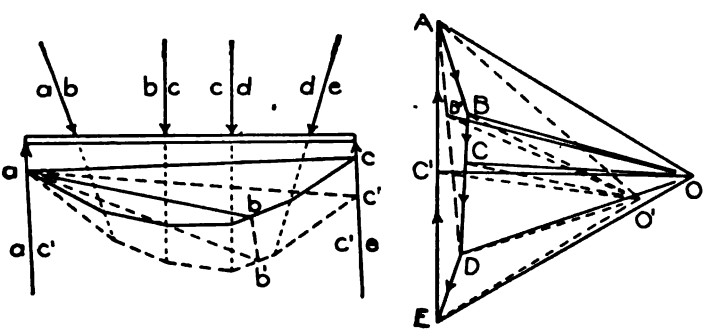


FIG. 21. FUNICULAR POLYGON THROUGH THREE POINTS.

Assume a system of forces to be acting upon a beam of such a length that the two reactions will pass through the two outside points. This length is determined by drawing lines through the two outside points parallel to the equilibrant of the given forces.

Take any pole O′, and draw the rays O′A, O′B, O′C, O′D, and O′E. Commencing at a, construct the funicular polygon a b′ c′, and close the polygon by drawing the closing line ac′. From the pole O′, draw the dividing ray O′C′, cutting EA, the equilibrant of the given forces, at C′. Then EC′ and C′A represent the two reactions whose lines of action are ec′ and c′a, and which pass through c and a, respectively. Now draw the line DA, which represents the equilibrant of the three forces to the left of b. Through b, draw the line bb′ parallel to DA. Connect the points a and b′, and the points a and b by the lines ab′ and ab, which are the closing strings of the funicular polygons for the forces to the left of b. Through O′, draw the dividing ray O′B′ parallel to ab′, cutting DA at B′. Then DB′ and B′A represent the reactions of the forces to the left of b, acting through the points b and a, respectively. Point C′ is common to all force polygons for the given system of forces, and point B′ is common to all force polygons for the forces to the left of b. From the points C′ and B′, draw lines parallel respectively to ac (the line connecting two of the given points a and c) and ab, intersecting at O. Then these lines are the dividing rays corresponding to the closing strings of the required funicular polygon, and their intersection O will determine the pole for this polygon. With this pole O, commence at a, and draw the required funicular polygon, which must pass through the three given points a, b, and c. For, if the pole O is on the line C′O, the funicular polygon must pass through the points a and c; and if it is on the line B′O, the funicular polygon must pass through the points a and b.

This construction and that shown in § 50 may be used to determine whether or not the line of pressure in a masonry arch remains within the middle third at all joints.

CHAPTER IV.

MOMENTS.

This chapter is divided into two articles, viz.: Art. 1, Moments of Forces and of Couples, and Art. 2, Graphic Moments. The first article treats of the general principles of the moments of forces and of couples; and the second treats of the graphic determination of moments.

ART. 1. MOMENTS OF FORCES AND OF COUPLES.

52. Moment of a Force. The moment of a force about any point is the product of the magnitude of the force into the perpendicular distance of its line of action from the given point. The point about which the moment is taken is called the *origin (or center) of moments;* and the perpendicular distance from the origin to the line of action of force is called the *arm* of the force.

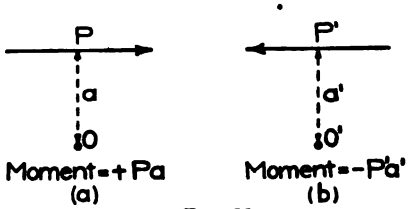


FIG. 22.

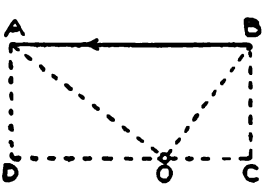


FIG. 23.

53. Positive and Negative Moments. Rotation may be in either of two opposite directions, viz.: in the direction of the hands of a clock, or opposite to the direction of the hands of a

clock. The first will be called positive; and the second, negative. The sign of the moment of the force is taken the same as that of the direction of the rotation it tends to produce. Thus the moment of the force P (Fig. *22*, a) about the point O is equal to $+Pa$, and the moment of P′ (Fig. *22*, b) about the point O′ is equal to $-P'a'$.

54. Moment Area. The moment of a force may be represented by double the area of a triangle, the vertex of which is at the origin of moments and the base a length in the line of action of the force equal to the magnitude of the force. Thus the moment of the force AB (Fig. *23*) about the point O is numerically represented by twice the area of the triangle OAB, which is equivalent to the area of the rectangle ABCD.

55. Transformation of Moment Area. In comparing the moments of forces, it is often convenient to transform their moment areas into equivalent areas having a common base. The moments are then to each other as the altitudes of their respective moment areas. Thus in Fig. *24*, let the moment area represented by ABCD be transformed into an equivalent area having its base equal to DC′.

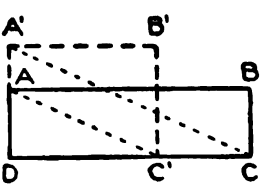


FIG. 24.

Connect A and C′, and from C draw CA′ parallel to C′A, and prolong it until it intersects DA, prolonged, at the point A′. Complete the rectangle A′B′C′D, which is the required moment area equivalent to ABCD. For, since the triangles ADC′ and A′DC are similar, AD:A′D::DC′:DC; hence, $AD \times DC = A'D \times DC'$, or in other words, the area ABCD= the area A′B′C′D.

56. Moment of the Resultant of Two Concurrent Forces. *Proposition.* The moment of the resultant of two concurrent forces about any point in their plane is equal to the algebraic sum of their separate moments about the same point.

Let DA (Fig. *25*) represent the resultant of the two concurrent forces BA and CA whose lines of action intersect at A, and let O be the origin of moments. It is required to

prove that the moment of DA about O is equal to the algebraic sum of the separate moments of BA and CA about the same point.

Connect the points O and A by the line OA, and from O, draw OE perpendicular to OA. Draw the lines DE, CF, and BG parallel to OA, and join the points O and B, O and C, and O and D by the lines OB, OC, and OD, respectively. Then (§ 54), the moment of BA about O is equal to twice the area of the triangle OBA $= +OA \times OG$. Likewise, the moment of CA about O is equal to twice the area of the triangle OCA $= +OA \times OF$; and the moment of DA about O is equal to twice the area of the triangle ODA $= +OA \times OE$. But $OE = OF + FE$, or since $FE = OG$ by construction, then $OE = OF + OG$. Therefore, $OA \times OE = OA\ (OF + OG)$, or the moment of DA about the point O is equal to the algebraic sum of the moments of BA and CA about the same point.

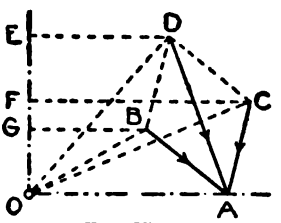


FIG. 25.

57. Moment of the Resultant of Any Number of Concurrent Forces. *Proposition.* The moment of the resultant of any number of concurrent forces about any point in their plane is equal to the algebraic sum of their separate moments about the same point.

The proof of this proposition follows directly from that given in § 56, and is simply an extension of that proof.

58. Moment of a Couple. The moment of a couple about any point in the plane of the couple is equal to the algebraic sum of the moments of the two forces composing the couple about the same point.

It will be shown that the moment of a couple is equal to the product of one of the forces into the perpendicular distance between the lines of action of the forces.

Let O (Fig. 26) be the origin of moments, and let P and P′ be the two equal forces of the couple.

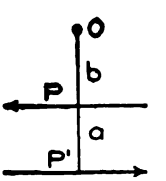


FIG. 26.

Then the moment of the couple is equal to $-P'(a+b)+Pb = -P'a$ (since P is equal to P'), or the moment of the couple is equal to the product of one of the forces into the perpendicular distance between the two forces. Since O is any point in the plane of the couple, it is evident that the moment of the couple is independent of the origin of moments.

59. **Moment of the Resultant of Any System of Forces.** *Proposition.* The moment of the resultant force, or resultant couple, of any system of coplanar forces about any point in their plane is equal to the algebraic sum of the separate moments of the forces composing the system about the same point.

Let AB, BC, CD, and DE (Fig. 27) represent in magnitude and direction any system of forces, and let ab, bc, cd, and de represent their lines of action, respectively.

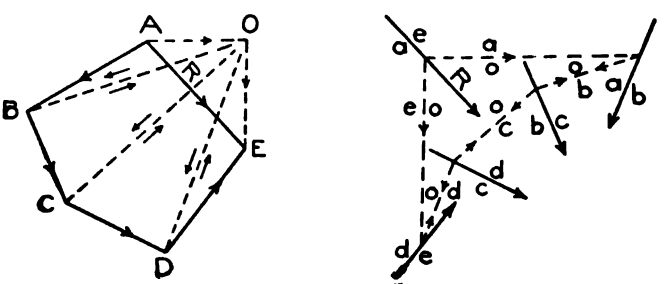


FIG. 27.

Assume any pole O, and draw the rays OA, OB, OC, OD, and OE. Also, draw the funicular polygon whose sides are oa, ob, oc, od, and oe. Now, AB may be replaced by AO and OB, acting along ao and ob; BC may be replaced by BO and OC, acting along bo and oc; CD may be replaced by CO and OD, acting along co and od; and DE may be replaced by DO and OE, acting along do and oe. (By AB, BC, etc., are meant the forces represented in magnitude and direction by AB, BC, etc.). Now (§ 56), no matter where the origin of moments is taken:

Moment of AB = moment of AO + moment of OB;
" " BC = " " BO + " " OC;
" " CD = " " CO + " " OD;
" " DE = " " DO + " " OE.

Since the forces represented by OB and BO are equal in magnitude, opposite in direction, and have the same line of action, their moments are equal but have opposite signs. In like manner, the moments of OC and CO, and of OD and DO are equal but have opposite signs. The addition of the above four equations shows that the sum of the moments of AB, BC, CD, and DE is equal to the sum of the moments of AO and OE. Now the resultant of the given system of forces may be either a resultant force or a resultant couple (§ 32 and § 35). In the former case, the resultant of the system is the resultant of AO and OE, which is AE; and its moment is equal to the algebraic sum of their moments, or to the moment of AE (§ 56). In the latter case, which occurs only when E coincides with A, the resultant is composed of the forces AO and OE. These forces are equal, act in opposite directions, and form a couple whose moment is equal to the algebraic sum of the moments of AO and OE. The proposition is therefore true in either case.

It should be noticed that the proof here given applies to any system of forces, whether parallel or non-parallel.

60. Moment of a System of Forces. *Definition.* The moment of a system of forces is equal to the algebraic sum of the moments of the forces composing the system.

61. Condition of Equilibrium. *Proposition.* If a given system of forces is in equilibrium, the algebraic sum of their moments about every origin must be equal to zero. For, in order that the given system of forces shown in Fig. 27 be in equilibrium, AO and OE must be equal, must act in opposite directions, and must have the same line of action. The sum of their moments, which is equal to the sum of the moments of the given system of forces, is therefore equal to zero. The converse of this proposition, if the algebraic sum of the moments about every origin is equal to zero, the system of

forces is in equilibrium, is also true. For, if the sum of their moments is not equal to zero, the system must have either a resultant force or a resultant couple. If the resultant is a force, then its moment is not equal to zero unless the origin is taken on its line of action; and if the resultant is a couple, then its moment is not equal to zero for any origin, no matter where taken. Therefore, if the sum of the moments about every origin is equal to zero, the system has neither a resultant force nor a resultant couple, and must be in equilibrium.

ART. 2. GRAPHIC MOMENTS.

62. Graphic Representation of the Moment of a Force. *Proposition.* If through any point in the space diagram a line is drawn parallel to a given force, the distance intercepted upon this line by the two strings corresponding to the components into which the given force is resolved by the rays multiplied by the pole distance of the force is equal to the moment of the force about the given point.

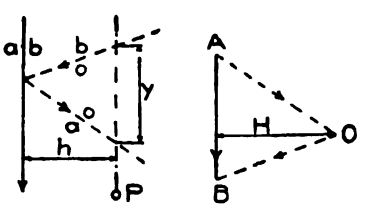


FIG. 28.

Let AB (Fig. 28) represent in magnitude and direction a force whose line of action is ab. Also, let P be the origin of moments, and H the pole distance of the given force. From the pole O, draw the rays OA and OB, and from any point on ab, draw the strings oa and ob parallel respectively to OA and OB. Also, from the origin P, draw a line parallel to the line of action of the given force AB, intersecting these strings and cutting off the intercept y. It is required to prove that the moment of the given force P is equal to H multiplied by y.

Let h be the perpendicular distance from P to ab. Then the moment of AB about P is equal to $-AB \times h$. From similar triangles, $H : h :: AB : y$; therefore $H \times y = AB \times h$, which proves the proposition.

It should be noticed that the intercept always represents a distance and is measured to the same scale as the distances in the space diagram, and that the pole distance always represents a force magnitude and is measured to the same scale as the forces in the force diagram.

63. Moment of Any System of Forces. *Proposition.* The moment of any system of coplanar forces about any origin in the plane is equal to the distance intercepted, on a line drawn through the origin of moments parallel to the resultant of all the forces, by the strings which meet upon the line of action of the resultant, multiplied by the pole distance of the resultant.

Let ABCDE (Fig. 29) be the force polygon, and let the polygon whose sides are oa, ob, oc, od, and oe be the funicular polygon for the given system of forces AB, BC, CD, and DE. Also, let R represent the resultant of the system, let H be the pole distance of this resultant, and y the distance intercepted, on a line drawn through the origin of moments P parallel to the resultant, by the strings meeting on the line of action of the resultant. It is required to prove that the moment M of the given system of forces is equal to $H \times y$.

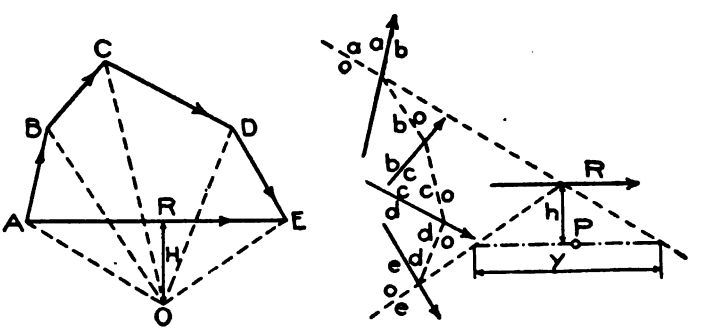


FIG. 29.

The moment of the resultant is equal to the moment of the given system of forces (§ 59). Then the moment M of the given system is equal to $R \times h$, where h is the perpendicular distance from the origin of moments to the line of action of

the resultant. From similar triangles, H : h :: R : y; therefore, $H \times y = R \times h$, which proves the proposition.

Referring to the space diagram (Fig. 29), it is seen that the resultant R tends to produce rotation in the direction of the hands of a clock, and is positive. The magnitude of this moment decreases as the origin approaches the line of action of the resultant, until it becomes zero when on the line of action of R. If the origin passes through this line of action, the moment is negative.

64. Moment of Parallel Forces. The method explained in § 62 and § 63 is especially useful when it is desired to find the moment of any, or all, of a system of parallel forces. For example, it is required to find the moment at any point in a beam caused by several loads on the beam.

Let M-N (Fig. 30) be a simple beam loaded with the loads AB, BC, CD, and DE. It is required to find the moment at any point P due to the forces to the left of P.

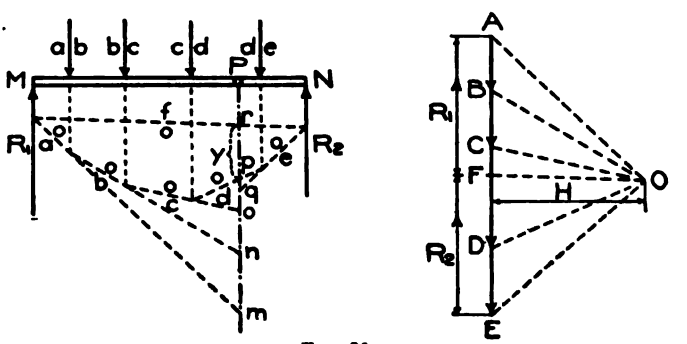


FIG. 30.

Construct the force and funicular polygons for the given system of forces, as shown in Fig. 30, and determine the magnitudes of the reactions R_1 and R_2. Produce each string of the funicular polygon until it intersects a line drawn through P parallel to the resultant of all the forces. Let H be the pole distance for the given forces, which in this case is the same for all the forces; since the force polygon is a straight line. Then (§ 62), considering the forces to the left of P:

Moment of R_1 about P $= +H \times mr$;
" " AB " P $= -H \times mn$;
" " BC " P $= -H \times no$;
" " CD " P $= -H \times op$.

The addition of these equations gives the sum of the moments of R_1, AB, BC, and CD $= H(+mr - mn - no - op) = +H \times y$, which is equal to the moment in the beam at the point P. In like manner, it may be shown that the moment of the forces to the right of P is equal to $-H \times y$. Since P is any point along the beam, it is seen that the moment at any point is equal to the ordinate of the funicular polygon, cut off by a line through the given point parallel to the resultant of all the forces, multiplied by the pole distance.

The summation of the moments of all the forces to the left of any point in the beam about that point is called the *bending moment.*

The ordinate represents a distance and is measured to the same scale as the beam, while the pole distance represents a force and is measured to the same scale as the forces in the force polygon.

65. Problems. *Problem 1.* Assume a system of five non-parallel, non-concurrent forces, and choose any origin in their plane. Determine their separate moments, and also the moment of their resultant by the method of § 62 and § 63.

Problem 2. Given the simple beam loaded as shown in Problem 1, § 47. Find the moment at a point 12 feet from the left end by the method of § 64.

Problem 3. Given the simple beam loaded at the same points as in Problem 1, § 47. The loads have the same numerical values as in Problem 1, but make the following angles with the axis of the beam (all angles being measured counter-clockwise): load at 4-ft. point, 315°; load at center, 270°; load at 10-ft. point, 240°; load at 13-ft. point, 225°. It is required to find the bending moments in the beam at points which are distant 6 ft., 11 ft., and 14 ft., respectively, from the left end of the beam. Both reactions are parallel to the resultant of all the loads.

CHAPTER V.

CENTER OF GRAVITY OF AREAS.

In designing static structures, it is frequently necessary to deal with the center of gravity of areas. The center of gravity of some areas may be found by simple geometrical constructions; while for others, it is convenient to divide the figure into elementary areas, and, treating these areas as a system of parallel forces, to locate the point of application of the resultant of the system. The point of application of the resultant of a system of parallel forces acting at fixed points is frequently referred to as the center of parallel forces, but is more appropriately called the *centroid;* and is the point through which the line of action of the resultant passes in whatever direction the parallel forces are assumed to act.

Methods will be given in this chapter for finding the center of gravity of some common geometrical areas, also of irregular areas.

66. Geometrical Areas. The center of gravity of some of the most common geometrical areas will now be located. The proofs of the constructions will not be given, as they may be found in any treatise on plane geometry; but the student should be able to supply the demonstrations.

(1) *Parallelogram.* The center of gravity of a parallelogram is at the point of intersection of the two bisectors of the opposite sides of the figure. Thus let ABCD (Fig. 31) be a parallelogram, and let ac and bd be the bisectors of the opposite sides; then the center of gravity of the figure is at G, the point of intersection of ac and bd.

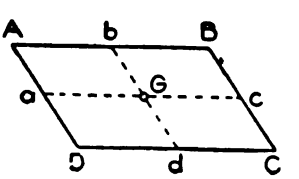


FIG. 31.

(2) *Triangle.* The center of gravity of a triangle lays on a line drawn from any vertex to the middle of the opposite side, and is therefore at the intersection of any two such lines. The center of gravity is at a distance from the vertex equal to two-thirds of the length of any bisector. Thus let Ba (Fig. 32) be one of the bisectors; then the center of gravity G divides the line Ba so that BG is equal to two-thirds Ba.

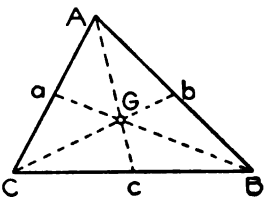


FIG. 32.

(3) *Quadrilateral.* Let ABCD (Fig. 33) be a quadrilateral whose center of gravity is required. Draw BD, and let E be its middle point. Join E with the points A and C. Make EM equal to one-third EA, and EN equal to one-third EC. Join the points M and N, and make MG = NH; then G is the center of gravity of the quadrilateral.

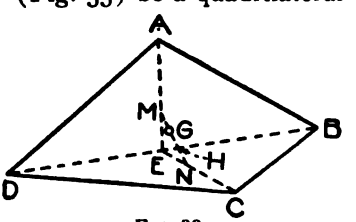


FIG. 33.

(4) *Circular Sector.* Let ABCO (Fig. 34) be a circular sector whose center of gravity is required. On AO make Aa equal to one-third AO, and describe the arc abc. Bisect the sector by the line OB. Draw bd tangent to the arc abc at b, and make bd equal to the length of one-half of the rectified arc abc. Join the points O and d, and from c draw ce parallel and from e draw eG perpendicular to OB. Then G is the center of gravity of the sector.

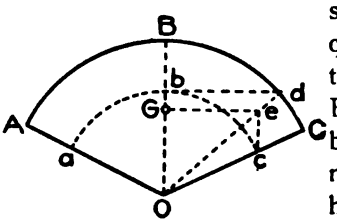


FIG 34.

(5) *Circular Segment.* Let ABC (Fig. 35) be a circular segment whose center of gravity is required. Draw the extreme radii AO and CO, thus completing the sector ABCO. Draw the middle radius BO. Locate the center of gravity of the triangle AOC, and that of the sector ABCO by the methods previously explained. Draw ab perpendicular to CO, and from g and g′, the centers of gravity of the triangle ACO and the sector

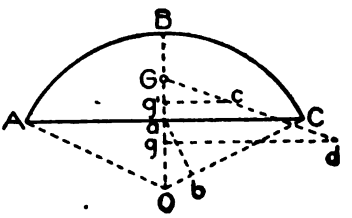


FIG 35.

ABCO, respectively, draw gd and g'c parallel, in any direction. Make g'c equal to ab and gd equal to the length of the rectified arc BC. Join the points d and c, and prolong the line dc to cut BO at G, which is the center of gravity of the sector.

67. **Irregular Areas.** The center of gravity of an irregular area may be found by dividing the figure into elementary areas, replacing these areas by parallel forces, and then locating the centroid of the resultant of the system of forces. The centroid of a system of parallel forces having fixed points of application has been defined as the point through which the line of action of the resultant of the system passes, in whatever direction the forces are assumed to act; and a method will now be given for finding this centroid.

68. **Determination of the Centroid of Parallel Forces.** Let ab and bc (Fig. 36) be the lines of action of the two parallel forces AB and BC, and let ac be the line of action of their resultant AC. Draw any line MN perpendicular to the lines of action of the given forces, intersecting them at the points M, O, and N. Since AC is the resultant of AB and BC, the moment of AC about any point in their plane is equal to the sum of the moments of AB and BC about the same point. If the origin of moments is taken on the line of action ac, then the moment of AC is equal to zero; and therefore $-AB \times MO + BC \times ON = 0$, or $AB \times MO = BC \times ON$. The latter equation shows that MN is divided into segments which are inversely proportional to AB and BC. Any other straight line cutting ab and bc will also be divided into segments which are inversely proportional to AB and BC. If the two forces act in opposite directions, then ac will lay outside of ab and bc; but the above statement will hold

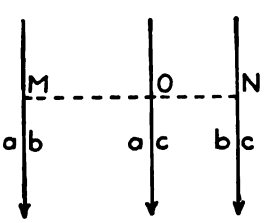


FIG. 36.

true. Now suppose the lines of action ab and bc to be turned through any angle about the points M and N, respectively; then the line of action ac will always pass through O. For, if the points M and N remain fixed, the line of action aç of the resultant will always divide the line MN into segments which are inversely proportional to the two forces AB and BC. Therefore, the point O will always remain fixed. The point O is called the *centroid* of the parallel forces AB and BC for the fixed points M and N.

The centroid of any number of parallel forces having fixed points of application may be located by first finding the centroid of any two of the forces, and then, taking the resultant of these two forces as acting at the centroid thus determined, by finding the centroid of this resultant and one of the remaining forces, etc.

69. Graphic Determination of the Centroid of a System of Parallel Forces. Since the centroid must be on the line of action of the resultant of the given system of forces, to locate this centroid it is only necessary to find the line of action of the resultant for each of two assumed directions of the forces; then the intersection of these two lines of action is the centroid of the given system. The forces may be assumed to be turned through any angle; but the greatest accuracy is secured if the two resultants are made to act at right angles to each other.

70. Center of Gravity of an Irregular Area. The method explained in § 69 may be used to find the center of gravity of an irregular area. The figure may be divided into areas whose centers of gravity are known, and these areas may then be taken as parallel forces acting through their respective centers of gravity. The problem then resolves itself into finding the centroid of a system of parallel forces having fixed points of application.

Let PQRSTUVW (Fig. 37) be the area whose center of gravity is required. Divide the figure into three rectangles, as shown, whose areas are represented by the forces AB, BC, and CD. Then take the point of application of these forces at the centers of gravity of their respective areas, and draw

their lines of action parallel to the side PQ. (The lines might have been drawn parallel in any direction, but are drawn as indicated for convenience). Construct the force and the funicular polygons for these forces, and locate the line of action of their resultant R_1. Then take the lines of action of the forces AB, BC, and CD parallel to the side PW, construct their force and funicular polygons, and locate the line of action of their resultant R_2. The point of intersection of the

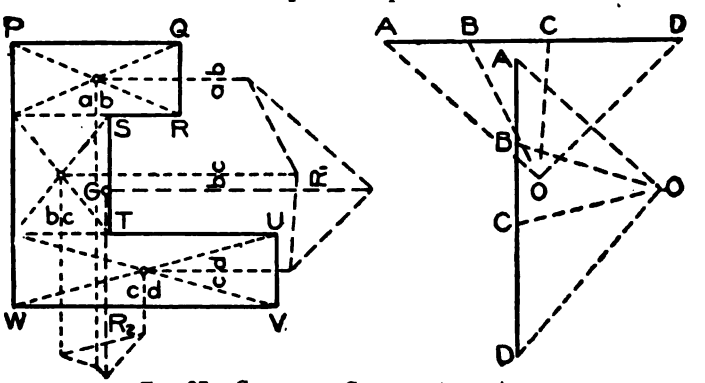


FIG. 37. CENTER OF GRAVITY OF AN AREA.

lines of action of the two resultants is the center of gravity of the area PQRSTUVW. For, the center of gravity of the forces which represent the respective areas must be on the line of action of every resultant, and is therefore at the point of intersection of any two of them.

If the given area has an axis of symmetry, then its center of gravity must be on this axis; and if it has two such axes, its center of gravity must be at their point of intersection.

CHAPTER VI.

MOMENT OF INERTIA.

The moments of inertia of most of the standard steel sections used in engineering have been computed, and may be found in tables published by the manufacturers of such sections. The moment of inertia of an irregular shaped section is often required, and the moment of inertia of such a section may be readily found by graphic methods. The elementary areas composing the section may be treated as parallel forces, and the moment of inertia of these forces may then be found by means of the force and the funicular polygons. This chapter will therefore be divided into two articles, as follows: Art. 1, Moment of Inertia of Parallel Forces, and Art. 2, Moment of Inertia of Areas.

ART. 1. MOMENT OF INERTIA OF PARALLEL FORCES.

71. **Definition.** The moment of inertia of a force with respect to any axis is the product of the magnitude of the force into the square of the distance of its point of application from the given axis. The moment of inertia of a system of parallel forces with respect to any axis is the sum of the moments of inertia of the separate forces composing the system about the same axis.

The student should notice that the moment of inertia as defined above is simply a mathematical product which frequently occurs in the solution of engineering problems, and in no sense involves the inertia of a body or mass. Since the

moment of inertia is so frequently required in the solution of problems, it is desirable to determine this product for all sections likely to occur in practice.

72. Moment of Inertia of a System of Parallel Forces. There are two methods in common use for finding the moment of inertia of forces, viz.: Culmann's, and Mohr's. In Culmann's method, the moment of inertia is determined by first finding the moment of the given forces, and then the moment of this moment, by means of the force and the funicular polygons. In Mohr's method, the moment of inertia is determined from the area of the funicular polygon.

73. Culmann's Method. Let the given system of parallel forces and the axis with respect to which their moment of inertia is to be determined lay in the same plane. The moment of inertia of any one of the given forces may be found as follows: First find the moment of the force about the given axis; then assume a force, equal in magnitude to this moment and acting in a direction corresponding to the sign of the moment, to act at the point of application of the original force, and find the moment of this new force about the given axis, which is the moment of inertia of the given force. The algebraic sum of the moments of inertia of all of the forces is the moment of inertia of the given system. The application of the above principles to a problem will now be shown.

Let ab, bc, cd, and de (Fig. 38) be the lines of action of the given system of forces AB, BC, CD, and DE whose magnitudes and directions are as shown in the force polygon. Also, let the line XX, which is drawn parallel to the lines of action of the given forces, be the axis with respect to which their moment of inertia I is required.

Construct the force polygon ABCDE, and the funicular polygon whose sides are oa, ob, oc, od, and oe, for the given forces. Then the moment of the force AB with respect to the axis XX is equal to $A'B' \times H$ (§ 62). Also, the moments of the forces BC, CD, and DE are respectively equal to $B'C' \times H$, $C'D' \times H$, and $D'E' \times H$. It is seen from Fig. 38 that the distinction between positive and negative moments is given if

the sequence of the letters is observed in reading the intercepts. Now take these moments as forces, applied along the lines of action of the original forces. Thus, let the force represented by $A'B' \times H$ be applied along ab, let the force represented by $B'C' \times H$ be applied along bc, etc. The line A'B'C'D'E' may be taken as the force polygon for the second system of forces; but it must be borne in mind that each force in this polygon must be multiplied by H to give its true magnitude. Assume any pole O', and construct a new funicular polygon whose sides are o'a', o'b', o'c', o'd', and o'e'. Since the moment of inertia of the system of forces is required, and since the sum of the separate intercepts is equal to the intercept between the extreme strings, it is only necessary to prolong the extreme strings o'a' and o'e' until they intersect the axis XX at the

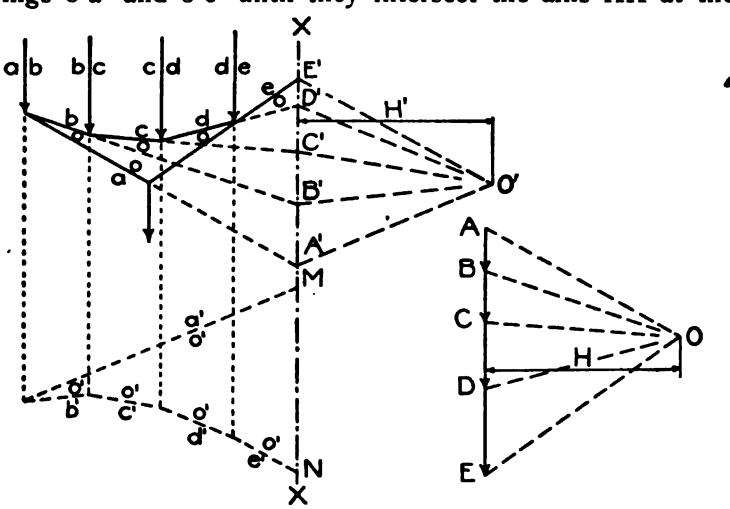


FIG. 38. MOMENT OF INERTIA—CULMANN'S METHOD.

points M and N, respectively. The moment of this second system of forces is then equal to $MN \times H \times H'$, which is the momenf of inertia of the given system. For, $A'E' \times H$ is the moment of the original system of forces AB, BC, CD, and DE (§ 63), and is equal to the summation of the products of each force into its distance from the axis XX; also $MN \times H \times H'$

is the moment of this moment, and is equal to the summation of the products of each force into the square of its distance from the axis XX, which by definition is the moment of inertia I of the given system of forces.

It should be noted that H is a line in the force diagram, therefore H represents a force and is measured to the same scale as the given forces; while H′ and MN are lines in the space diagram, represent distances, and are measured to the same scale as the distances in the space diagram. In choosing H and H′, they should be taken of such units of length that the numerical force which H represents and the distance which H′ represents may be easily multiplied together.

74. Mohr's Method. Let ab, bc, cd, and de (Fig. 39) be the lines of action of the given forces AB, BC, CD, and DE, whose magnitudes and directions are as shown in the force polygon. It is required to find the moment of inertia of the system about the axis XX, which axis is parallel to the lines of action of the given forces.

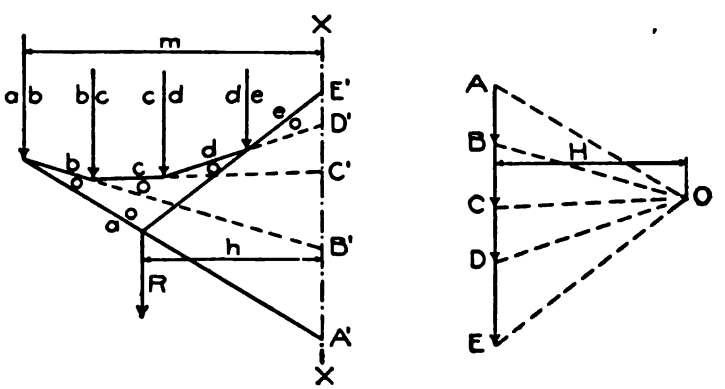


FIG. 39. MOMENT OF INERTIA—MOHR'S METHOD.

Construct the force polygon ABCDE, and with the pole O and the pole distance H, draw the funicular polygon whose strings are oa, ob, oc, od, and oe. Prolong these strings until they intersect the axis XX at the points A′, B′, C′, D′, and E′, respectively. Then the moment of AB about XX is equal

to the intercept A′B′ multiplied by the pole distance H; and the moment of inertia of AB about the same axis is equal to the moment of this moment, which is equal to A′B′ × H × m. But A′B′ × m equals twice the area of the triangle whose base is A′B′ and whose vertex is on ab; and therefore the moment of inertia of AB about the axis XX is equal to the area of this triangle multiplied by 2H. In like manner, it may be shown that the moment of inertia of BC about the axis XX is equal to the area of the triangle whose base is B′C′ and whose vertex is on bc, multiplied by 2H; also that the moment of inertia of CD is equal to the area of the triangle whose base is C′D′ and whose vertex is on cd, multiplied by 2H; and further that the moment of inertia of DE is equal to the area of the triangle whose base is D′E′ and whose vertex is on de, multiplied by 2H. Adding the moments of inertia of these forces, it is seen that the moment of inertia of the given system is equal to the area of the funicular polygon multiplied by twice the pole distance, i.e., I is equal to the area of the polygon whose sides are oa, ob, oc, od, oe, and E′A′, multiplied by 2H.

75. Relation Between Moments of Inertia About Parallel Axes. A graphic proof will now be given for the proposition that the moment of inertia I of a system of parallel forces about any axis parallel to the forces is equal to the moment of inertia $I_{c.g.}$ about an axis through their centroid plus the moment of inertia I_r of the resultant of the system about the given axis.

The moment of the resultant R of the system of forces shown in Fig. 39 about the axis XX is equal to A′E′ × H; and the moment of inertia I_r of R about XX is equal to the moment of this moment, which is equal to A′E′ × H × h (where h is the perpendicular distance between XX and the line of action of R). But A′E′ × h is equal to twice the area of the triangle whose base is A′E′ and whose vertex is on the line of action of the resultant R. Therefore, the moment of inertia I_r of R about XX is equal to the area of the funicular triangle whose base is A′E′ and whose vertex is on the line of action of R, multiplied by 2H. In like manner, if the axis is taken to coincide with the line of action of R, i.e., through

the centroid of the system, it may be shown that the moment of inertia $I_{c.g.}$ is equal to the area of the funicular polygon whose sides are oa, ob, oc, od, and oe, multiplied by 2H. Now the area of the triangle plus the area of the polygon is equal to the total area of the figure whose sides are oa, ob, oc, od, oe, and E′A′. But the total area of this figure multiplied by 2H is equal to the moment of inertia I of the given system of forces about the axis XX; therefore, $I = I_{c.g.} + I_r$.

It will be shown that the moment of inertia of a system of forces about any axis is equal to the moment of inertia of the given system about a parallel axis through its centroid plus the product of the magnitude of the resultant of the system into the square of the distance between the two axes. For, by definition, the moment of inertia I_r of the resultant of the system about the axis XX is equal to Rh^2 (where h is the perpendicular distance from the line of action of R to the axis XX). Substituting this value of I_r in the equation $I = I_{c.g.} + I_r$ gives $I = I_{c.g.} + Rh^2$, which proves the proposition.

By taking the axis through the centroid of the system and then moving it, first to one side, then to the other side of this centroid, it is readily seen (Fig. 39) that any movement of the axis out of the centroid increases the total area included between the extreme strings meeting on the resultant. Therefore, the moment of inertia of a given system of parallel forces is a minimum about an axis through the centroid of the system.

76. Radius of Gyration. The radius of gyration of a system of parallel forces about any axis is the distance from the axis to a point through which the resultant of the system must act in order that the moment of inertia may be the same as that of the given system. An equation will now be derived for the radius of gyration in terms of the moment of inertia and the resultant of the given system. Let R, the resultant of the system, be substituted for the given forces, and let this resultant act at such a distance r (which by definition is the radius of gyration of the system) from the axis that its

moment of inertia I_1 remains the same as that of the original system of forces, I. Then $I_1 = I = Rr^2$,

or $r^2 = \frac{I}{R}$. which gives $r = \sqrt{\frac{I}{R}}$.

77. Graphic Determination of the Radius of Gyration. The moment of inertia of the system of forces shown in Fig. 38 is equal to $H \times H' \times MN$. If r is the radius of gyration of the given system, then (§ 76), $r^2 = \frac{I}{R} = \frac{H \times H' \times MN}{R}$. Now if H (Fig. 38) is taken equal to $AE = R$, then the preceding equation becomes $r^2 = H' \times MN$, which gives $r = \sqrt{H' \times MN}$. The length of this radius of gyration may be found, graphically, as follows: Draw $AB = MN$, and $BC = H'$. With AC as a diameter, construct the semi-circle ADC; and from B, draw the line BD perpendicular to AC, intersecting the semi-circle at D. Then from geometry, AB : BD :: BD : BC, or $AB \times BC = \overline{BD}^2$. Substituting the values of AB and BC, we have $MN \times H' = \overline{BD}$, or $BD = \sqrt{H' \times MN}$, which has been shown equal to the radius of gyration r.

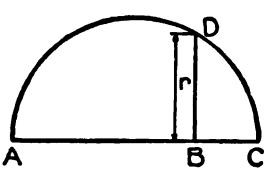


FIG. 40.

In the above construction, H has been taken equal to $AE = R$. If H′ had been taken equal to nR, then AB (Fig. 40) should have been taken equal to nMN, or BC equal to nH′.

ART. 2. MOMENT OF INERTIA OF AREAS.

78. Moment of Inertia of an Area. The moment of inertia of an area about any axis is equal to the summation of the products of the differential areas composing the area into the squares of the distances of these differential areas from the axis. The moment of inertia of an area may be found by dividing the given area into elementary areas, treating these elements as parallel forces, and finding the moment of inertia

of the forces. There are two somewhat similar graphic methods for finding the moment of inertia of an area, one of which gives an approximate value, and the other an accurate value.

79. **Approximate Method for Finding the Moment of Inertia of an Area.** An approximate value for the moment of inertia of an area about any axis may be obtained as follows: Divide the given area into small elements by lines drawn parallel to the axis, and let these elementary areas be replaced by forces numerically equal to them, acting at their respective centers of gravity. Then find the moment of inertia of this system of forces, which will be approximately equal to that of the given area. The smaller these elementary areas are taken, the more nearly will the moment of inertia of the parallel forces which represent them approximate that of the given area; and if they could be taken as infinitesimal in size, their moment of inertia would be the true value for the moment of inertia of the area.

The application of the above principles to the determination of the moment of inertia of an area will now be shown.

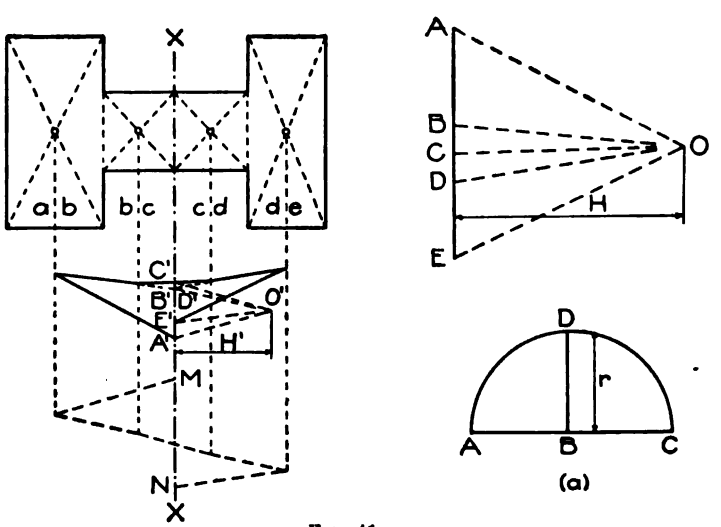


FIG. 41.

Let the area shown in Fig. 41 be the area whose moment of inertia about the axis XX is required.

Divide the section into four rectangular areas as shown, and assume the forces AB, BC, CD, and DE, numerically equal to these areas and parallel to the axis XX, to act at the centers of gravity of the respective areas. Construct the force polygon ABCDE for these forces, and with the pole distance H (in this case H is taken equal to AE) and the pole O, draw the funicular polygon corresponding to this pole. Prolong the strings of the funicular polygon until they intersect the axis at the points A′, B′, C′, D′, and E′, and with the pole distance H′ and the pole O′, draw the second funicular polygon, using the same lines of action as before. Prolong the first and last strings of this funicular polygon until they intersect the axis XX at the points M and N. Then $H \times H' \times MN$ is equal to the moment of inertia of the forces AB, BC, CD, and DE, (§ 73), and is approximately equal to the moment of inertia of the given area.

A more nearly correct value for the moment of inertia of the area might have been obtained if the area had been divided into smaller strips by lines drawn parallel to XX; as the distance of each elementary area from the axis would then have been more nearly equal to the distance of its center of gravity from the axis.

80. Radius of Gyration of an Area. The radius of gyration of an area about any axis is the distance from the axis to a point at which if all the area was concentrated the moment of inertia would be the same as that of the given area.

An approximate value for the radius of gyration of the area shown in Fig. 41 may be obtained by applying the method explained in § 77. Thus, lay off BC (Fig. 41, a) equal to MN, and AB equal to H′. Then on AC as a diameter, construct a semi-circle, and from B draw the line BD perpendicular to AC, cutting the semi-circle at the point D. Then BD is approximately equal to the radius of gyration of the given area. It should be remembered that the pole distance H (Fig. 41) was taken equal to AE. If H had been taken equal to $n \times AE$,

then AB should have been taken equal to nH′, or BC equal to nMN.

81. Accurate Method for Finding the Moment of Inertia of an Area. An accurate value for the moment of inertia of an area about any axis may be obtained if it is possible to divide the area into elementary areas whose radii of gyration about axes through their centers of gravity and parallel to the given axis are known. For, if the radius of gyration of each elementary area about an axis through its center of gravity and also the distance of its center of gravity from the given axis are known, then its radius of gyration about the given axis may be determined. Now, if forces numerically equal to these areas and acting parallel to the axis at distances equal to their respective radii of gyration about the given axis are substituted for these areas, and the moment of inertia of these forces is determined, the result will be the moment of inertia of the given area. For, by definition, the radius of gyration of an area is the distance from the axis to a point at which the entire area must be concentrated in order that the moment of inertia may remain the same as about the given axis.

The application of the above method to the determination of the moment of inertia of an area will now be shown by a problem. Let the area shown in Fig. 42 be the area whose moment of inertia about XX is required, and let AB, BC, CD, and DE represent in magnitude the elementary areas into which the section is divided.

The lines of action of these forces may be found as follows: Draw PR (Fig. 42) perpendicular to XX and equal to the distance from XX to the center of gravity of the elementary area shown; also, at the point R, draw QR perpendicular to PR and equal to the radius of gyration of this area about an axis through its center of gravity and parallel to XX. Connect the points P and Q, and with the line PQ as a radius, draw the arc QS. Then PS = PQ is the radius of gyration of the elementary area about XX, and is the distance from the axis to the point of application of the force AB. For, the moment of inertia of the elementary area about an axis through its

center of gravity and parallel to XX is equal to $AB \times \overline{QR}^2$ (from the definition of the radius of gyration), and the moment of inertia of this area about the axis XX is equal to $AB \times \overline{QR}^2 + AB \times \overline{PR}^2 = AB(\overline{QR}^2 + \overline{PR}^2) = AB \times \overline{PQ}^2 = AB \times \overline{PS}^2$ (§ 75). In like manner, the lines of action of the forces BC, CD, and DE, which represent the other elementary

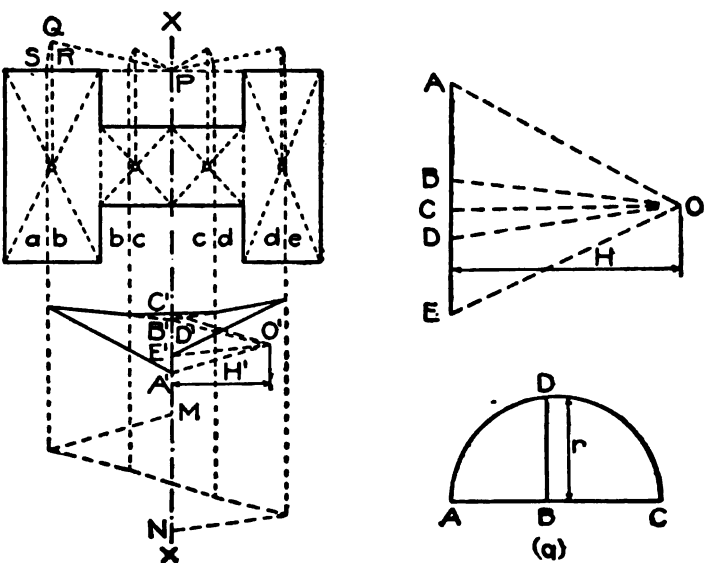


FIG. 42.

areas, may be found. Now apply the method explained in § 73 and § 79, and find the moment of inertia of these forces (see Fig. 42). Then the moment of inertia of these forces, which is equal to the moment of inertia of the given area, is equal to $H \times H' \times MN$.

The correct value for the radius of gyration of the area shown in Fig. 42 is given by the line BD (Fig. 42, a). The construction for finding this radius of gyration is similar to that used for Fig. 41, a, which is explained in § 80, except that the values for H′ and MN, which have been found by the construction of Fig. 42, are used in Fig. 42, a.

82. The table shown in Fig. 42, b gives the algebraic formulæ for finding the moments of inertia and radii of gyration of several common sections.

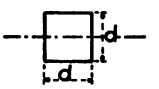
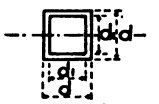
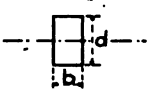
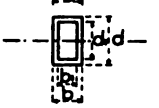
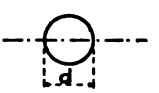
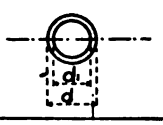
SECTION	MOMENT OF INERTIA	RADIUS OF GYRATION
	$\frac{d^4}{12}$	$\frac{d}{\sqrt{12}}$
	$\frac{d^4-d_1^4}{12}$	$\sqrt{\frac{d^2+d_1^2}{12}}$
	$\frac{bd^3}{12}$	$\frac{d}{\sqrt{12}}$
	$\frac{bd^3-b_1d_1^3}{12}$	$\sqrt{\frac{bd^3-b_1d_1^3}{12(bd-b_1d_1)}}$
	$0.049d^4$	$\frac{d}{4}$
	$0.049(d^4-d_1^4)$	$\sqrt{\frac{d^2+d_1^2}{4}}$

FIG. 42, b.

83. Relation Between the Moments of Inertia of an Area About Parallel Axes. If the areas are used instead of the forces which represent them, then the relation shown in § 75 for the moment of inertia of a force about any axis, in terms of its moment of inertia about a parallel axis through its center of gravity, becomes, the moment of inertia of an area about any axis is equal to its moment of inertia about a parallel axis through its center of gravity plus the product of the area into the square of the distance between the two axes. The application of this relation is of great importance in designing sections.

84. Moment of Inertia of an Area Determined from the Area of the Funicular Polygon (Mohr's Method). In the

examples that have been given for finding the moment of inertia of an area, only Culmann's method has been used, but Mohr's method might have been employed instead. If the lines of action of the forces had been taken as acting at the centers of gravity of the elementary areas, an approximate value for the moment of inertia would have been found; and if each force had been taken as acting at a distance from the given axis equal to the radius of gyration of the elementary area about the given axis, then an accurate value would have been found from the area of the funicular polygon.

PART II.

FRAMED STRUCTURES—ROOF TRUSSES.

CHAPTER VII.

DEFINITIONS.

85. Framed Structure. A framed structure is a structure composed of a series of straight members fastened together at their ends in such a manner as to make the entire frame act as a rigid body.

The only geometrical figure which is incapable of any change of shape without a change in the length of its sides is the triangle; and it therefore follows that the triangle is the basis of the arrangement of the members in a framed structure.

86. Types of Framed Structures. Framed structures may be divided into the three following classes: (a) complete framed structures, (b) incomplete framed structures, and (c) redundant framed structures.

(a) *Complete Framed Structure.* A complete framed structure is one which is made up of the minimum number of members required to form the structure wholly of triangles. This is the type which is usually employed, and which will receive the most attention in this work. A simple form of a complete framed structure is shown in Fig. 43.

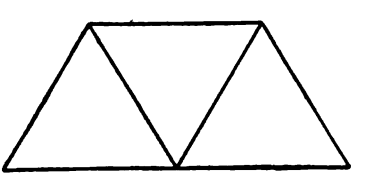


FIG. 43. COMPLETE.

(b) *Incomplete Framed Structure.* An incomplete framed structure is one which is not wholly composed of triangles. A simple form of an incomplete framed structure is shown in Fig. 44. Such a structure is stable only under symmetrical or specially arranged loads.

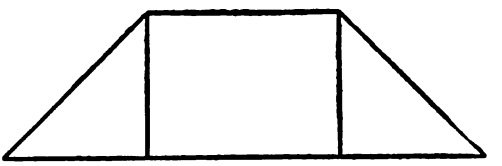


FIG. 44. INCOMPLETE.

(c) *Redundant Framed Structure.* A redundant framed structure is one which contains a greater number of members than is required to form the structure wholly of triangles. If the second diagonal is added to a quadrilateral, then the added diagonal is a redundant member; but if the member is capable of resisting only one kind of stress, then the redundancy is only apparent. Fig. 44, a shows a redundant structure; but if the diagonals are made of rods and can therefore take tension only, then the redundancy is only apparent; as only one diagonal will act at a time.

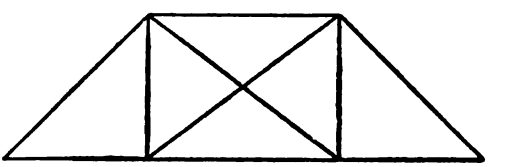


FIG. 44, a. REDUNDANT.

87. **Roof Truss.** A simple roof truss is a framed structure whose plane is vertical and which is supported at its

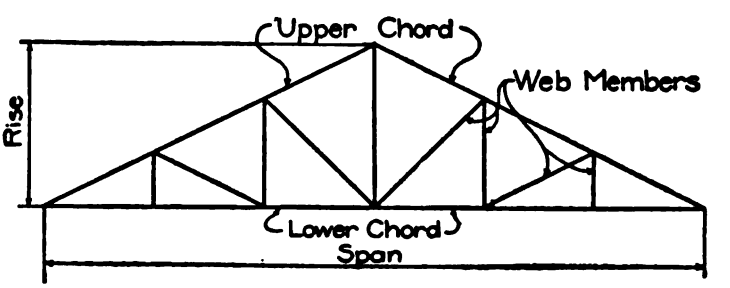


FIG. 45. ROOF TRUSS.

ends. The ends of the truss may be supported upon side walls, or upon columns. A common form of a simple roof truss is shown in Fig. 45.

Span. The span of a truss is the distance between the end joints, or the centers of the supports, of the truss.

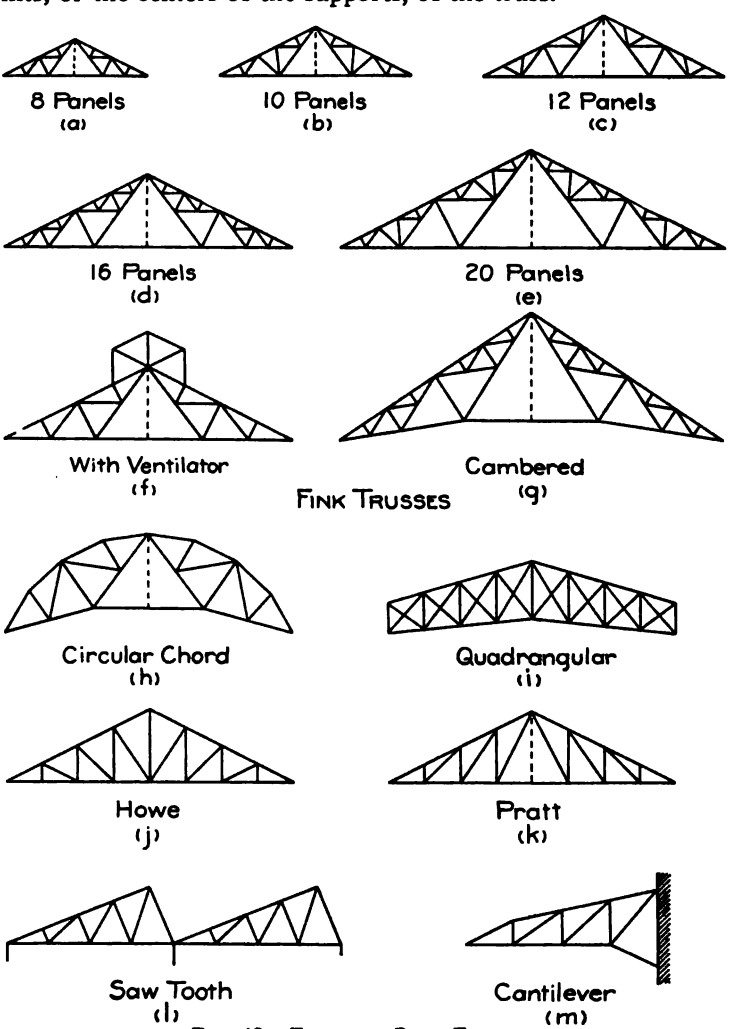


FIG. 46. TYPES OF ROOF TRUSSES.

Rise. The rise is the distance from the apex, or highest point of the truss, to the line joining the points of support of the truss.

Pitch. The pitch is the ratio of the rise of the truss to its span.

Upper and Lower Chords. The *upper chord* consists of the upper line of members of the truss, and extends from one support to the other, through the apex. The *lower chord* consists of the lower line of members, and extends from one support to the other.

Web Members. The web members connect the joints of the upper chord with those of the lower chord. A web member which is subject to compression is called a *strut,* and one which is subject to tension is called a *tie.*

Pin-connected and Riveted Trusses. A pin-connected truss is one in which the members are connected with each other by means of pins. A riveted truss is one in which the members are fastened together by means of plates and rivets. The latter type is the more common, while the former is used for long spans.

88. Types of Roof Trusses. Several types of roof trusses are shown in Fig. 46.

When a building is to be designed, the conditions governing the design should be carefully studied before deciding upon the type of roof truss to be used; as a truss which is economical for one building may not prove so for another.

The Fink truss is very commonly used, and is well adapted to different spans; as the number of panels may be easily increased.

The Quadrangular truss shown in Fig. 46, i is well adapted to the use of counterbracing.

The Howe and Pratt trusses shown in Fig. 46, j and Fig. 46, k, respectively, are common type, and may be used with a small rise.

As a rule, trusses with long compression members are not economical.

CHAPTER VIII.

LOADS.

The loads that must be considered in the discussion of a roof truss may be classed as follows: (1) dead loads; (2) snow loads; and (3) wind loads. A discussion of these loads will be given in the three following articles.

Instead of considering separately the dead, snow, and wind loads, an equivalent vertical load is sometimes taken. This method is efficient in some cases if intelligently used, but should not be used by the beginner.

ART. 1. DEAD LOAD.

A short description of the common type of roof construction, together with the terms employed, will now be given, preliminary to the discussion of dead loads.

89. Construction of a Roof. A roof includes the covering and the framework. There are a number of materials used for roof coverings, among the more common being slate, tiles, tar and gravel, tin, and corrugated steel. *Sheathing* is commonly used in connection with roof coverings, but it is often omitted when corrugated steel is used.

The covering is usually supported by members called *jack-rafters,* which in turn are supported by other members called *purlins.* The purlins run longitudinally with the building and are connected to the trusses, generally at panel points. The jack-rafters are usually made of wood, while the purlins may be either of wood, or of steel. If the distance between the trusses is great, the purlins may be trussed.

A system of *sway bracing* is generally used to give rigidity to the structure. This bracing may be in the plane of the upper chord, in the plane of the lower chord, or in both planes. Sometimes the sway bracing is made continuous throughout the entire length of the building, and at other times the trusses are connected in pairs, depending upon the rigidity required.

The trusses may be supported upon masonry walls, upon masonry piers, or upon columns. When the trusses rest upon masonry and the span is short, the expansion and contraction of the trusses are provided for by a planed base plate; but for long spans, rollers are usually employed.

90. **Dead Load.** The dead load includes the weights of the following items: (1) roof covering; (2) purlins, rafters, and bracing; (3) roof trusses; and (4) permanent loads supported by the trusses.

(1) *Roof Covering.* The weight of the roof covering varies greatly, depending upon the materials employed; but it may be closely estimated if the materials used in its construction are known. The approximate weights of some of the common roof coverings are given in the following table:

Slate, without sheathing.....	8 to 10 lbs. per sq. ft.
Tiles, flat.....	15 to 20 lbs. per sq. ft.
Tiles, corrugated.....	8 to 10 lbs. per sq. ft.
Tar and gravel, without sheathing..	8 to 10 lbs. per sq. ft.
Tin, without sheathing.....	1 to 1.5 lbs. per sq. ft.
Wooden shingles, without sheathing	2 to 3 lbs. per sq. ft.
Corrugated steel, without sheathing	1 to 3 lbs. per sq. ft.
Sheathing, 1 inch thick.....	3 to 4 lbs. per sq. ft.

(2) *Purlins, Rafters, and Bracing. Wooden purlins* will weigh from 1.5 to 3 pounds, and *steel purlins* from 1.5 to 4 pounds per square foot of roof surface.

Rafters will weigh from 1.5 to 3 pounds per square foot of roof surface.

The weight of the *bracing* is a variable quantity, depending upon the rigidity required. If bracing is used in the planes of

both the upper and lower chords, its weight will be from 0.5 to 1 pound per square foot of roof surface.

(3) *Roof Trusses.* The roof trusses may be either of wood or of steel. Steel trusses are now commonly used for spans of considerable length, although wooden trusses are still used for short spans; but the rapidly increasing cost of wood, and the difficulty of framing the joints so as to develop the entire strength of the members, has led to a general use of steel for trusses.

The weights of wooden trusses are given by the following formula, taken from Merriman's "Roofs and Bridges," which is based upon data given in Ricker's "Construction of Trussed Roofs."

$$W = \frac{1}{2} AL \left(1 + \frac{1}{10} L\right), \qquad (1)$$

where W = the total weight of one truss in pounds,
A = the distance between adjacent trusses in feet,
L = the span of the truss in feet.

The weights of steel roof trusses vary with the span, the pitch, the distance between trusses, and the capacity or load carried by the trusses. The following formula is given in Ketchum's "Steel Mill Buildings," and is based upon the actual shipping weights of trusses:

$$W = \frac{P}{45} AL \left(1 + \frac{L}{5\sqrt{A}}\right), \qquad (2)$$

where W = weight of the steel roof truss in pounds,
P = capacity of the truss in pounds per square foot of horizontal projection of the roof,
A = distance, center to center of trusses, in feet,
L = span of truss in feet.

These trusses were designed for a tensile stress of 15 000 lbs. per sq. in., and a compressive stress of $15\,000 - 55\frac{l}{r}$ lbs. per sq. in.; where l = length, and r = radius of gyration of the member, both in inches. The minimum sized angle used was

$2'' \times 2'' \times \frac{1}{4}''$, and the minimum thickness of plate was one-fourth inch.

Dividing equation (2) by AL, we have

$$w = \frac{P}{45}\left(1 + \frac{L}{5\sqrt{A}}\right), \qquad (3)$$

where w is the weight in lbs. per sq. ft. of the horizontal projection of the roof.

Short span simple roof trusses may weigh somewhat less than the values given by the above formulæ, especially if the minimum thickness and minimum size of angles are used, but such trusses are too light to give good service.

(4) *Permanent Loads Supported by the Trusses.* It is impossible to give figures which will be of much value for the weights of permanent loads supported by the trusses. If the roof truss supports a plastered ceiling, the weight of the ceiling may be taken at about 10 lbs. per sq. ft. If other loads are supported by the truss, they should be carefully considered in estimating the weight of the truss.

Art. 2. Snow Load.

91. Snow Load. The amount of the snow load to be considered varies greatly for different localities. The weight of the snow which must be taken into account is a function of both the latitude and the humidity, and those factors should be carefully considered in determining the weight to be used for any particular locality. The maximum wind load and the maximum snow load will probably never occur at the same time; and if the maximum wind is taken, then a smaller snow load should be used. For a latitude of about 35 to 40 degrees, the writer recommends a minimum snow load of 10 pounds, and a maximum snow load of 20 pounds per square foot of the horizontal projection of the roof; the former to be used in connection with a maximum wind load, and the latter when the wind load is not considered.

ART. 3. WIND LOAD.

92. Wind Pressure. The pressure of the wind against a roof surface depends upon the velocity and direction of the wind, and upon the inclination of the roof. The wind is assumed to move horizontally, and the pressure against a flat surface, normal to the direction of the wind, may be found by the formula

$$P = 0.004\,V^2, \tag{4}$$

where P is the pressure in pounds per square foot on a flat normal surface, and V is the velocity of the wind in miles per hour.

The pressure on a flat vertical surface is usually taken at about 30 pounds per square foot, which is equivalent to a wind velocity of 87 miles per hour. This assumed pressure is sufficient for all except the most exposed positions.

93. Wind Pressure on Inclined Surfaces. The normal wind pressure upon an inclined surface varies with the inclination of the roof; and several formulæ have been derived for finding the normal component. The best known of these are Duchemin's, Hutton's, and The Straight Line formulæ.

Duchemin's formula is

$$P_n = P\frac{2\sin A}{1+\sin^2 A}, \tag{5}$$

where P_n is the normal compohent of the wind pressure,
P is the pressure per sq. ft. on a vertical surface,
A is the angle which the roof surface makes with the horizontal, in degrees.

Hutton's formula is

$$P_n = P \sin A,^{1.842 \cos A - 1} \tag{6}$$

where P_n, P, and A are the same as in Duchemin's formula.

The Straight Line formula is

$$P_n = \frac{P}{45} A, \tag{7}$$

where P_n, P, and A are the same as in Duchemin's formula.

Duchemin's formula is based upon carefully conducted experiments, gives larger values, and is considered more reliable than Hutton's. The Straight Line formula is preferred by many on account of its simplicity, and gives results which agree quite closely with experiments.

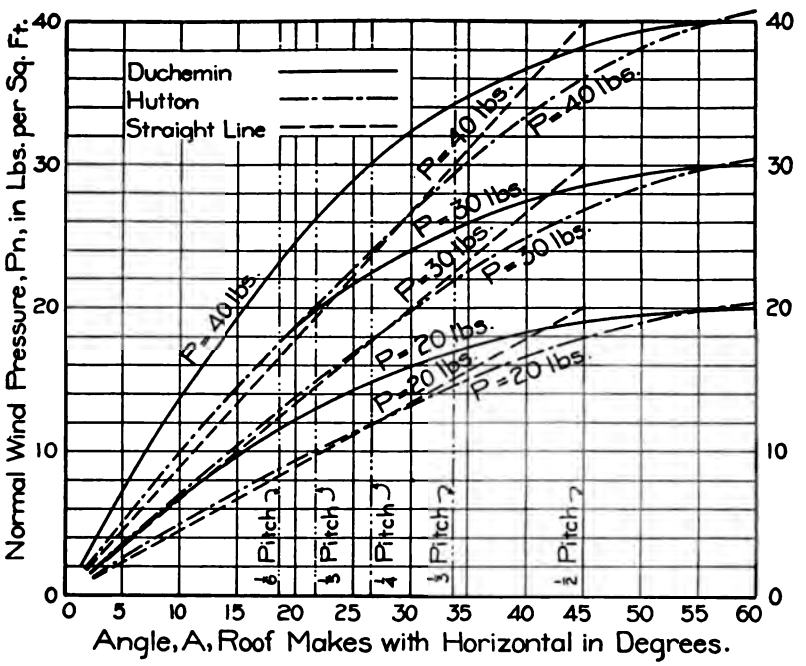


FIG. 47. NORMAL WIND LOAD ON ROOF BY DIFFERENT FORMULAE.

Fig. 47 gives values for the normal wind pressure P_n in pounds per square foot, in terms of the pressure on a vertical surface and of the angle which the roof surface makes with the horizontal. The use of this diagram will greatly lessen the work required to find the normal wind pressure when the pressure on a vertical surface and the inclination of the roof are known.

CHAPTER IX.

REACTIONS.

The reactions are the forces which if applied at the center of the bearings of a truss would hold in equilibrium the weight of the truss and the loads supported by it. The reactions are numerically equal to the pressures exerted by the truss against its supports. As the method of finding the reactions for vertical loads differs somewhat from that for inclined loads, the determination of these reactions will be considered in separate articles, the first article treating of the reactions for dead and snow loads, and the second of the reactions for wind loads.

ART. 1. REACTIONS FOR DEAD AND SNOW LOADS.

Before the reactions can be determined, it is necessary to find the loads that are supported by the truss. The purlins are usually placed at the panel points of the upper chord, and the loads are considered to act at these panel points. The method of finding the joint loads and the dead load reactions will now be shown by the solution of the following problem.

94. Problem. It is required to find the dead load reactions for the truss shown in Fig. 48. The truss has a span of 48 feet, a rise of 12 feet, and the distance apart, center to center of trusses, is 14 feet. The roof is of slate, laid on sheathing, which is supported by wooden rafters. The purlins and the truss are of steel.

The solution will be divided into two parts; the determination of (a) the joint loads, and (b) the reactions.

(a) *Joint Loads.* Referring to the table given in § 90 (1), it is seen that the weight of the slate covering may be taken at 10 pounds, and the weight of the sheathing at 3 pounds per square foot of roof surface. From § 90 (2), it is seen that the weight of the purlins and bracing may be taken at 3 pounds, and the weight of the rafters at 2.5 pounds per square foot of roof surface. This gives a total weight of 18.5 pounds per square foot of roof surface, exclusive of the weight of the truss itself.

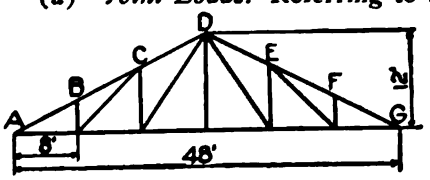


FIG. 48.

The length of one-half of the upper chord $= \sqrt{24^2 + 12^2}$ feet $= 26.83$ feet. This length is divided into three equal parts by the web bracing, making the length of each panel of the upper chord equal to $26.83 \div 3 = 8.94$ feet. Since the joint loads are taken at panel points, and the trusses are spaced 14 feet apart, the joint load supported by the truss, exclusive of the weight of the truss, $= 8.94 \times 14 \times 18.5 = 2315$ pounds.

The weight of the roof truss per square foot of horizontal projection for a capacity P of 40 pounds (which is about right for the given roof truss) is given by formula (3), § 90, and is about 3 pounds. This is equivalent to a joint load of $8 \times 14 \times 3 = 336$ pounds. The total joint load is therefore equal to $2315 + 336 = 2651$ pounds, or say 2650 pounds. Referring to Fig. 48, it is seen that the loads acting at the joints B, C, D, E, and F are full panel loads; while those acting at A and G support but half the area, and are half panel loads. The loads acting at B, C, D, E, and F are each equal to 2650 pounds; while those acting at A and G are each equal to 1325 pounds.

(b) *Reactions.* The problem now is to find the reactions at the ends of the truss loaded as shown in Fig. 49, or in other words, it is required to find the two forces acting at the ends of the truss, which will hold in equilibrium the given loads.

Construct the force polygon ABCDEFGH for the given loads on the truss. In this case, the force polygon is a straight line, and is called the *load line.* Assume any pole O, construct

the funicular polygon, and draw the closing string of the polygon. Then the dividing ray, drawn through the pole O parallel to this closing string, will determine the magnitudes of the two reactions, HM being the magnitude of the right reaction R_2, and MA that of the left reaction R_1 (§ 43). The lines of

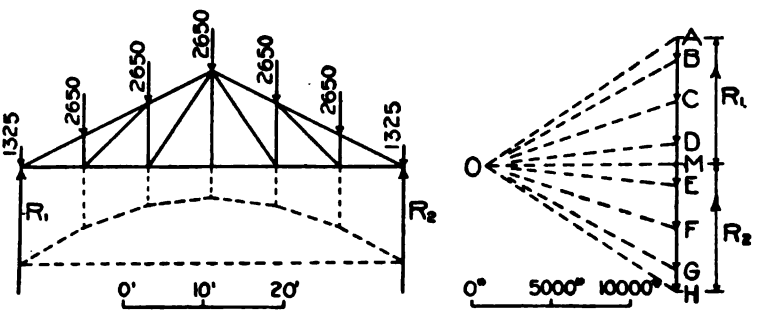


FIG. 49. DEAD LOAD REACTIONS.

action of these reactions will be parallel to the load line—which is vertical. By scaling the lines HM and MA, it is found that these reactions are each equal to 7950 pounds.

The reactions might have been found, algebraically, by taking moments about the supports. Thus, to find the value of the left reaction, take moments about the right support. Then the equation of moments is $+ R \times 48 - 1325 \times 48 - 2650 \times 40 - 2650 \times 32 - 2650 \times 24 - 2650 \times 16 - 2650 \times 8 = 0$. Solving this equation gives $R_1 = 7950$. Since the two reactions must hold the given loads in equilibrium, the right reaction is therefore equal to the total load minus the left reaction $= 15\,900 - 7950 = 7950$ pounds.

If the loads are symmetrical with respect to the center line of the truss, it is evident that the reactions are equal, and that each is equal to $15\,900 \div 2 = 7950$ pounds.

95. Snow Load Reactions. The reactions and the stresses for snow loads are usually considered separately from those due to dead loads. The same methods may be used for finding the snow load reactions as have been explained for finding the dead load reactions, and need no further explanation.

96. Effective Reactions. It is seen from Fig. 49 that the half joint loads at the ends are transferred directly to the supports without causing any stress in the truss. These half loads act through the same lines as the total reactions, and subtract from these reactions. The resulting forces are called the *effective* reactions. If the effective reactions are used, the half loads at the ends of the truss are neglected in computing the stresses in the truss. The effective reactions for the truss loaded as shown in Fig. 49 are each equal to $7950 - 1325 = 6625$ pounds.

ART. 2. REACTIONS FOR WIND LOADS.

The magnitudes and lines of action of the wind load reactions for any given truss depend upon the condition of the ends of the truss. If the span is small, or if the truss is made of wood, the ends are usually fixed to the supports by anchor bolts. For large spans, the changes of temperature and the deflections due to the loads cause the truss to expand or contract a considerable amount. If the ends are fixed, the temperature changes and the loads may produce large stresses in the truss. The usual method of providing for the changes in length is to place one end of the truss upon a planed base plate or upon rollers.

97. Wind Load Reactions. The wind load reactions will be determined for each of the following assumptions: (1) that both ends of the truss are fixed; and (2) that one end of the truss is supported upon rollers. In the example that will be given, a triangular form of truss will be used, although the methods employed are applicable to any type of simple truss.

98. (1) Truss Fixed at Both Supports. The problem of finding the lines of action of the wind load reactions for a truss fixed at both ends is indeterminate; but assumptions may be made which will give approximately correct results. The vertical components of the reactions are independent of any assumptions; but the horizontal components depend upon the

condition of the ends of the truss. If the roof is comparatively flat, i. e., if the resultant of the normal components of the wind loads is nearly vertical, the reactions may be taken parallel to this resultant; as each reaction will then be approximately vertical. The above assumption gives erroneous results when the roof makes a large angle with the horizontal. The assumption that the horizontal components of the reactions are equal more nearly approximates actual conditions; and this is particularly true if the ends of the truss are fastened to the supports in the same manner and the supports are equally elastic. The method of finding the reactions of a truss with fixed ends will now be shown by a problem, assuming (a) that the reactions are parallel, and (b) that the horizontal components of the reactions are equal.

(a) *Reactions Parallel.* It is required to find the wind load reactions for the truss shown in Fig. 48. This truss has a span of 48 feet and a rise of 12 feet, the distance between trusses being 14 feet. The wind pressure on a vertical surface will be taken at 30 pounds per square foot, and Duchemin's formula will be used for finding its normal component. Referring to Fig. 47, it is seen that, for $P = 30$ pounds and for a pitch of one-fourth, the component normal to the roof is about 22 pounds. The length of a panel of the upper chord is 8.94 feet (see § 94, a). The panel load is therefore equal to $8.94 \times 14 \times 22 = 2753$, or say 2750 pounds. The half panel loads at the ends of the truss and at the apex are each equal to 1375 pounds.

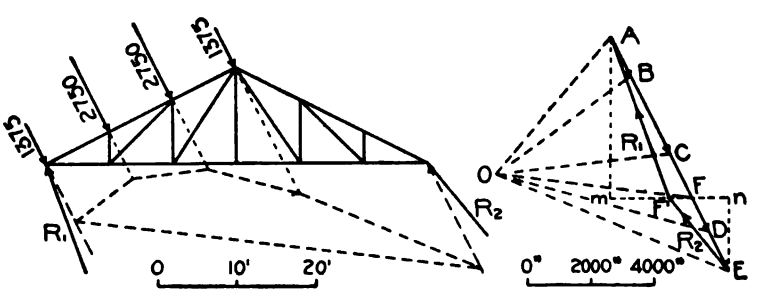


FIG. 50. WIND LOAD REACTIONS—ENDS FIXED.

The truss with its loads is shown in Fig. 50. To determine the reactions, construct the force polygon or load line ABCDE, assume a pole, and draw the funicular polygon. The dividing ray OF, drawn through the pole O parallel to the closing string of the funicular polygon, will give the magnitudes of the two reactions, FA being the magnitude of the left reaction, and EF that of the right. The lines of action of these reactions are given by drawing, through the supports, lines parallel to the load line AE. By scaling the lines FA and EF, it is seen that the left reaction is equal to 5670 pounds, and the right reaction to 2580 pounds. The values given for the reactions were actually taken from a diagram four times as large as that shown in Fig. 50; as the scale used here is too small to give accurate results.

(b) *Horizontal Components of Reactions Equal.* The reactions will be found for the same truss and loads as in Fig. 50, assuming that the horizontal components of the reactions are equal. First find the reactions, as in § 98 (a), assuming that their lines of action are parallel. The vertical components of these reactions are correct; since they are independent of the ends of the truss. Now draw a horizontal line through the point F (Fig. 50), and make mn equal to the horizontal component of all the loads on the truss. This may be done by drawing vertical lines through A and E, intersecting the horizontal line at the points m and n, respectively. Bisect mn, making $mF' = F'n$, and draw the lines F'A and EF', which represent the magnitudes of the required reactions. The lines of action of these reactions are given by drawing lines through the left and also through the right points of support of the truss parallel respectively to F'A and EF'. By scaling the lines F'A and EF', it is seen that the left reaction is equal to 5400 pounds, and the right reaction to 2960 pounds.

99. (2) One End of Truss Supported on Rollers. If one end of the truss is supported on rollers, then the roller end can take no horizontal component of the wind loads (neglecting the friction of the rollers); for there would be a continued movement of the rollers if a horizontal force was applied at

this end of the truss. The reaction at the roller end is therefore vertical, and hence all the horizontal component of the wind loads must be taken up at the fixed end of the truss. Since the wind may come from either of two opposite directions, the rollers may be under the leeward side, or under the windward side of the truss. The method for finding the reactions will now be shown by a problem, considering (a) that the rollers are under the leeward end of the truss, and (b) that the rollers are under the windward end.

(a) *Rollers Under Leeward End of Truss.* It is required to find the wind load reactions for the truss shown in Fig. 48, the leeward end being on rollers. The wind loads are the same as those shown in Fig. 50. In this problem, the line of action of the reaction at the roller end is vertical; while the line of action of the reaction at the fixed end is unknown, its point of application, only, being known. To determine the reactions for this case, apply the method explained in § 44.

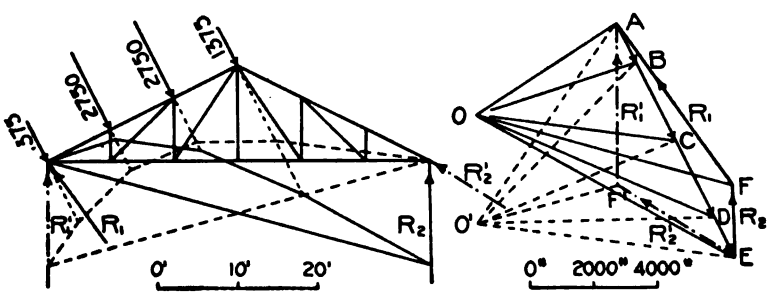


FIG. 51. WIND LOAD REACTIONS—ONE END ON ROLLERS.

Draw the load line ABCDE (Fig. 51) for the given loads, assume any pole O, and draw the funicular polygon (shown by full lines), starting the polygon at the only known point on the left reaction—its point of application at the left end of the truss. From the pole O, draw the dividing ray OF parallel to the closing string of the funicular polygon to meet a vertical line drawn through E parallel to the known direction of the right reaction. Connect the points A and F; then EF represents the magnitude of the right reaction, and FA that of the

left reaction. The lines of action of these reactions, which are represented by R_1 and R_2, are given by drawing lines through the ends of the truss parallel respectively to EF and FA (§ 44). By scaling the lines EF and FA, it is seen that the right reaction is equal to 2310 pounds, and the left reaction to 6270 pounds.

The reactions might also have been found by a slightly different method, as follows: Find the reactions for the truss fixed at both ends, assuming that their lines of action are parallel (see § 98 (a), and Fig. 50). EF and FA represent the magnitudes of these reactions, their vertical components being independent of the condition of the ends of the truss. Now since the right reaction can have no horizontal component, draw the line En through E to meet the horizontal line through F; then En represents the magnitude of this reaction. Now all the horizontal component of the wind loads must be taken up at the left end of the truss, this component being represented by the line mn. The vertical component of the left reaction is represented by the line mA; therefore the resultant of these two components, or the line nA (not drawn), represents the magnitude of the left reaction.

(b) *Rollers Under Windward End of Truss.* If the rollers are under the windward, instead of the leeward end of the truss, then the left reaction is vertical and the right is inclined, the only known point on the right reaction being its point of application at the right support. For this case, the funicular polygon must start at the right support, the remainder of the solution being similar to that explained in § 99 (a). The construction for this solution is shown by the dotted lines in Fig. 51, EF′ and F′A representing the magnitudes of the two reactions, their lines of action R_2' and R_1' passing through the right point of support and the left point, respectively. By scaling the lines EF′ and F′A, it is seen that the right reaction is equal to 4350 pounds, and the left reaction to 5070 pounds.

The reactions might also have been found by a method similar to that explained in the last paragraph of § 99 (a), mA and Em (not drawn) representing these reactions (see Fig. 50).

100. Fig. 52 shows to scale the wind loads and the wind load reactions for different assumptions as to the condition of the ends of the truss. It is seen from the figure that the verti-

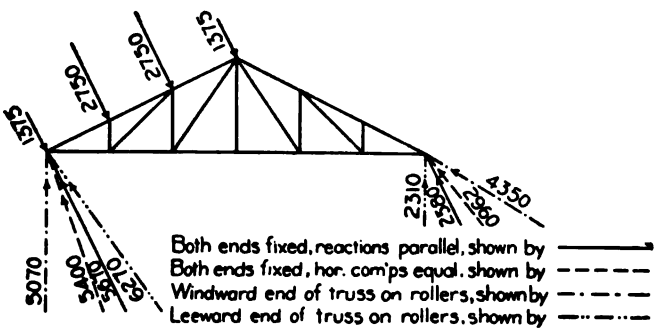


FIG. 52. WIND LOAD REACTIONS FOR DIFFERENT ASSUMPTIONS AS TO CONDITION OF ENDS OF TRUSS.

cal components of the reactions are independent of, and that the horizontal components are dependent upon, the assumptions as to the condition of the ends of the truss. If the roof had made a greater angle with the horizontal than that shown in the figure, then the differences in the reactions, due to the assumptions as to the condition of the ends of the truss, would have been more apparent.

101. In determining the reactions for a truss, great care should be exercised in drawing the truss, and in locating the panel points accurately. The truss and diagrams should be drawn to a large scale to insure good results. Care should be taken in laying out the load line, and the pole should be chosen in such a position that acute intersections are avoided.

CHAPTER X.

STRESSES IN ROOF TRUSSES.

The determination of the external forces acting upon a truss has been taken up in Chapter VIII and Chapter IX. The dead and wind loads acting upon the roof are transferred to the supports through the members of the truss, and this chapter will treat of the determination of the stresses in the members due to these external forces.

ART. 1. DEFINITIONS AND GENERAL METHODS FOR DETERMINING STRESSES.

102. Definitions. The external loads tend to distort the truss, i.e., to shorten some of the members, and to lengthen others; and since the materials used in the construction of the truss are not entirely inelastic, the members are actually distorted. The deformation in any member caused by the loads is called *strain,* and the internal force which is developed in the member and which tends to resist the deformation, or strain, is called *stress.* There are three kinds of stress which may be developed by the external forces, viz.: (a) tension, (b) compression, and (c) shear.

(a) *Tension.* A member is subjected to a tensile stress, or is in tension, if there is a tendency for the particles of the member to be pulled apart in a direction normal to the surface of separation. In this case, the external forces causing the tension act in a direction from the center toward the ends; and therefore the internal forces, or stresses, act from the ends toward the center.

(b) *Compression.* A member is subjected to a compressive

stress, or is in compression, if there is a tendency for the particles of the member to move toward each other. If the member is in compression, the external forces causing the compression act in a direction from the ends toward the center; and therefore the internal forces, or stresses, act from the center toward the ends.

(c) *Shear.* A member is subjected to a shearing stress, or is in shear, if there is a tendency for the particles of the member to slide past each other.

Since the loads are generally applied at the joints, the members are usually subjected to a longitudinal stress only, and are either in tension or in compression; although a shearing stress may be developed at the connections of the members.

103. General Methods for Determining Stresses. It has been shown in the preceding chapter that the reactions of a truss may be determined, graphically, by means of the force and the funicular polygons. They may also be determined, algebraically, by the three fundamental equations of equilibrium:

$$\Sigma \textit{ horizontal components of forces} = 0, \qquad (1)$$

$$\Sigma \textit{ vertical components of forces} = 0, \qquad (2)$$

$$\Sigma \textit{ moment of forces about any point} = 0. \qquad (3)$$

Having found the reactions (see Chapter IX), the stresses may be determined either by equations (1) and (2) or by equation (3). The first two equations involve the resolution of forces, and they may be solved either algebraically or graphically. The third equation involves the moments of forces, and it may also be solved either algebraically or graphically. There are, therefore, four methods for determining the stresses in a truss.

Moment of Forces:

Algebraic Method, (a)

Graphic Method. (b)

Resolution of Forces:

Algebraic Method, (c)

Graphic Method. (d)

It is possible to solve the stresses in the members of any

simple truss by using any one of the above four methods; but all are not equally well suited to any particular case. It is usually the case that a certain one of the four methods is better suited to a particular problem than are any of the other three; and in solving stresses, the problem should be studied in order that the simplest solution may be found.

104. Notation. The notation that will be used to designate the members of a truss is known as Bow's notation, and is shown in Fig. 53. Referring to this figure, it is seen that the upper chord members, beginning at the left end of the truss, may be designated by X-1, X-2, X-4, X-4′, X-2′, and X-1′; the lower chord members by Y-1, Y-3, Y-5, Y-5′, Y-3′, and Y-1′; and the web members by 1-2, 2-3, 3-4, 4-5, 5-5′, 5′-4′, 4′-3′, 3′-2′, and 2′-1′. The numerical value of the stress will generally be written directly on the member.

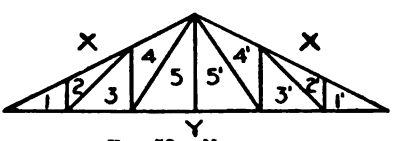


FIG. 53. NOTATION.

A *tensile* stress will be denoted by prefixing a *plus* (+) sign, and a *compressive* stress by prefixing a *minus* (—) sign before the number representing the stress. This use of these signs is not universal, but the above designation was adopted as it is more often employed.

105. It is the object of this work to deal with graphic—rather than with algebraic methods; and since the method of graphic moments is not well suited to the determination of the stresses in a truss, except to explain other methods, the first three methods will be treated only briefly, while this text will deal chiefly with the determination of stresses by the fourth method—graphic resolution. These four methods will now be taken up in the order shown in § 103.

ART. 2. STRESSES BY ALGEBRAIC MOMENTS.

The method of algebraic moments furnishes a convenient means of finding stresses, especially when the upper and lower chords of the truss are not parallel.

106. **Method of Computing Stresses by Algebraic Moments.** To obtain the stress in any particular member, cut the truss by a section; and replace the stresses in the members cut, by external forces. These forces are equal to the stresses in the members cut, but act in an opposite direction. The section should be so taken that the member whose stress is required is cut by it; and if possible, so that the other members cut by the section (excepting the one whose stress is required) pass through a common point, which point is taken as the center of moments. To determine the sign of the moment and of the resulting stress, assume the unknown external force to act away from the cut section, i. e., to cause tension. Write the equation of moments, considering the external forces which replace the members cut and those on one side of the section only, equate to zero (see § 103, equation 3), and solve for the unknown stress. The sign of the result will then indicate the kind of stress. If it is positive, the assumed direction of the unknown force is correct, and the stress is tension. If the result is negative, the assumed direction is incorrect, and the stress is compression, i. e., the

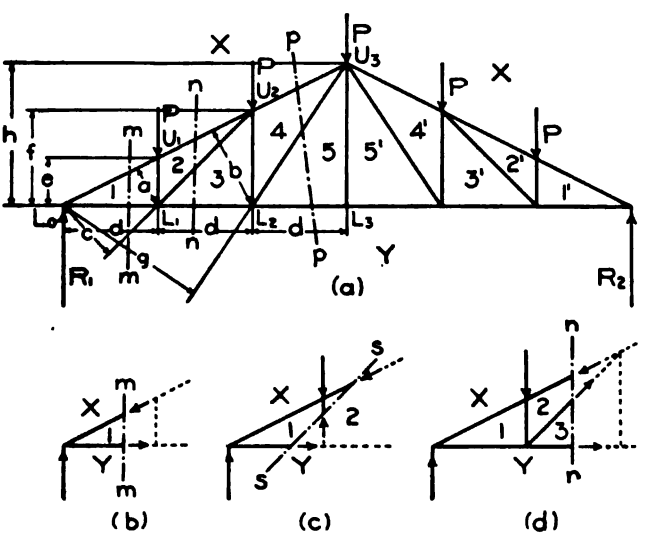


FIG. 54. STRESSES BY ALGEBRAIC MOMENTS.

equation of moments is not equal to zero unless the assumed direction of the external force is reversed.

The application of the method of algebraic moments to the determination of the stresses in a truss will now be shown.

107. Problem. It is required to find the stresses, due to the dead load, in the members of the truss shown in Fig. 54.

To determine the stress in the member X-1, cut the members X-1 and Y-1 by the section m-m (see Fig. 54, a), replace these members by external forces, assume that the unknown external force replacing the required stress X-1 acts away from the cut section, and take moments about the joint L_1 (Fig. 54, a). Then

$$+ R_1 \times d + X\text{-}1 \times a = 0,$$

or
$$X\text{-}1 = -\frac{R_1 \times d}{a} \qquad \text{(compression)}. \qquad (1)$$

Since the sign of the result is negative, the equation shows that the external force acts in an opposite direction to that assumed, i. e., that it acts in the direction shown in Fig. 54, b, and causes compression.

To find the stress in the member Y–1, use the same section and take moments about U_1, assuming that the external force replacing the stress in Y-1 acts away from the cut section. Then

$$+ R_1 \times d - Y\text{-}1 \times e = 0,$$

or
$$Y\text{-}1 = +\frac{R_1 \times d}{e} \qquad \text{(tension)}. \qquad (2)$$

Since the result is positive, the equation shows that the external force acts in the direction assumed, i. e., that it acts in the direction shown in Fig. 54, b.

To find the stress in the member 1-2, cut the members X-2, 1-2, and Y-1 by the section s-s (see Fig. 54, c), and take moments about L_0, assuming that the external force which replaces the stress in 1-2 acts away from the cut section. Then

$$+ P \times d + 1\text{-}2 \times d = 0,$$

or
$$1\text{-}2 = -\frac{P \times d}{d} = -P \qquad \text{(compression)}. \qquad (3)$$

To find the stress in X-2, cut the members X-2, 2-3, and Y-3 by the section n–n (see Fig. 54, a and Fig. 54, d), and take the moments about L_1, the intersection of 2-3 and Y-3. Then

$$+ R_1 \times d + X\text{-}2 \times a = 0,$$

or
$$X\text{-}2 = -\frac{R_1 \times d}{a}. \qquad \text{(compression).} \qquad (4)$$

To determine the stress in 2-3, cut the members by the section n-n, assume the external force replacing the stress in 2-3 to act away from the cut section, and take the moments about L_0. Then

$$+ P \times d - 2\text{-}3 \times c = 0,$$

or
$$2\text{-}3 = + \frac{P \times d}{c} \qquad \text{(tension).} \qquad (5)$$

This equation shows that the assumed direction of the external force replacing 2-3 was correct, i. e., that this force acts away from the cut section, as shown in Fig. 54, d.

For the stress in Y-3, cut the members by the section n-n, and take moments about U_2. Then

$$+ R_1 \times 2d - P \times d - Y\text{-}3 \times f = 0,$$

or
$$Y\text{-}3 = \frac{+ R_1 \times 2d - P \times d}{f} \qquad \text{(tension).} \qquad (6)$$

For the stress in 3–4, take a section (not shown in the figure) cutting X–4, 3–4, and Y–3, and take moments about L_0. Then

$$+ P \times d + P \times 2d + 3\text{-}4 \times 2d = 0,$$

or
$$3\text{-}4 = \frac{- P \times d - P \times 2d}{2d} = -\frac{3}{2} P \quad \text{(compression).} \quad (7)$$

For the stress in X–4, take the section p–p, and the center of moments at L_2. Then

$$+ R_1 \times 2d - P \times d + X\text{-}4 \times b = 0,$$

or
$$X\text{-}4 = \frac{- R_1 \times 2d + P \times d}{b} \quad \text{(compression).} \quad (8)$$

For the stress in 4–5, take the section p–p, and the center of moment at L_0. Then

$$+ P \times d + P \times 2d - 4\text{-}5 \times g = 0,$$

$$\text{or } 4\text{-}5 = \frac{+ P \times d + P \times 2d}{g} = \frac{+ 3\,Pd}{g} \quad \text{(tension).} \qquad (9)$$

For the stress in Y–5, take the section p–p, and the center of moments at U_3. Then

$$+ R_1 \times 3d - P \times 2d - P \times d - Y\text{-}5 \times h = 0, \quad \text{or } Y\text{-}5 =$$

$$\frac{+ R_1 \times 3d - P \times 2d - P \times d}{h} = \frac{+ 3d\,(R_1 - P)}{h} \text{(tens'n).} \quad (10)$$

The stress in the member $5\text{-}5' = 0$, as may be shown by taking a circular section cutting Y–5, 5–5′, and Y–5′, and the center of moments at L_2.

Since the truss and the loads are symmetrical about the center line, it is evident that it is necessary to find the stresses for only one-half of the truss.

In some trusses, it is impossible to take a section so that all of the members except the one whose stress is required will pass through the center of the moments. If another member cut by the section does not pass through the center of the moments, it is necessary to first solve for the stress in this member, and then replace this stress by an external force. If the stress in this member is tension, the external force is taken as acting away from the cut section; and if it is compression, toward the section.

If the moment arms for the forces are computed algebraically, considerable work is required; and these arms may generally be most easily found by drawing the truss to a large scale, and scaling the moment arms from the diagram.

One of the most important advantages of the method of moments is that the stress in any particular member may be found independently of that in any other member.

108. By noting the results obtained for the stresses in the preceding problem, several conclusions may be drawn as to the nature of the stresses in the different members of the truss. Referring to equations (1) and (4), § 107, it is seen that the stress in X–1 is equal to that in X–2; and referring to equation (3), it is seen that 1–2 is an auxiliary member whose func-

tion is to transfer the load P to the joint L_1 (by compression). Further, if there is no load at U_1, the stress in 1–2 is zero. Since the load is transferred to L_1 by the member 1–2, the stress in X–1 must be equal to that in X–2; as the load has no horizontal component and all of its vertical component is taken up by the member 1–2. There can be no stress in the member 5–5′ unless there is a load at L_3; and if there is a load at that point, the stress in 5–5′ is tension, and is equal to the load. The member 5–5′ is usually put in, its functions being merely to support the lower chord and prevent deflection.

ART. 3. STRESSES BY GRAPHIC MOMENTS.

The stresses in the members of a truss may be found by graphic moments, although this method is not generally the simplest that may be used.

109. Stresses by Graphic Moments. This method is somewhat similar to that of algebraic moments, as explained in Art. 2; except that in this method, the moment of the external forces is found graphically instead of algebraically. Since the structure is in equilibrium, the moment of the known external forces must be equal to the moment of the forces which replace the stresses in the members cut by the section. If all the members cut, except one, pass through the center of moments; then the moment of the external force replacing the stress in this member must be equal to the moment of the known external forces. To determine the stress in any particular member, cut the member by a section, and take moments about the intersection of the other members cut. To find the moment of the known external forces, construct the force and the funicular polygons for these forces. Then the moment is equal to the pole distance multiplied by the intercept. This intercept is measured on a line drawn through the center of moments parallel to the resultant of the external forces on one side of the section; and is the distance on this line cut off by the strings drawn parallel to the rays meeting

on the extremities of the line representing the magnitude of the resultant in the force polygon. If all the members cut by the section, except the one whose stress is required, pass through the center of moments, the algebraic sum of the moments of the unknown force replacing the stress in this member and of the known external forces on one side of the section must be equal to zero.

The solution of the following problem shows the application of the above method to the determination of the stresses in the members of a truss.

110. Problem. It is required to find the stresses in the members of the truss loaded as shown in Fig. 55.

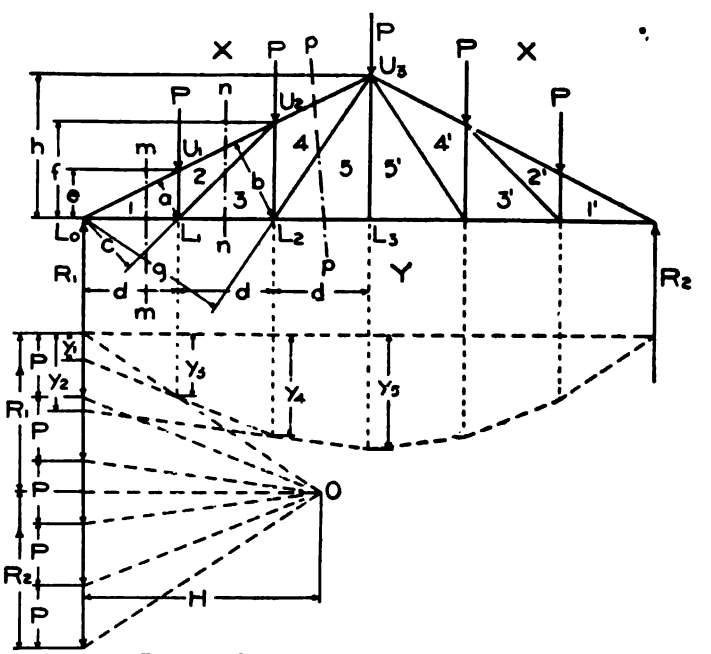


FIG. 55. STRESSES BY GRAPHIC MOMENTS.

To determine the stress in the member X–1, cut the member by the section m–m, and take the center of moments at L_1. Then the moment of the left reaction is equal to $+ H \times y_2$,

and the equation of moments (assuming the unknown external force to act away from the cut section) is

$$+ H \times y_3 + X\text{-}1 \times a = 0,$$

or
$$X\text{-}1 = - \frac{H \times y_3}{a}. \qquad (1)$$

A plus sign placed before the stress indicates tension, and a minus sign indicates compression.

To find the stress in Y-1, cut the member by the section m-m, and take the center of moments at U_1. Then

$$+ H \times y_3 - Y\text{-}1 \times e = 0,$$

or
$$Y\text{-}1 = + \frac{H \times y_3}{e}. \qquad (2)$$

To find the stress in 1-2, take a section (not shown in the figure) cutting X-2, 1-2, and Y-1, and take the center of moments at L_0. Then

$$+ H \times y_1 + 1\text{-}2 \times d = 0,$$

or
$$1\text{-}2 = - \frac{H \times y_1}{d}. \qquad (3)$$

To find the stress in X-2, take the section n-n, and the center of moments at L_1. Then

$$+ H \times y_3 + X\text{-}2 \times a = 0,$$

or
$$X\text{-}2 = - \frac{H \times y_3}{a}. \qquad (4)$$

To find the stress in 2-3, take the section n-n, and the center of moments at L_0. Then

$$+ H \times y_1 - 2\text{-}3 \times c = 0,$$

or
$$2\text{-}3 = + \frac{H \times y_1}{c}. \qquad (5)$$

To find the stress in Y-3, take the section n-n, and the center of moments at U_2. Then

$$+ H \times y_4 - Y\text{-}3 \times f = 0,$$

or
$$Y\text{-}3 = + \frac{H \times y_4}{f}. \qquad (6)$$

To find the stress in 3–4, take a section (not shown in the figure), and the center of moments at L_0. Then

$$+ H \times y_2 + 3\text{–}4 \times 2d = 0,$$

or
$$3\text{–}4 = - \frac{H \times y_2}{2d}. \qquad (7)$$

To find the stress in X–4, take the section p–p, and the center of moments at L_2. Then

$$+ H \times y_4 + X\text{–}4 \times b = 0,$$

or
$$X\text{–}4 = - \frac{H \times y_4}{b}. \qquad (8)$$

To find the stress in 4–5, take section p–p, and the center of moments at L_0. Then

$$+ H \times y_2 - 4\text{–}5 \times g = 0,$$

or
$$4\text{–}5 = + \frac{H \times y_2}{g}. \qquad (9)$$

To find the stress in Y–5, take the section p–p, and the center of moments at U_3. Then

$$+ H \times y_5 - Y\text{–}5 \times h = 0,$$

or
$$Y\text{–}5 = + \frac{H \times y_5}{h}. \qquad (10)$$

Referring to the equations of moments given in § 110, it is seen that the sign of the moment of the external forces to the left of any section is always positive. Keeping this in mind, it is possible to write the value for the stress in any member, together with its proper sign, without first writing the equation of moments. The stress in any particular member—irrespective of whether it is tension or compression—is equal to the moment of the external forces to the left of the section cutting the member divided by the arm of the force which replaces the unknown stress in the member. The sign of this stress is opposite to that of the moment of the unknown external force replacing the stress in the member, assuming this stress to act away from the cut section, i. e., if the sign of the moment of this external force is negative, the stress is

tension; and if the sign is positive, the stress is compression. It is seen that this follows directly from equations of moments.

ART. 4. STRESSES BY ALGEBRAIC RESOLUTION.

111. **Stresses by Algebraic Resolution.** The stresses in the members of a truss may be found by the application of equations (1) and (2), § 103, which are

$$\Sigma \textit{ horizontal components of forces} = 0, \quad (1)$$

$$\Sigma \textit{ vertical components of forces} = 0. \quad (2)$$

The above equations may be applied either (a) to the forces at a joint, or (b) to the forces on one side of a section, including those replacing the stresses in the members cut by the section. The method of algebraic resolution is not applicable if more than two of the forces at a joint or at a section are unknown; since there are but two fundamental equations of resolution. As it is necessary to find the stresses in some of the members before those in other members may be found, this method may be most easily explained by solving a particular problem rather than a general one.

In writing the equations, forces acting upward and to the right will be considered positive, and those acting downward and to the left, negative.

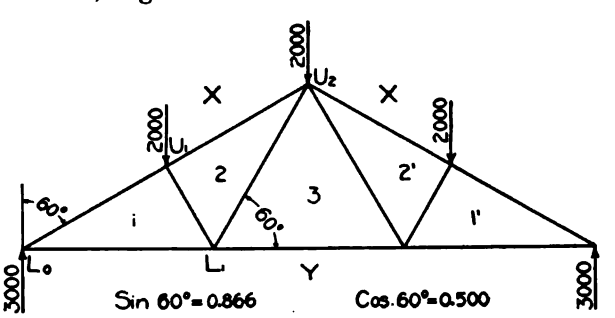


FIG. 56.

(a) *Forces at a Joint.* To find the stresses in the members meeting at a joint, apply equations (1) and (2), § 111, assum-

ing that the unknown forces replacing the stresses act away from the joint, i. e., that they cause tension. If all the forces are known except two, these may be found by solving the above equations. The signs of the results will determine the kind of stress, i. e., a plus sign indicates that the assumed direction is correct and that the stress is tension, while a minus sign indicates that the assumed direction is incorrect and that the stress is compression.

Problem. It is required to find the stresses in the members of the truss loaded as shown in Fig. 56, all loads being given in pounds.

To find the stresses in X–1 and Y–1, apply equations (1) and (2) to the forces at the joint L_0, assuming that these forces act away from the joint. Then

$$+ \text{X–1} \sin 60° + \text{Y–1} = 0,$$

and $$+ \text{X–1} \cos 60° + 3000 = 0.$$

Substituting the values of sin 60° (0.866) and cos 60° (0.500), and solving the equations for X–1 and Y–1, we have X–1 = — 6000 (compression), and Y–1 = + 5196 (tension).

To find the stresses in X–2 and 1–2, apply equations (1) and (2) to the forces at the joint U_1. Then

$$+ \text{X–1} \sin 60° + \text{X–2} \sin 60° + \text{1–2} \cos 60° = 0,$$

and $$+ \text{X–1} \cos 60° + \text{X–2} \cos 60° - \text{1–2} \sin 60° - 2000 = 0.$$

Substituting the values of sin 60° and cos 60°, also the value already found for X–1, and solving these equations for 1–2 and X–2, we have

$$\text{1–2} = - 1732, \qquad \text{and} \qquad \text{X–2} = - 5000.$$

To find the stresses in 2–3 and Y–3, apply equations (1) and (2) to the forces at the joint L_1. Then

$$- \text{Y–1} + \text{1–2} \cos 60° + \text{2–3} \cos 60° + \text{Y–3} = 0,$$

and $$- \text{1–2} \sin 60° + \text{2–3} \sin 60° = 0.$$

Substituting the values of sin 60° and cos 60°, also the values of Y–1 and 1–2, and solving these equations for 2–3 and Y–3, we have

$$\text{2–3} = + 1732, \qquad \text{and} \qquad \text{Y–3} = + 3464.$$

Fig. 56, a, shows the truss just solved, together with its loads and stresses. This figure illustrates the method of writing the stresses on the members of the truss.

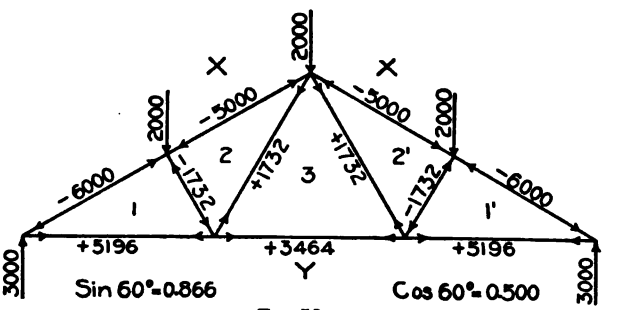


FIG. 56, a.

(b) *Forces on One Side of a Section.* The method of algebraic resolution may also be applied to the forces on one side of a section, including those replacing the stresses in the members cut by the section. Since the forces replacing the stresses in the members cut and the external forces on one side of a section must be in equilibrium, we may apply the two fundamental equations (1) and (2), § 103, and find the unknown stresses, provided not more than two of the stresses are unknown. The kind of stress may be determined by assuming the unknown external forces replacing the stresses in the members cut to act away from the section. A plus

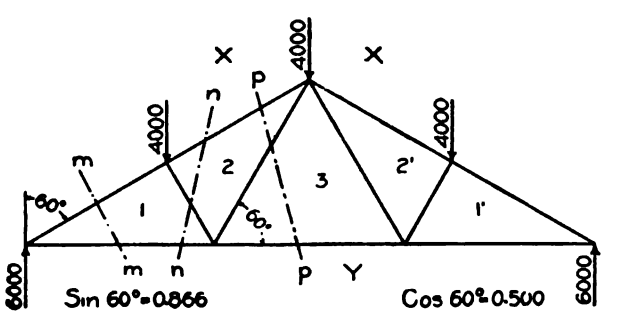


FIG. 57.

sign for the result indicates tension, and a minus sign, compression.

Problem. It is required to find the stresses in the members of the truss loaded as shown in Fig. 57, all loads being given in pounds.

To find the stresses in the members X–1 and Y–1, take the section m–m (Fig. 57), and assume that the unknown forces replacing the stresses in X–1 and Y–1 act away from the cut section. If a known stress is considered in any equation, the external force replacing it is taken as acting in its true direction. Then

$$+ \text{X–1} \sin 60° + \text{Y–1} = 0,$$

and

$$+ \text{X–1} \cos 60° + 6000 = 0.$$

Substituting the values of sin 60° (0.866) and cos 60° (0.500) in these equations, and solving for X–1 and Y–1, we have

$$\text{X–1} = -12\,000, \quad \text{and} \quad \text{Y–1} = +10\,392.$$

To find the stresses in 1–2 and X–2, take the section n–n, and assume that the unknown external forces replacing the stresses in X-2 and 1-2 act away from the section (the direction of the force replacing the stress in Y-1 has already been found to act away from the section). Then

$$+ \text{X-2} \sin 60° + \text{1-2} \cos 60° + \text{Y-1} = 0,$$

and

$$+ \text{X-2} \cos 60° - \text{1-2} \sin 60° + 6000 - 4000 = 0.$$

Substituting the values of sin 60° and cos 60°, also the value already found for Y-1, and solving these equations for 1-2 and X-2, we have

$$\text{1–2} = -3464, \text{ and } \text{X–2} = -10\,000.$$

To find the stresses in 2-3 and Y-3, take the section p-p, and assume that the unknown external forces replacing the stresses in 2-3 and Y-3 act away from the cut section (the force replacing the stress in X–2 has already been found to act towards the section). Then

$$- \text{X-2} \sin 60° + \text{2-3} \cos 60° + \text{Y-3} = 0,$$

and

$$- \text{X-2} \cos 60° + \text{2-3} \sin 60° + 6000 - 4000 = 0$$

Substituting the values for sin 60° and cos 60°, also the value already found for X-2, and solving for 2-3 and Y-3, we have

$$2\text{-}3 = +\ 3464, \text{ and } Y\text{-}3 = +\ 6928.$$

ART. 5. STRESSES BY GRAPHIC RESOLUTION.

The method of graphic resolution is usually the most convenient one for finding the stresses in roof trusses; since it is rapid and has the advantage of furnishing a check on the work. It consists of the application of the principle of the force polygon to the external forces and stresses acting at each joint of the truss. Since the external forces and stresses at each joint are in equilibrium, the force polygon must close, and the forces must act in the same direction around the polygon (see § 28). As the lines of action of all the forces are known, it follows that if a sufficient number of the forces at a joint are completely known to permit of the drawing of a closed polygon, the magnitudes and directions of the unknown stresses may be determined.

The reactions may be found by means of the force and funicular polygons, as explained in Chap. II. As soon as these reactions are determined, all the external forces acting on the truss are known, and the stresses may then be found.

112. Stresses by Graphic Resolution. *Loads on Upper Chord.* It is required to find the stresses in the truss loaded as shown in Fig. 58.

The reactions R_1 and R_2 are first found by constructing the force polygon (Fig. 58, b) for the given loads (see § 94, b). If both the truss and the loads are symmetrical about the center line, it is unnecessary to construct the funicular polygon; as each reaction is equal to one-half of the total load. The stresses may then be determined by applying the principle of the force polygon to each joint of the truss. Referring to the joint L_0 (Fig. 58, a), of the three forces acting at this joint, it is seen that the reaction R_1 is completely known,

while the internal forces or stresses X–1 and Y–1 are unknown in magnitude and direction. These forces are shown in Fig. 58, c. The force polygon for these forces is constructed by

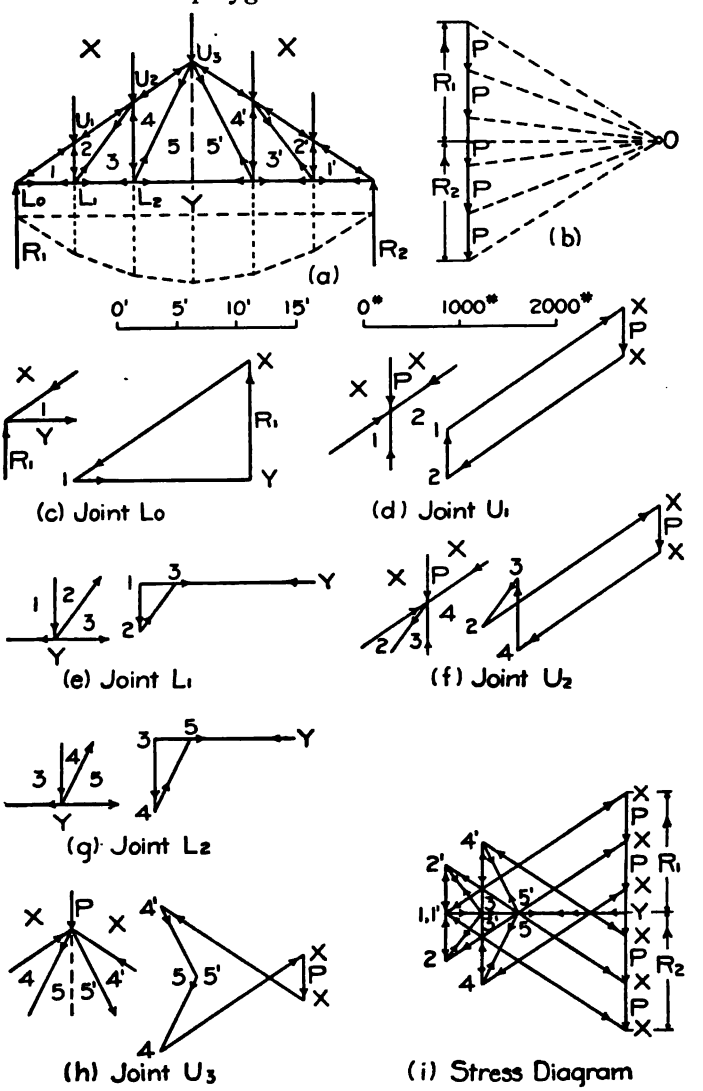


FIG. 58. STRESSES BY GRAPHIC RESOLUTION.

drawing R_1 (Fig. 58, c), acting upward, equal and parallel to the known left reaction; and, from the extremities X and Y of this line, drawing the lines X–1 and Y–1 parallel respectively to the members of the truss X–1 and Y–1, meeting at the point 1. Then X–1 and Y–1 represent to scale the magnitudes of the stresses in the members X–1 and Y–1. These forces are in equilibrium, so they must act around the polygon in the same direction; and by applying the forces to the joint L_0, it is seen that the stress X–1 acts towards the joint and is compression, while the stress Y–1 acts away from the joint and is tension.

The forces at the joint U_1 are next taken, instead of those at L_1; since at the former there are but two unknown forces, while at the latter there are three. The forces at U_1 are shown in Fig. 58, d. Since the stress X–1 has been found to be compression, it must act towards the joint U_1. The force P is also known, while the internal forces X–2 and 1–2 are known in lines of action only. The force polygon for these forces (Fig. 58, d) is constructed by drawing X–1, acting upward and to the right, equal and parallel to the known force X–1, and the force P, acting downward, equal and parallel to the known load P. The force polygon is then closed by drawing from the point X the line X–2 parallel to the member X–2, and from the point 1, by drawing 1–2 parallel to the member 1–2. Then X–2 and 1–2 represent the magnitudes of the unknown forces, X–2 acting downward and to the left, and 1–2 acting upward. Both X–2 and 1–2 are compression; as may be seen by applying these forces to the joint U_1 (Fig. 58, a).

The forces acting at the joint L_1 are next considered. These forces are shown in Fig. 58, e, the forces 2–3 and Y–3 being unknown. The unknown forces are found by constructing the force polygon, as shown in Fig. 58, e. Applying the forces in this polygon to the joint L_1, it is seen that both 2–3 and Y–3 act away from the joint, and therefore the stresses are tension.

The forces acting at the joint U_2 are shown in Fig. 58, f. Of these, the forces P, X–2, and 2–3 are known, while 3–4 and

X–4 are unknown. The unknown forces are found by constructing the force polygon shown in Fig. 58, f. Applying the unknown forces to the joint U_2, it is seen that both 3–4 and X–4 are compression.

The forces acting at L_2 are shown in Fig. 58, g. The unknown forces 4–5 and Y–5 are found by constructing the force polygon shown in Fig. 58, g. Both forces act away from the joint, and therefore the stresses are tension.

The forces acting at U_3, together with their force polygon, are shown in Fig. 58, h. The stress in X–4′ is compression, while that in 4′–5′ is tension. The stress in 5–5′ is zero.

Since the truss and loads are symmetrical about the center line, the force polygons for the joints to the right of the center need not be drawn; as they are the same as those for the left.

Stress Diagram. Referring to the separate force polygons shown in Fig. 58, it is seen that some of the forces in one polygon are repeated in another, i. e., it is necessary to find some of the stresses in one polygon and use these stresses in drawing the next polygon. It is thus seen that these separate force polygons may be grouped together in such a manner that none of the lines are drawn twice. Again referring to the separate diagrams, it is seen that the stress in any member acts in one direction in a particular polygon and in an opposite direction when repeated in another polygon. If these force polygons are combined, the line representing the stress will therefore have two arrows pointing in opposite directions. The force polygon for all the joints of the truss are grouped together into the *stress diagram* shown in Fig. 58, i. If the diagram is drawn for the forces at all the joints, as in this figure, the last polygon in the stress diagram must check with one side on the line representing the known right reaction.

As the stress diagram is being drawn, it is usually most convenient to put the arrows representing the directions of the stresses at any joint directly upon the diagram of the truss; as shown in Fig. 58, a. If this is done, they may be omitted in the stress diagram.

The student should follow through the separate force polygons in the stress diagram shown in Fig. 58, i, paying particular attention to the fact that the forces at a joint, whose magnitudes are represented by a closed polygon in the stress diagram, must act in the same direction around the polygon. Great care should be used in drawing the truss, and in transferring the lines from the truss to the corresponding lines in the stress diagram to secure accuracy.

113. **Stresses by Graphic Resolution.** *Loads on Lower Chord.* In the problem given in § 112, and in the preceding problems in this chapter, the loads have all been applied at the upper chord panel points; as it is the usual practice to consider all the dead load on the upper chord. In some buildings, however, the ceiling loads and other miscellaneous loads are supported at the lower chord panel points; and the follow-

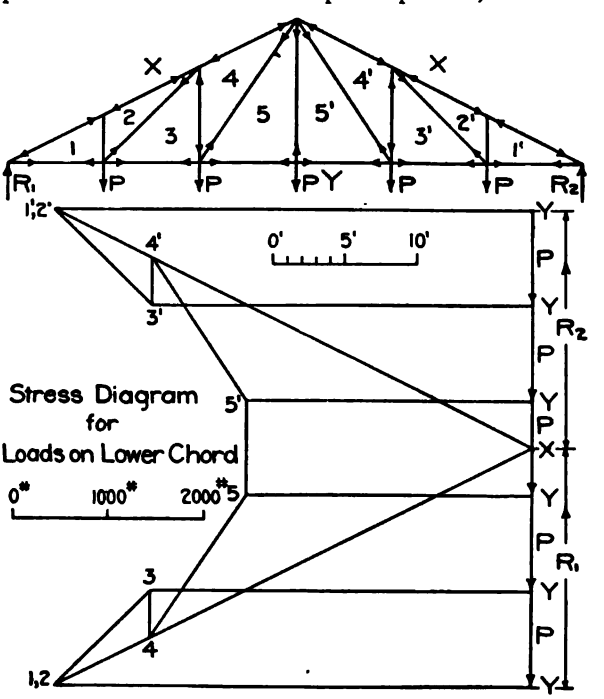


FIG. 59. STRESS DIAGRAM—LOADS ON LOWER CHORD.

ing problem will show the methods used in drawing the stress diagram for a truss loaded at the lower chord panel points only.

Problem. It is required to find the stresses due to ceiling loads in all the members of the truss shown in Fig. 59. The trusses are spaced 15 ft. apart, each truss having a span of 40 ft. and a rise of 10 ft. The ceiling load is 10 lbs. per sq. ft.

The panel load P is equal to $\frac{40}{6} \times 15 \times 10 = 1000$ lbs., and the effective reactions are each equal to $2\frac{1}{2} \times 1000 = 2500$ lbs.

To draw the stress diagram, lay off the reactions R_1 and R_2 on the load line, as shown in Fig. 59, and start the diagram with the forces acting at the left end of the truss. The stress diagram is drawn considering the forces at the joints in the following order: L_0, U_1, L_1, U_2, L_2, L_3, U_3, L_2', U_2', L_1', U_1', and and L_0'. The arrows indicating the kind of stress are placed at each panel point of the truss, as the portion of the stress diagram for that panel point is drawn. The arrows, with the exception of those in the load line, are omitted in the stress diagram. Since the truss and the loads are symmetrical about the center line, it is seen that the stresses are all determined as soon as the diagram is drawn for the forces up to and including those acting at L_3. It is therefore unnecessary to draw the diagram for the right half of the truss.

CHAPTER XI.

WIND LOAD STRESSES.

This chapter will treat of the stresses in roof trusses due to wind loads for different conditions of the ends of the trusses. The method for finding the wind load stresses in roof trusses by graphic resolution will be shown for the four following cases: (a) Both Ends of Truss Fixed—Reactions Parallel; (b) Both Ends of Truss Fixed—Horizontal Components of Reactions Equal; (c) Leeward End of Truss on Rollers; (d) Windward End of Truss on Rollers.

ART. 1. BOTH ENDS OF TRUSS FIXED—REACTIONS PARALLEL.

When the roof truss is comparatively flat, it is usually customary to assume that both reactions are parallel to the resultant of the wind loads. The wind load stresses in the members of a roof truss will be found by the method of graphic resolution, assuming that the reactions are parallel.

114. **Problem.** It is required to find the wind load stresses in all the members of the truss shown in Fig. 60. The truss has a span of 60 ft., a rise of 15 ft., and the trusses are spaced 15 ft. apart, center to center. The lower chord has a camber of 2 ft., and the normal component of the wind is taken at 23 lbs. per sq. ft. (§ 93).

After computing the loads, the reactions are found by means of the force and the funicular polygons (see § 98, a). The left reaction is represented by R_1, and the right reaction by R_2 (Fig. 60).

The stress diagram is started by drawing the force polygon for the forces at the joint L_0 (Fig. 60). The stresses in the members X–1 and Y–1 are unknown, while the wind load at L_0 and the left reaction are known. The unknown stresses at this joint are determined by drawing the force polygon $\frac{P}{2}$, X, 1, Y, R_1, noting particularly that the polygon closes, and that the forces act in the same direction around the polygon.

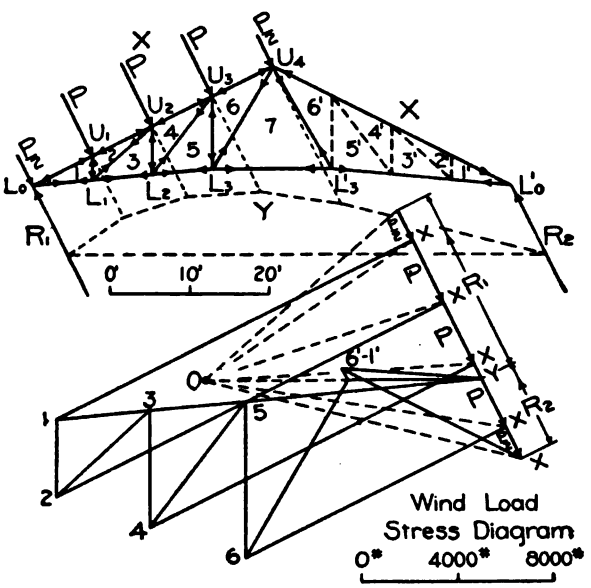


FIG. 60. ENDS FIXED—REACTIONS PARALLEL.

The lines X–1 and Y–1 in the force polygon represent the stresses in the members X–1 and Y–1. By placing the arrows on the members at the joint L_0, corresponding to the directions of the forces in the force polygon, it is seen that X–1 is compression and that Y–1 is tension.

The force polygon, or stress diagram, for the forces at the joint U_1 is X, X, 2, 1, X, the sequence of the letters and figures denoting the directions of the forces. The kind of stress in

the unknown members X–2 and 1–2 is determined by placing arrows on the members meeting at the joint U_1. The stresses in both these members are compression.

The stress diagram for the forces at the joint L_1 is Y, 1, 2, 3, Y, the sequence of the letters and figures denoting the directions of the forces, which directions are shown by arrows placed on the members at the joint L_1. The stresses in both 2–3 and Y–3 are thus found to be tension.

The unknown stresses at the joint U_2 are X–4 and 3–4, and the stress diagram for the forces at the joint U_2 is X, X, 4, 3, 2, X.

The unknown stresses at the joint L_2 are 4–5 and Y–5, and the stress diagram for the forces at L_2 is Y, 3, 4, 5, Y.

The unknown stresses at U_3 are X–6 and 5–6, and the stress diagram for the forces at this joint is X, X, 6, 5, 4, X.

At the joint L_3, the unknown stresses are 6–7 and Y–7, and the stress diagram for the forces at this joint is Y, 5, 6, 7, Y.

At the joint U_4, the unknown stresses are X–(6′–1′) and 6′–7, and the stress diagram for the forces at this joint is X, X, 6′–1′, 7, 6, X.

The unknown stress at the joint L_3' is Y–(6′–1′), and the stress diagram for the forces at this joint is Y, 7, 6′–1′, Y.

The stress diagram for the forces at the joint L_0 is Y, (6′–1′), X, Y.

The numerical values for the stresses in all the members of the truss may be found by scaling the corresponding lines in the stress diagram to the given scale.

It is seen from the stress diagram that there are no stresses in the members 6′–5′, 5′–4′, 4′–3′, 3′–2′, and 2′–1′. This follows, since there are no intermediate loads between the joints U_4 and L_0', and the members X–(6′–1′), 7–6′, and Y–(6′–1′) form a triangle. It is further seen that it is necessary to draw the stress diagram for the complete truss, and that the members X–(6′–1′) and Y–(6′–1′) must form a triangle, one side

of which is the known reaction R_2, which furnishes a check on the work.

Art. 2. Both Ends of Truss Fixed—Horizontal Components of Reactions Equal.

When the horizontal component of the wind loads is large and the supports are equally elastic, actual conditions are most nearly approximated by taking the horizontal components of the reactions equal. The horizontal component of the wind loads may be large if the roof is steep, or if a ventilator such as shown in Fig. 61 is used. The wind load stresses in a roof truss having a "monitor" ventilator will now be found, assuming that the reactions have equal horizontal components.

115. Problem. It is required to find the wind load stresses in all the members of the truss shown in Fig. 61. The truss has a span of 50 ft., a pitch of one-fourth, and has a monitor

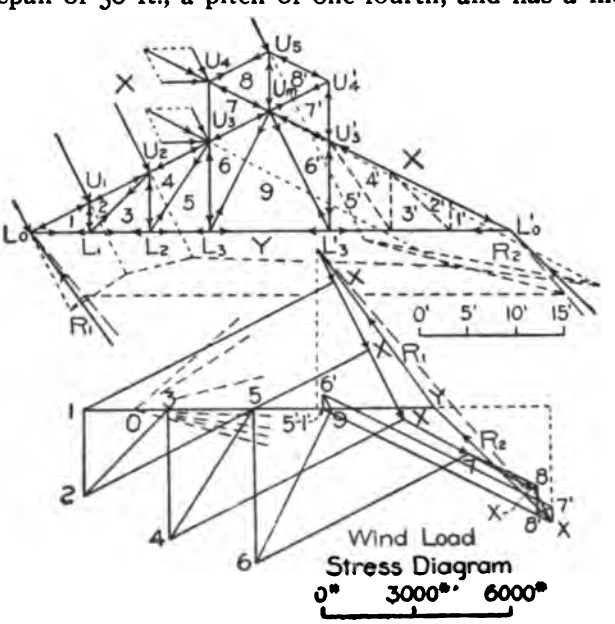


Fig. 61. Ends Fixed—Hor. Comp. of Reactions Equal.

ventilator, as shown in Fig. 61. The trusses are spaced 16 ft. apart, center to center. The wind load on the vertical surface of the ventilator is taken at 30 lbs. per sq. ft., and the component of the wind, normal to the roof surface, is taken at 23 lbs. per sq. ft.

The wind loads at the joints U_3 and U_4 are found by taking the resultants of the horizontal and inclined loads acting at these joints (see Fig. 61). The reactions R_1 and R_2 are determined by the methods explained in § 98, b, assuming first that the reactions are parallel and finding these reactions by means of the force and the funicular polygons; and then making their horizontal components equal.

The stresses in the members of the truss are found by drawing the force polygons for the forces at each joint and then combining these polygons, taking the joints in the following order: L_0, U_1, L_1, U_2, L_2, U_3, L_3, U_4, U_5, U_4', U_m, U_3', L_3', and L_0'. The complete stress diagram is shown in Fig. 61. When the stress diagram has been drawn for the forces up to those acting at U_3, it is seen that there are three unknown stresses at this joint, viz.: 5–6, 6–7, and X–7. The stress in X–7 is taken as zero, and the load acting at U_4 is held in equilibrium by the stresses in the two members X–8 and 7–8. There are no stresses in the members $5'$–$4'$, $4'$–$3'$, $3'$–$2'$, and $2'$–$1'$. The numerical value of the stresses in the members may be found by scaling the corresponding lines in the stress diagram to the given scale. The kind of stress, whether tension or compression, is given by the arrows placed on the members of the truss, as shown in Fig. 61. If the arrows act away from the center of the member, i. e., toward the joints, the stress is compression; and if they act toward the center of the member, i. e., away from the joints, the stress is tension.

The student should carefully follow through the construction of the stress diagram, placing the arrows, which show the directions of the forces at the joint, on the members, as the force polygon for that joint is drawn.

Art. 3. Leeward End of Truss on Rollers.

If the span of the truss is large, the change in its length due to the loads and to the temperature variations is usually adjusted by placing one end of the truss on rollers. The stresses in a roof truss with its leeward end supported on rollers will now be determined.

116. Problem. It is required to find the wind load stresses in all the members of the "Fink" truss shown in Fig. 62, the leeward end of the truss being supported on rollers. The span of the truss is 60 ft., the rise 20 ft., and the lower chord is cambered 3 ft. The trusses are spaced 16 ft. apart, and the normal component of the wind is taken at 26 lbs. per sq. ft.

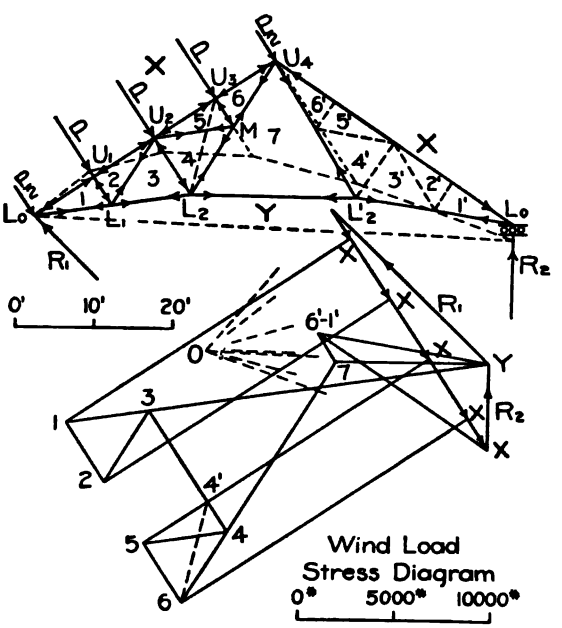


Fig. 62. Leeward End of Truss on Rollers.

The reactions R_1 and R_2 (Fig. 62) are found by means of the force and funicular polygons, the funicular polygon being

started at the left point of support of the truss (see § 99, a). The stresses in the members intersecting at the joints L_0, U_1, and L_1 are found by drawing the stress diagram for these forces, as in the preceding articles. Referring to the forces at the joint U_2, it is seen that there are three unknown stresses at this joint, viz.: X–5, 4–5, and 3–4; and therefore the stress diagram cannot be drawn for this joint. There are also three unknown stresses at L_2. The unknown stresses at U_2 may, however, be found as follows: Replace the members 4–5 and 5–6 by the auxiliary dotted member connecting the joints L_2 and U_3 (Fig. 62). Now draw the stress diagram for the forces at the joint U_2, this diagram being X,X,4′,3,2,X. Next draw the stress diagram for the forces at U_3, which is X,X,6,4′,X. Now the line X–6 in the stress diagram represents the true stress in the member X–6. After this stress is determined, remove the dotted auxiliary member, and replace it by the original members 4–5 and 5–6. Two of the forces (P, and X–6) at U_3 are now known, and the stress diagram may be drawn for this joint, this diagram being X,X,6,5,X. Four of the forces at U_2 are now known, the unknown forces being 3–4 and 4–5. The stress diagram for the forces at this joint is X,X,5,4,3,2,X. The stress diagram for the forces at the remaining joints may be drawn in the following order: L_2, M, U_4, L_2', L_0'. There are no stresses in the members 6′–5′, 5′–4′, 4′–3′, 3′–2′, and 2′–1′. The complete stress diagram for all the members of the truss is shown in Fig. 62. The kind of stress, whether tension or compression, is denoted by the arrows placed on the members of the truss; and the numerical values of the stresses may be found by scaling the lines in the stress diagram to the given scale.

Art. 4. Windward End of Truss on Rollers.

Since the wind may act from either of two opposite directions, it is necessary to find the stresses in the members of the truss when the rollers are under the leeward end of the truss

and also when the rollers are under the windward end. The stresses in a roof truss with its windward end on rollers will now be found.

117. **Problem.** It is required to find the wind load stresses in all the members of the "Camels Back" truss shown in Fig. 63, the windward end of the truss being supported on rollers. The span of the truss is 60 ft., the rise 14 ft., and the trusses are spaced 15 ft. apart.

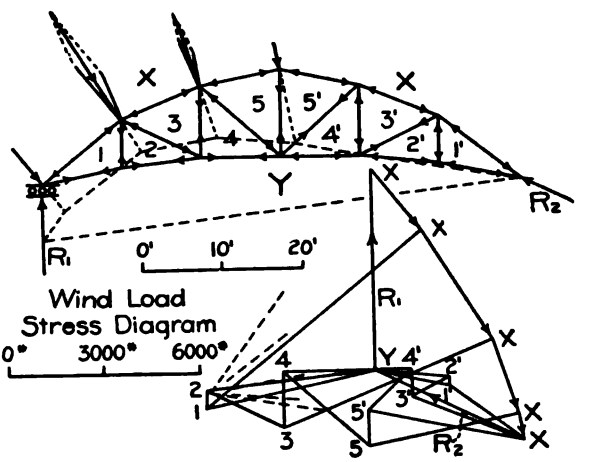


FIG. 63. WINDWARD END OF TRUSS ON ROLLERS.

The normal components of the wind loads are found from the diagram based on Duchemin's formula (see Fig. 47), assuming that P equals 30 lbs. per sq. ft. The reactions R_1 and R_2 (Fig. 63) are found by means of the force and funicular polygons (see § 99, b).

The complete stress diagram for all the members of the truss is shown in Fig. 63. The numerical values of the stresses may be found by scaling the lines in the stress diagram to the given scale. The kind of stress in each member, whether tension or compression, is shown by the arrows placed on the members of the truss. Referring to the arrows on the truss, it is seen that the left half of the lower chord is in tension, while the right half is in compression. It is also seen that the mem-

bers 1–2, 3–4, and 5–5′ are in tension, while the corresponding members 1′–2′ and 3′–4′ are in compression; and further that the members 2–3 and 4–5 are in compression, while 2′–3′ and 4′–5′ are in tension. Since the wind may act from either direction, it is seen that these members will be subjected to reversals of stress.

CHAPTER XII.

STRESSES IN CANTILEVER AND UNSYMMETRICAL TRUSSES—MAXIMUM STRESSES.

This chapter will be divided into two articles. The first article will treat of the application of the method of graphic resolution to the solution of stresses in cantilever and unsymmetrical trusses, showing a method for drawing a combined stress diagram, and the second will treat of the determination of the maximum stresses in trusses.

ART. 1. STRESSES IN CANTILEVER AND UNSYMMETRICAL TRUSSES.

A cantilever truss is one which is supported at one end only, the other end being entirely free. Such a truss is often used to project over platforms and entrances to buildings. The cantilever truss may be fastened to the walls of the building or to the columns supporting the main trusses.

118. Stresses in a Cantilever Truss. *Problem.* It is required to find the dead load stresses in the cantilever truss shown in Fig. 64. The span of the truss is 24 ft. The trusses are spaced 15 ft. apart, center to center, and support a dead load of 12 lbs. per sq. ft. of the horizontal projection of the roof.

The point of application A (Fig. 64) and the line of action of the reaction R_2 are known; while the point of application, only, of the reaction R_1 is known. The line of action of R_1 may be found by applying the principle that, if a body is in equilibrium under the action of three external forces, these

forces must all intersect at a common point. One of these three forces is the resultant of the loads acting on the truss, the other two being the two reactions.

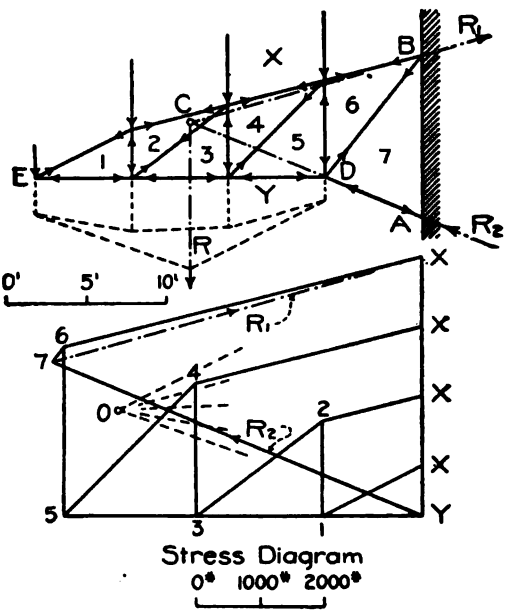


FIG. 64. STRESS DIAGRAM—CANTILEVER TRUSS.

The resultant R of the loads acting on the truss is found by drawing the force and funicular polygons for these loads, as shown in Fig. 64. Now produce the lines of action of R and R_2 until they intersect at the point C. Then the line of action of R_1 must pass through the points B and C. The magnitudes of these reactions are found by drawing the force triangle for the loads and the two reactions (see Fig. 64). Then Y–7 represents the magnitude of the reaction R_2, and X–7 that of R_1.

The stresses in the members of the truss are found by drawing the stress diagram, starting the diagram with the forces acting at the point B. The forces acting at D are then taken, noting that the stress in the member Y–7 is equal to

the known reaction R_2. The complete stress diagram is shown in Fig. 64. The numerical values of the stresses may be found by scaling the lines in the stress diagram to the given scale.

The stress diagram might also have been drawn without first finding the reactions by starting the diagram with the forces acting at E, the left end of the truss.

Referring to the truss and to the stress diagram (Fig. 64), it is seen that the stress in the member 6–7 would be zero if the inclination of the member Y–7 was changed so that the lines of action of R and R_2 would intersect on the member X–2; and further, that the stress in 6–7 would be compression if the lines of action of R and R_2 intersected above the member X–2.

119. Unsymmetrical Truss—Combined Stress Diagram. In the problems that have been given (excepting that given in § 113), the loads have all been applied at the panel points of the upper chord of the truss; and separate stress diagrams have been drawn for the dead and for the wind loads. This section will treat of the loads on both the upper and lower chords; and the stresses due to the dead loads and to the wind loads will be found by drawing a single diagram. The method for drawing the combined stress diagram will now be shown by the following problem.

Problem. It is required to find the stresses in the members of the truss shown in Fig. 65, the leeward end of the truss being supported upon rollers. The span of the truss is 50 ft., the rise 16⅔ ft., and the adjacent trusses are spaced 14 ft. apart, center to center. The dead load is taken at 12 lbs. per sq. ft. of horizontal projection, the ceiling load at 10 lbs. per sq. ft. of horizontal projection, and the wind load at 26 lbs. per sq. ft. of roof surface.

The resultants of the dead and wind loads at each panel point of the upper chord are first found. These resultants are shown on the truss diagram (Fig. 65). The reactions R_1 and R_2, considering the right end of the truss on rollers, are found for these loads by means of the force and funicular polygons. These reactions are shown in the force polygon, but are not shown on the diagram of the truss. The reactions due to the

loads on the lower chord are then found, these reactions being each numerically equal to two lower chord panel loads.

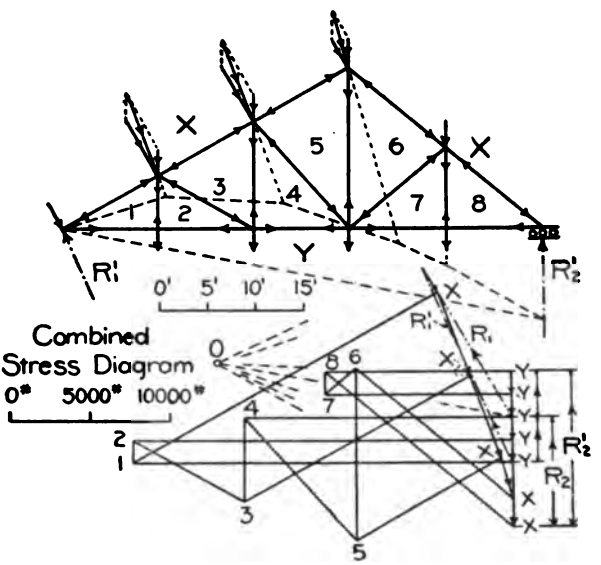


FIG. 65. UNSYMMETRICAL TRUSS—COMBINED STRESS DIAGRAM.

The left reaction due to all the loads is then determined by finding the resultant of the reactions due to the dead and wind loads on the upper chord and to the ceiling loads on the lower chord. This reaction is represented by R_1' (Fig. 65). The right reaction is determined in the same manner, and is represented by R_2'.

The stresses in the members of the truss are now found by drawing the stress diagram, as shown in Fig. 65. The numerical values of the stresses may be found by scaling the lines in the stress diagram to the given scale.

To determine which condition gives the largest stresses, it is necessary to take the wind as acting both from the right and from the left; and further, to assume that the rollers may be under either end of the truss.

The above method of laying off the loads in the load line,

and of determining the reactions, is applicable to all trusses carrying loads on both the upper and lower chords. It is seen that this method places the loads in their proper order in the load line.

ART. 2. MAXIMUM STRESSES.

120. **Maximum Stresses.** In the preceding articles, methods have been given for finding the stresses in various types of trusses, due separately to the dead load, to the snow load, and to the wind load. In this article, the maximum stresses that may occur in the members of a truss due to the combined effect of the different loadings will be found. The stresses will first be found separately for the different loadings, and then combined to determine the maximum stresses.

If the truss and also the loads are symmetrical about the center line, each reaction is equal to one-half of the total load on the truss (neglecting the half loads at the ends). In this case, it is unnecessary to draw the funicular polygon to determine the reactions. The dead load stresses may then be found by drawing the stress diagram for the dead loads.

Since the snow load is always taken at so much per square foot of the horizontal projection of the roof, it is seen that it is unnecessary to draw a stress diagram for the snow loads; but that the snow load stresses may be determined directly by proportion from the known dead load stresses. The minimum snow load in this work will be taken at 10 pounds and the maximum snow load at 20 pounds per square foot of the horizontal projection of the roof.

If the truss is symmetrical and is fixed at both ends, the wind need only be taken as acting from one side; since the stresses in the corresponding members would be the same as if the wind was taken from the other side. If the truss has rollers under one end, the wind must be taken as acting both from the roller side and from the fixed side of the truss. The condition which gives the larger stresses is then considered in making the combinations for maximum stresses.

The dead load is always acting upon the truss, and therefore it must be used in all the combinations for maximum stresses. If the maximum snow load is taken, then the wind load will be neglected; as it is improbable that the maximum wind and the maximum snow will ever occur at the same time. If the maximum wind is considered, then the minimum snow load will be used.

From the above discussion, it is seen that the following combinations should be made to obtain the maximum stresses in the members of a truss when there are no reversals of stress:

(a) Dead load stress plus maximum snow load stress.

(b) Dead load stress plus minimum snow load stress plus wind load stress (rollers under leeward end of truss).

(c) Dead load stress plus minimum snow load stress plus wind load stress (rollers under windward end of truss).

The method for finding the maximum stresses in the members of a truss will now be shown by the solution of a problem.

121. Problem. It is required to find the maximum stresses in the members of the Fink truss shown in Fig. 66. The dimensions of the truss, together with the loadings taken, are shown in the following table:

Span of truss = 80 ft.

Rise of truss = 20 ft.

Distance between trusses = 16 ft.

Dead load taken at 12 lbs. per sq. ft. hor. proj.

Minimum snow load taken at 10 lbs. per sq. ft. hor. proj.

Maximum snow load taken at 20 lbs. per sq. ft. hor. proj.

Wind load taken at 23 lbs. per sq. ft. of roof surface.

Since one-half of the upper chord is divided by the web members into eight equal parts, preliminary computations give the following panel loads:

Dead panel load.....	960 lbs.
Minimum snow panel load.....	800 lbs.
Maximum snow panel load.....	1600 lbs.
Wind panel load.....	2060 lbs.
Wind loads at end and apex.....	1030 lbs.

Since the dead loads are symmetrical about the center line, the effective dead load reactions are each equal to one-half of the total dead load (neglecting the half loads at the ends of the truss). The dead load stresses are found by drawing the stress diagram, as shown in Fig. 66, b.

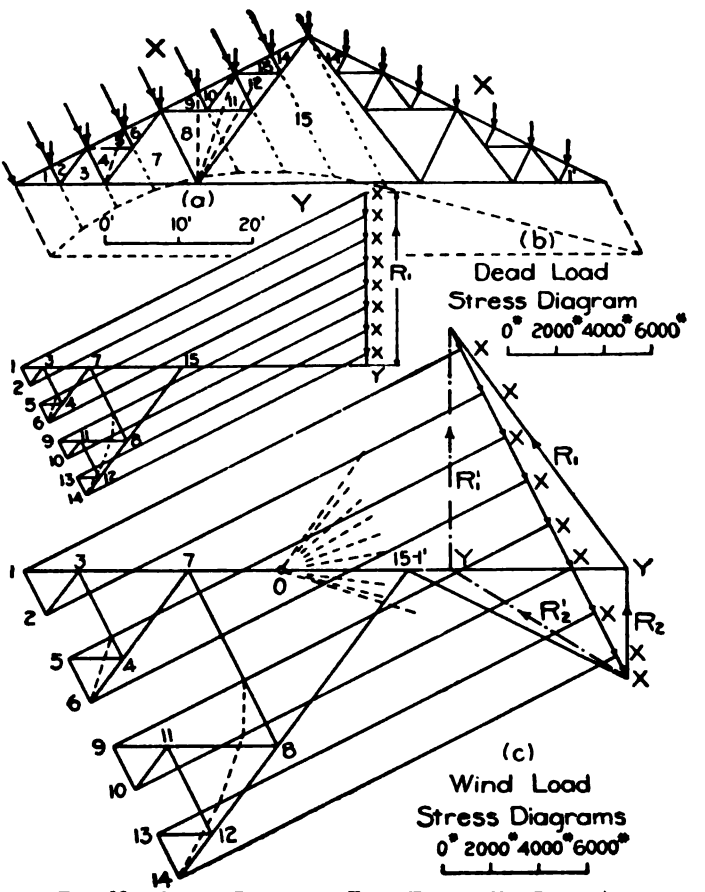


FIG. 66. STRESS DIAGRAMS—FINK TRUSS (NO CAMBER)

The minimum and maximum snow load stresses are obtained from the dead load stresses without drawing another stress diagram. The ratio of the minimum snow load to the

dead load is as 5 is to 6, and that of the maximum snow load to the dead load is as 5 is to 3. The minimum and maximum snow load stresses are obtained by multiplying the corresponding dead load stresses by these ratios.

The wind load reactions, considering the leeward end of the truss on rollers, are found by means of the force and funicular polygons. These reactions are represented by R_1 and R_2 (Fig. 66, c). The wind load stress diagram is shown in Fig. 66, c. The apparent ambiguity at some of the joints is overcome by substituting the dotted members in the truss diagram and the corresponding ones in the stress diagram, as is shown in Fig. 66, a and Fig. 66, c (see § 116).

The wind load reactions, considering the windward end of the truss on rollers, are represented by R_1' and R_2' (Fig. 66, c). The wind load stress diagram for this case is also shown in Fig. 66, c. It is seen that the stresses in all the upper chord and web members for this case are the same, and that the stresses in the lower chord members are smaller than they were when the rollers were considered under the leeward end.

The stresses in the members of this truss due to the different loadings, together with the maximum stresses, are shown in tabular form in the upper half of the table given in Fig. 68. It is seen that, for this particular truss, there are no reversals of stress in any of the members.

122. Problem. It is required to find the maximum stresses in the members of the truss shown in Fig. 67, the lower chord being cambered two feet. The other dimensions of the truss, together with the dead, snow, and wind loads, are the same as for the problem given in the preceding section.

The minimum and maximum snow load stresses are obtained by proportion from the dead load stresses (see § 121).

The dead load stress diagram is shown in Fig. 67, b.

The wind load stress diagram, considering the leeward end of the truss on rollers, is shown by the full lines in Fig. 67, c. The wind load stress diagram, considering the windward end of the truss on rollers, is shown by the broken lines in Fig. 67, c.

The stresses in the members of the truss due to the different loadings, together with the maximum stresses, are shown in the lower half of the table given in Fig. 68. It is seen that, for this truss, there are no reversals of stress in any of the members.

By comparing the stresses given in Fig. 68, it is seen that even a small camber in the lower chord increases the stresses

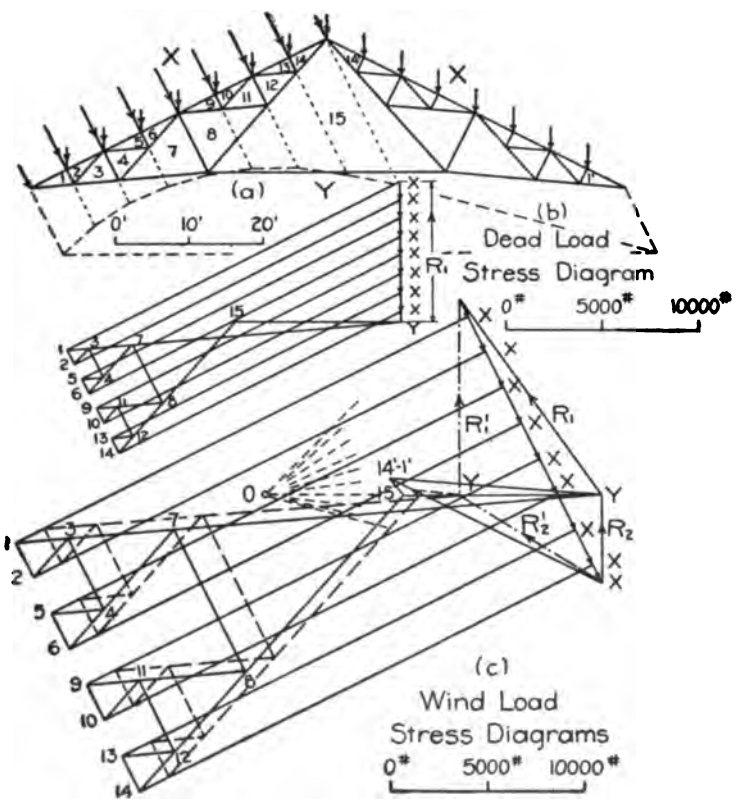


FIG 67. STRESS DIAGRAMS—FINK TRUSS (WITH CAMBER).

in most of the members a considerable amount. A small camber, however, improves the appearance of the truss, and also increases the clearance in the building. If no camber is

STRESSES IN A FINK TRUSS (No Camber).

Truss Member	Dead Load Stress	Snow Load Stress		Wind Load Stress		Maximum Stress
		Min.	Max.	Rollers Leeward	Rollers Windward	
X-1	-16100	-13400	-26800	-20600	-20600	-50100
X-2	-15700	-13100	-26200	-20600	-20600	-49400
X-5	-15300	-12700	-25500	-20600	-20600	-48600
X-6	-14800	-12300	-24700	-20600	-20600	-47700
X-9	-14400	-12000	-24000	-20600	-20600	-47000
X-10	-14000	-11700	-23300	-20600	-20600	-46300
X-13	-13600	-11300	-22700	-20600	-20600	-45500
X-14	-13100	-10900	-21800	-20600	-20600	-44600
X-(14'-1')				-10300	-10300	
Y-1	+14400	+12000	+24000	+25400	+18000	+51800
Y-3	+13500	+11300	+22500	+23000	+15700	+47800
Y-7	+11600	+9700	+19300	+18400	+11100	+39700
Y-15	+7800	+6500	+13000	+9200	+1900	+23500
Y-(15-1')				+9200	+1900	
1-2, 5-6, 9-10, 13-14	-800	-700	-1300	-2100	-2100	-3600
2-3, 4-5, 10-11, 12-13	+900	+800	+1500	+2300	+2300	+4000
3-4, 11-12	-1700	-1400	-2800	-4100	-4100	-7200
4-7, 8-11	+1400	+1200	+2300	+4600	+4600	+7200
6-7, 8-9	+2800	+2300	+4700	+6900	+6900	+12000
7-8	-3400	-2800	-5700	-8200	-8200	-14400
8-15	+3800	+3200	+6300	+9200	+9200	+16200
12-15	+5700	+4800	+9500	+13800	+13800	+24300
14-15	+6700	+5600	+11200	+16100	+16100	+28400

STRESSES IN A FINK TRUSS (With Camber).

X-1	-19300	-16100	-32200	-26200	-24500	-61600
X-2	-18800	-15700	-31400	-26200	-24500	-60700
X-5	-18400	-15300	-30700	-26200	-24500	-59900
X-6	-18000	-15000	-30000	-26200	-24500	-59200
X-9	-17600	-14700	-29300	-26200	-24500	-58500
X-10	-17200	-14300	-28600	-26200	-24500	-57700
X-13	-16700	-13900	-27800	-26200	-24500	-56800
X-14	-16300	-13600	-27200	-26200	-24500	-56100
X-(14'-1')				-12300	-10700	
Y-1	+17300	+14400	+28800	+30400	+21600	+62100
Y-3	+16200	+13500	+27000	+27700	+18800	+57400
Y-7	+13900	+11600	+23200	+22200	+13300	+47700
Y-15	+8600	+7200	+14300	+10200	+2000	+26000
Y-(7½'-1')				+11100	+2200	
1-2, 5-6, 9-10, 13-14	-800	-700	-1300	-2100	-2100	-3600
2-3, 4-5, 10-11, 12-13	+1100	+900	+1800	+2800	+2800	+4800
3-4, 11-12	-1700	-1400	-2800	-4100	-4100	-7200
4-7, 8-11	+2300	+1900	+3800	+5500	+5500	+9700
6-7, 8-9	+3400	+2800	+5700	+8300	+8300	+14500
7-8	-3400	-2800	-5700	-8200	-8200	-14400
8-15	+5600	+4700	+9300	+12300	+11300	+22600
12-15	+7900	+6600	+13200	+17800	+16800	+32300
14-15	+9000	+7500	+15000	+20600	+19600	+37100
15-(14'-8')				+1200	+200	

FIG. 68. TABLE OF STRESSES—MAXIMUM STRESSES.

used and the span is long, the lower chord has the appearance of sagging.

123. **Maximum and Minimum Stresses.** By the terms "maximum and minimum stresses" are meant the greatest ranges of stress that may occur in any member due to the different loadings. If there is no reversal of stress in any member, the maximum stress is the greatest numerical stress, and the minimum is the smallest numerical stress that may ever occur in any member. If there is a reversal of stress, the maximum is the greatest numerical stress, and the minimum is the greatest numerical stress of an opposite kind that may ever occur.

In finding maximum and minimum stresses, it must be borne in mind that the dead load is always acting, and that there is no reversal of stress in any member unless the wind load stress, or stress due to another condition of loading, in that member is greater in amount and is of an opposite kind to that caused by the dead load.

In designing, there is no need of finding minimum stresses unless there are reversals of stress; since if there are no reversals, the members must be designed for their maximum stresses, while if there are reversals, they must be designed for each kind of stress.

If both the maximum and minimum stresses are required, the following combinations should be made:

(a) Dead load stress alone.

(b) Dead load stress plus wind load stress (rollers leeward).

(c) Dead load stress plus wind load stress (rollers windward).

(d) Dead load stress plus maximum snow load stress.

(e) Dead load stress plus minimum snow load stress plus wind load stress (rollers leeward).

(f) Dead load stress plus minimum snow load stress plus wind load stress (rollers windward).

CHAPTER XIII.

COUNTERBRACING.

The use of counterbracing adds considerably to the work required to find the maximum stresses in a truss; since the same diagonals are not always stressed. In many cases, however, its use is more economical than to design the member for a reversal of stress. There are two methods which may be used to determine the stresses in a truss with counterbracing, viz.: (a) by the use of separate stress diagrams, and (b) by the use of combined stress diagrams. The former method may be used to advantage when several different combinations must be made to determine the maximum stresses, while the latter is useful when but few combinations are required.

The determination of stresses in trusses with counterbracing will now be taken up in three articles, as follows: Art. 1, Definitions and Notation; Art. 2, Stresses in Trusses with Counterbracing—Separate Stress Diagrams; and Art. 3, Stresses in Trusses with Counterbracing—Combined Stress Diagrams.

ART. 1. DEFINITIONS AND NOTATION.

124. Definitions. The triangle is the only geometrical figure which is incapable of any change in shape without a change in the length of one or more of its sides. The triangle is therefore the elementary form of truss; and the trusses in common use consist of a series of triangles so arranged as to form a rigid body.

A polygonal figure which is composed of more than three

sides and which is free to turn at its joints may be distorted without changing the length of any of its sides. For example, take the quadrilateral frame shown in Fig. 69, a, the frame being acted upon by an external force. This figure may be distorted without changing the length of any of its sides, as is shown by the dotted lines in the figure.

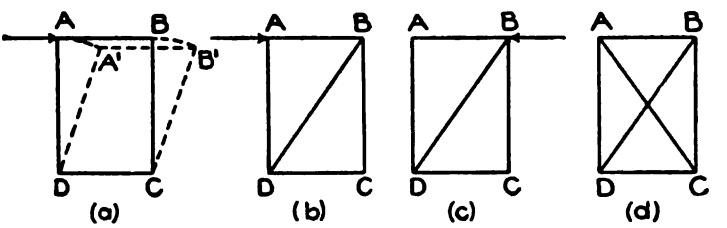


FIG. 69.

Now suppose a diagonal, connecting the points B and D, is added to the frame ABCD, as shown in Fig. 69, b. The figure is then composed of two triangles, and any force acting towards the right will tend to distort the frame, and will produce tension in the diagonal BD. The tension produced in this diagonal will prevent any distortion of the frame. Now suppose that the external force acts towards the left, as shown in Fig. 69, c. The distortion of the frame will be prevented by the diagonal BD, which will be in compression.

It is thus seen that the diagonal BD, which is capable of resisting both tension and compression, will prevent any distortion in the frame ABCD. The same is true of the diagonal AC. If the diagonal member is capable of resisting tension only, or compression only; then two diagonals will be required to prevent distortion, as shown in Fig. 69, d. If two diagonals are used, and both are designed to take the same kind of stress, it is evident that only one acts at a time. Referring to Fig. 69, d, if both the diagonals are rods and therefore can resist tension only, it is seen that BD is in tension when the external force acts towards the right and that there is no stress in AC. It is further seen that AC is in

tension when the force acts towards the left and that there is no stress in BD. If two diagonals are used, and each can resist both compression and tension, the problem is indeterminate by static methods. This case will not be considered in this work.

Let the quadrilateral ABCD (Fig. 70) be one panel of a Pratt truss loaded with dead load, as shown in the figure. In this type of truss, the intermediate diagonals are tension members. The members AC and A'C, which are in tension under the dead load, are called *main diagonals or main braces;* while the members BD and BD', which are not stressed by the dead load, are called *counterbraces* or *counters.* The counters may be stressed either under the action of wind loads or of unsymmetrical loads.

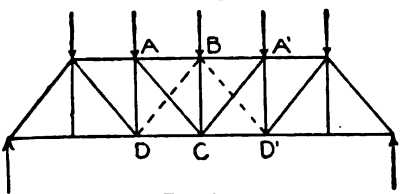


FIG. 70.

125. Counterbracing. The two tension or two compression diagonals in the same panel cannot act at the same time unless they are subjected to initial stress. A truss will not be in equilibrium under the action of initial stress unless external forces are applied, or unless the initial stress is held in equilibrium by the resisting moment of some of the members of the truss. Since initial stress will not be considered in this work, it is evident that if the diagonals can resist only one kind of stress, but one diagonal in each panel will act at a time.

The method for determining whether the main member or the counter is under stress due to any system of loading, together with the magnitude of the resulting stress, will now be given. Referring to Fig. 70, and noting the fact that the dead load is always acting upon the truss, it may be readily shown that there will be a tensile stress in the diagonal AC when the dead load alone is acting. The kind of stress in this member may be determined by drawing the stress diagram, or

by a method which will be explained in the following article. Now suppose that the snow load covers the right half of the truss only, or that the wind is acting from the right. Either of these conditions will tend to cause a compressive stress in the diagonal A′C. If this compression is less than the dead load tension in the member, then the tension already in the member will be reduced by an amount equal to the magnitude of the compressive stress. The resultant of these stresses will be the actual stress in the member due to the combined loads. If this compression is exactly equal to the dead load tension, the resulting stress in the member is zero. Now suppose that the compression caused by the unsymmetrical loading is greater than the dead load tension. In this case, the member A′C can only take enough compression to neutralize the dead load tension. If more than this amount of compression is thrown into A′C, the member will tend to be distorted, and the counter BD′ will be thrown into tension to resist this distortion.

If the chords are parallel and the diagonals have the same inclination, as in the truss shown in Fig. 70, the magnitude of the tension in the counter BD′ is equal to the difference between the stress in A′C caused by the wind or unsymmetrical load and that caused by the dead load. If the chords are not parallel, i. e., if the main member and the counter in any panel have different inclinations, the stress in the counter is equal to the difference in shears in the panel resolved in the direction of the counter.

126. Notation. When counterbracing is used, the system of notation must be slightly modified; since either diagonal in a panel may be stressed. A convenient system of notation is shown in Fig. 71. In this system, one diagonal in each panel is shown as a dotted line. The diagonals shown as full lines are designated by the figures 2–3 and 4–5; while those shown as dotted lines are designated by the same figures accented, 2′–3′ and 4′–5′.

To illustrate this system of notation, if the diagonals

stressed are 2′–3′ and 4–5, then the upper chord members are X–1, X–3′, X–5, and X–6; the lower chord members are Y–1, Y–2′, Y–4, and Y–6; and the web members are 1–3′, 2′–3′, 2′–5, 4–5, and 4–6. If the dead load alone is acting, the diagonals 2–3 and 4–5 are stressed, and the web members are 1–2, 2–3, 3–5, 4–5, and 4–6.

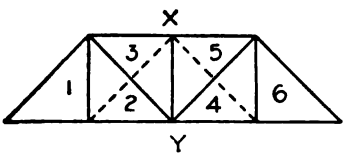


FIG. 71. NOTATION.

ART. 2. STRESSES IN TRUSSES WITH COUNTERBRACING—SEPARATE STRESS DIAGRAMS.

127. Determination of Stresses in Trusses with Counterbracing—Separate Stress Diagrams. In the explanations that follow, the diagonals will be assumed to be tension members, as this is the more common case; although the same general principles will apply if they are compression members. The method for determining the maximum stresses in trusses with counterbracing, when separate stress diagrams are used, involves the following steps:

(1). Construct the dead and snow load stress diagrams, assuming that the diagonals all slope in the same direction. If the truss is symmetrical about its center line, it will only be necessary to construct the diagrams for one-half of the truss. The snow load stresses may be found by direct proportion from the dead load stresses without the use of another diagram.

(2). From these diagrams, determine in which panels, if any, the diagonals will be subjected to compression, and draw in the second diagonals in these panels. Revise the stress diagrams to include the added diagonals. The revised diagrams will now contain the actual stresses in all the members due to vertical loads. The main members (those stressed by the dead load) are represented by full lines, and the counters by dotted lines.

(3). Construct the wind load stress diagrams, using those

diagonals which have been found to be in tension due to the dead load. If the truss is symmetrical, it will only be necessary to consider the wind as acting from one direction; while if the truss is unsymmetrical, it will be necessary to consider the wind as acting from both directions.

(4). From the wind load stress diagram, determine which diagonals, if any, will be in compression. Draw counters in these panels. Revise the stress diagrams to include the added diagonals.

(5). From the stress diagrams, determine the stresses due to the different loadings, combine these stresses to determine which diagonals are stressed, and then find the maximum stresses in all the members of the truss. In making any particular combination to determine the maximum stress in any member, it is necessary to first find which diagonals are acting at that time, and then to combine the stresses found in that particular member when these diagonals are acting. In making the combinations for maximum stresses, it is necessary to consider, not only the member itself, but also the corresponding member on the other side of the center of the truss.

The following combinations will be considered in this work in determining maximum stresses in roof trusses.

(a) Dead load plus maximum snow load.

(b) Dead load plus minimum snow load plus wind load (wind acting from either direction).

(c) Dead load plus wind load (wind acting from either direction).

The method outlined above will now be explained by the solution of two problems. The first problem will be to determine the maximum stresses in a truss with parallel chords, and the second to determine the maximum stresses in a truss with non-parallel chords.

128. Problem 1. Truss With Parallel Chords. It is required to find the maximum stresses in all the members of the Pratt truss with counterbracing shown in Fig. 72. The span of the truss is 40 ft.; the height, 7.5 ft.; and the trusses are spaced 15 ft. apart. The dead load will be taken at 10 lbs.

per sq. ft. of hor. proj.; the minimum snow load, at 10 lbs. per sq. ft. of hor. proj.; the maximum snow load, at 20 lbs. per sq. ft. of hor. proj.; and the component of the wind normal to the roof surface, at 27 lbs. per sq. ft. of roof surface. The wind load reactions will be assumed parallel to the resultant of all the wind loads.

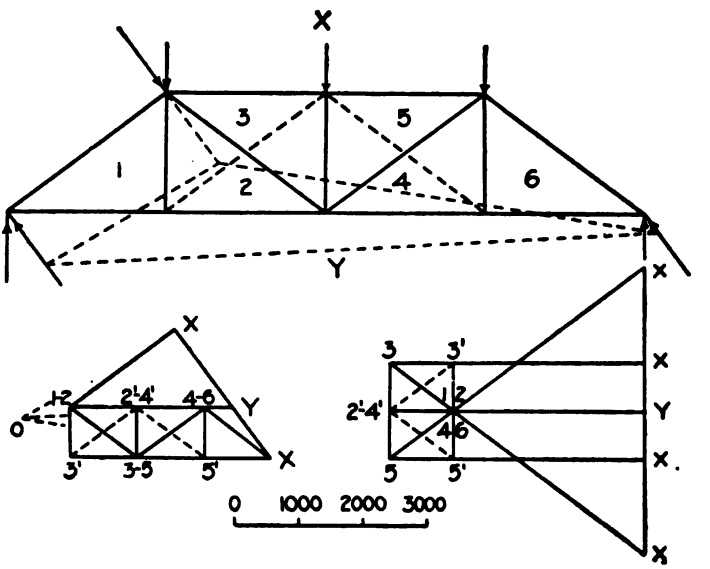


FIG. 72. STRESS DIAGRAMS—TRUSS WITH PARALLEL CHORDS.

The dead load stress diagram (see Fig. 72) is first drawn, assuming that all the diagonals slope in the same direction, i. e., that 2–3 and 4′–5′ are acting. From this diagram it is found that 4′–5′, if acting, would be in compression; therefore, the other diagonal, 4–5, in the same panel is acting due to the dead load. The stress diagram is now revised to include the diagonal 4–5, also the diagonal 2′–3′. The dead load stress diagram for the entire truss is shown in Fig. 72. The dead load stresses when the different diagonals are acting are shown in the table in Fig. 73.

The minimum and maximum snow load stresses are deter-

mined by direct proportion from the dead load stresses without constructing any new diagrams, and are shown in Fig. 73.

Truss Member	Dead Load Stress	Snow Load Stress		Wind Load Stress	Maximum Stress
		Min.	Max.		
X-1	− 3750	− 3750	− 7500	− 2070	−11250*
X-3'	− 3000	− 3000	− 6000	− 3160	—
X-3	− 4000	− 4000	− 8000	− 2110	−12000*
X-5	− 4000	− 4000	− 8000	− 2110	−12000
X-5'	− 3000	− 3000	− 6000	− 1060	—
X-6	− 3750	− 3750	− 7500	− 1320	−11250
Y-1	+ 3000	+ 3000	+ 6000	+ 2580	+ 9000*
Y-2'	+ 4000	+ 4000	+ 8000	+ 1530	—
Y-2	+ 3000	+ 3000	+ 6000	+ 2580	+ 9000*
Y-4'	+ 4000	+ 4000	+ 8000	+ 1530	—
Y-4	+ 3000	+ 3000	+ 6000	+ 480	+ 9000
Y-6	+ 3000	+ 3000	+ 6000	+ 480	+ 9000
1-3'	+ 750	+ 750	+ 1500	− 790	− 40*
1-2	0	0	0	0	0
2'-3'	− 1250	− 1250	− 2500	+ 1320	+ 70*
2-3	+ 1250	+ 1250	+ 2500	− 1320	+ 3750
3-5	− 1500	− 1500	− 3000	0	− 4500*
2'-5	− 750	− 750	− 1500	− 790	—
4'-5'	− 1250	− 1250	− 2500	− 1320	—
4-5	+ 1250	+ 1250	+ 2500	+ 1320	+ 3820*
4-6	0	0	0	0	0
5'-6	+ 750	+ 750	+ 1500	− 790	—

FIG. 73. TABLE OF STRESSES—TRUSS WITH PARALLEL CHORDS.

The wind load stress diagram for the wind acting from the left is also shown in Fig. 72. The diagram is first drawn using the diagonals which are found to be in tension for dead load, viz.: 2–3 and 4–5. The stresses due to the wind load are shown in Fig. 73. By comparing the dead and wind load stresses, it is seen that the wind tends to cause compression in 2–3, and further that the wind load compression in this member is greater than the dead load tension; therefore, the counter 2′–3′ will act. The stress diagram (Fig. 72) is now revised to include the counter 2′–3′, also the counter 4′–5′; since the latter member will be stressed when the wind acts from the right. Since all the members which may ever act are included in the stress diagram and the truss is symmetri-

cal, it is evident that it is unnecessary to construct the wind load stress diagram for the wind acting from the right. The wind load stresses when the different diagonals are acting are given in Fig. 73. The maximum stresses in all the members are determined by making the combinations indicated in § 127, and are shown in the last column of Fig. 73. From this table, it is seen that the counters $2'$–$3'$ and $4'$–$5'$ are required. These counters are shown by dotted lines in the truss and stress diagrams. In determining the maximum stresses, it is seen that not only the member itself, but also the corresponding member on the other side of the truss must be considered. The stresses shown with a star after them in Fig. 73 are maximum stresses.

For a truss with parallel chords, it is evident that, if the diagonals 2–3 and $2'$–$3'$ act separately, the stresses in them are numerically equal but have opposite signs, i. e., if one is tension, the other will be compression. It is therefore seen that it would not be necessary to actually use the member $2'$–$3'$ in the stress diagrams.

129. Problem 2. Truss With Non-parallel Chords. It is required to find the maximum stresses in all the members of the truss with counterbracing shown in Fig. 74. The span of the truss is 100 ft.; the total height, 37.5 ft.; the height of the vertical sides, 12.5 ft.; the pitch of the roof, one-fourth; and the trusses are spaced 15 ft. apart. The panel points of the lower chord lay on the circumference of a circle of 106.25 ft. radius. The dead load is taken at 10 lbs. per sq. ft. of hor. proj.; the minimum snow load, at 10 lbs. per sq. ft. of hor. proj.; the maximum snow load, at 20 lbs. per sq. ft. of hor. proj.; the wind on the vertical sides of the truss, at 30 lbs. per sq. ft. of surface; and the component of the wind normal to the roof surface, at 23 lbs. per sq. ft. of roof surface. The wind load reactions are assumed parallel to the resultant of all the wind loads.

The dead load stress diagram for one-half of the truss is shown in Fig. 74. In constructing this diagram, it was first assumed that all the diagonals sloped downward toward the

right. From the stress diagram, it is seen that the dead load tends to cause compression in 5′–6′ and 7′–8′; therefore, 5–6 and 7–8 are stressed by the dead load. The stress diagram was then revised to include these members.

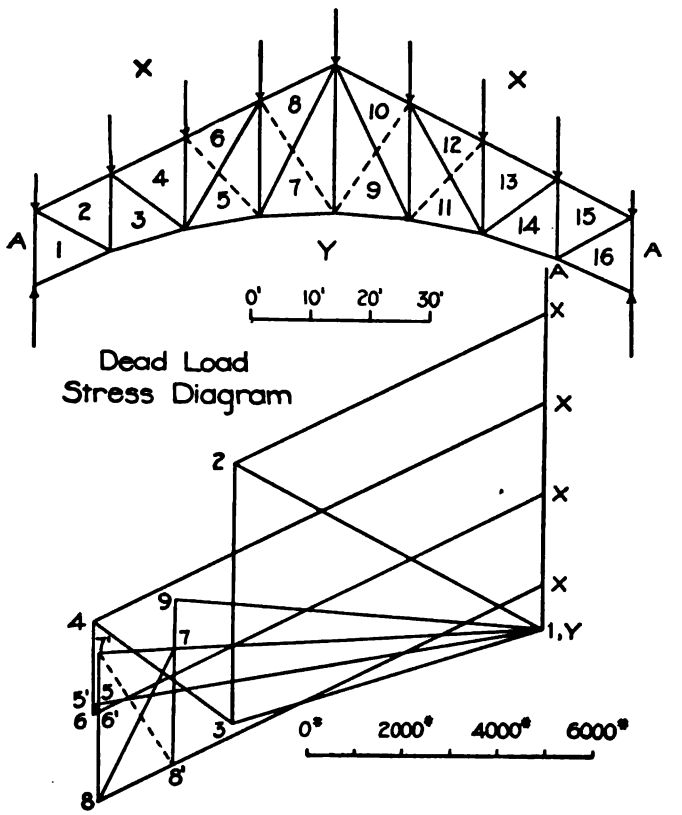


FIG. 74. NON-PARALLEL CHORDS—DEAD LOAD STRESS DIAGRAM.

The minimum and maximum snow load stresses are found by proportion from the dead load stresses without constructing additional diagrams.

The wind load stress diagram, assuming the wind to act upon the left side of the truss, is shown in Fig. 75. This dia-

gram was constructed by first using the diagonals which are in tension due to the dead load.

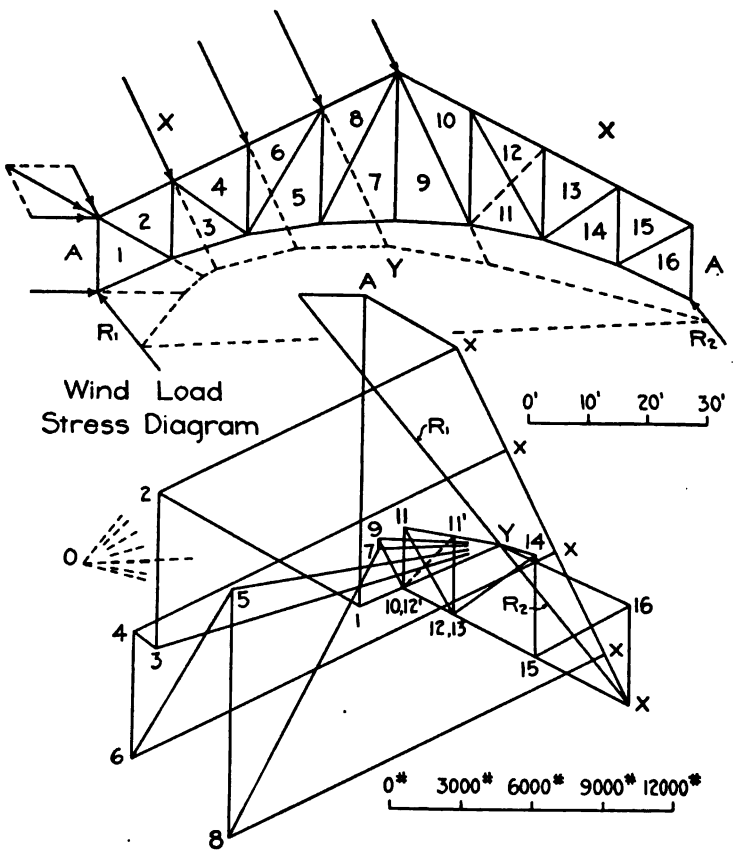


FIG. 75. NON-PARALLEL CHORDS—WIND LOAD STRESS DIAGRAM.

The table of stresses is shown in Fig. 76, the stresses being determined from the diagrams in Fig. 74 and Fig. 75. This table is constructed as follows: First record the stresses in the diagonals, starting with the member 1–2. The dead and wind load stresses are obtained from the stress diagrams, and the minimum and maximum snow load stresses are obtained by direct proportion from the dead load stresses. The dead

and wind loads produce the same kind of stress in the diagonals 1–2, 3–4, 5–6, and 7–8; therefore no counters are required in the left half of the truss when the wind acts towards the right. The dead load stress in 9–10 is + 3500, and the wind load stress in that member is — 2400; therefore no counter is required in that panel. The dead load stress in 11–12 is + 300, and the wind load stress in that member is — 4300. Since the member 11–12 can only take enough wind load compression to neutralize the dead load tension already in it, it is seen that the counter 11′–12′ will be thrown into action. Revise the wind load stress diagram to include the counter 11′–12′, as shown in Fig. 75. Also revise the dead load stress diagram to include the corresponding counter 5′–6′ (if not already included). Record the member 11′–12′ in the table, and tabulate the dead, snow, and wind load stresses. Also record the stresses in the diagonals 13–14 and 15–16. It is seen that the dead and wind loads produce the same kind of stress in these members, therefore no counters are required in these panels. It is thus seen that 11′–12′ is the only counter required in the truss when the wind acts towards the right.

Likewise, tabulate the stresses in the verticals. The stresses in the verticals adjacent to the diagonal 11–12 should be considered, both when the main member 11–12 and when the counter 11′–12′ act, i. e., when 11–12 acts, the verticals are 10–11 and 12–13, and when 11′–12′ acts, the verticals are 10–12′ and 11′–13.

Also, tabulate the stresses in the upper and lower chord members. The stresses in the chord members in the same panel as 11–12 should be considered, both when 11–12 and when 11′–12′ act.

Combine the stresses to determine the maximum stress in each member, as shown in Fig. 76. In making the combinations, it must be borne in mind that, to get the stress in any member, it is necessary to first determine which diagonals are acting for each combination, and to combine the stresses in that particular member when these diagonals are acting. The combinations considered are those indicated in § 127. Since

Truss Member		Dead Load Stress	Snow Load Stress Min.	Snow Load Stress Max.	Wind Load Stress	Maximum Stress
1-2	Diagonals	+ 7200	+ 7200	+ 14400	+ 9700	+ 24100*
3-4	Diagonals	+ 3600	+ 3600	+ 7200	+ 1200	+ 10800
5-6	Diagonals	+ 300	+ 300	+ 600	+ 8400	+ 9000*
7-8	Diagonals	+ 3500	+ 3500	+ 7000	+ 13700	+ 20700*
9-10	Diagonals	+ 3500	+ 3500	+ 7000	− 2400	+ 10500
11-12	Diagonals	+ 300	+ 300	+ 600	− 4300	+ 900
11'-12'	Diagonals	− 200	− 200	− 400	+ 3200	+ 3000*
13-14	Diagonals	+ 3600	+ 3600	+ 7200	+ 4200	+ 11400*
15-16	Diagonals	+ 7200	+ 7200	+ 14400	+ 4500	+ 21600
A-1	Verticals	− 7600	− 7600	− 15200	− 13100	− 28300*
2-3	Verticals	− 5500	− 5500	− 11000	− 6600	− 17600*
4-6	Verticals	− 1900	− 1900	− 3800	− 5400	− 9200*
5-8	Verticals	− 2100	− 2100	− 4200	− 10600	− 14800*
7-9	Verticals	+ 1000	+ 1000	+ 2000	+ 500	+ 3000*
10-11	Verticals	− 2100	− 2100	− 4200	+ 2700	− 6300
10-12'	Verticals	− 1900	− 1900	− 3800	0	− 5700
12-13	Verticals	− 1900	− 1900	− 3800	0	− 5700
11'-13	Verticals	− 1700	− 1700	− 3400	− 3400	− 6800
14-15	Verticals	− 5500	− 5500	− 11000	− 4200	− 16500
A-16	Verticals	− 7600	− 7600	− 15200	− 4200	− 22800
X-2	Upper Chord	− 7100	− 7100	− 14200	− 14000	− 28200*
X-4	Upper Chord	− 10300	− 10300	− 20600	− 17400	− 38000*
X-6	Upper Chord	− 10500	− 10500	− 21000	− 19800	− 40800*
X-8	Upper Chord	− 10300	− 10300	− 20600	− 17500	− 38100*
X-10	Upper Chord	− 10300	− 10300	− 20600	− 10900	− 31500
X-12	Upper Chord	− 10500	− 10500	− 21000	− 8400	− 31500
X-12'	Upper Chord	− 10300	− 10300	− 20600	− 10900	− 31500
X-13	Upper Chord	− 10300	− 10300	− 20600	− 8400	− 30900
X-15	Upper Chord	− 7100	− 7100	− 14200	− 4500	− 21300
Y-1	Lower Chord	0	0	0	+ 6300	+ 6300*
Y-3	Lower Chord	+ 6700	+ 6700	+ 13400	+ 15100	+ 28500*
Y-5	Lower Chord	+ 9300	+ 9300	+ 18600	+ 11300	+ 29900*
Y-7	Lower Chord	+ 7600	+ 7600	+ 15200	+ 5200	+ 22800*
Y-9	Lower Chord	+ 7600	+ 7600	+ 15200	+ 5200	+ 22800
Y-11	Lower Chord	+ 9300	+ 9300	+ 18600	+ 4100	+ 27900
Y-11'	Lower Chord	+ 9500	+ 9500	+ 19000	+ 1900	+ 20900
Y-14	Lower Chord	+ 6700	+ 6700	+ 13400	− 1700	+ 20100
Y-16	Lower Chord	0	0	0	− 6300	− 6300*

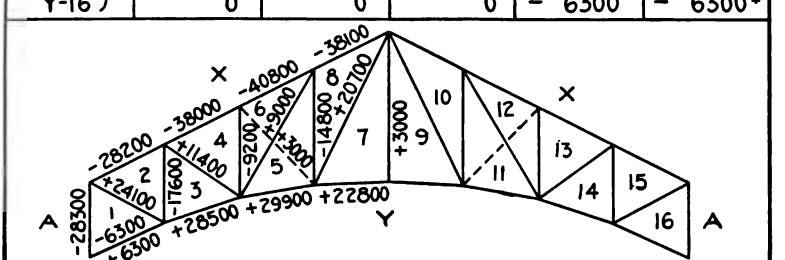


FIG. 76. TABLE OF STRESSES—TRUSS WITH COUNTERBRACING.

the wind is taken as acting towards the right, only, it is necessary to consider the member itself and also the corresponding member on the other side of the center line. The maximum stresses are indicated by stars in Fig. 76.

Referring to the table of stresses, it is seen that the maximum stress in 1–2 is given by the combination of dead, minimum snow, and wind loads; that the maximum stress in 7–9 is given by the combination of dead and maximum snow loads; and that the maximum stress in the counter 11′–12′ is given by the combination of dead and wind loads. The attention of the student is called to the stress Y–11′, which is the stress in the lower chord when the counter 11′–12′ is acting. Unless care is taken to determine which diagonal is acting, the student is liable to make the mistake of combining the dead and maximum snow load stresses, which would seem to give a stress of + 28 500 in Y–11′. However, this stress can never occur, as the main diagonal 11–12 is acting for this combination. When the dead, minimum snow, and wind loads are on the truss, the counter 11′–12′ is acting, and the stress in Y–11′ is then + 20 900, which is the maximum stress in that member when the counter is acting. When the main diagonal 11–12 is acting, the combination of dead and maximum snow loads gives a stress of + 27 900 in Y–11. However, the maximum stress occurs in the corresponding member Y–5 when the dead, minimum snow, and wind loads are acting, and is + 29 900. It is seen that the stress in Y–11 is also + 29 900 when the wind acts towards the left. Referring to Y–1 and Y–16, it is seen that the stresses in these members are + 6300 and — 6300, respectively.

If the direction of the wind is changed and is made to act towards the left, it is seen that Y–1 and Y–16 are subjected to reversals of stress. It is further seen that the counter 5′–6′ is then thrown into action. The diagonals 5′–6′ and 11′–12′ are the only counters required, and Y–1 and Y–16 are the only members which have reversals of stress.

The maximum stresses are shown on the truss diagram in Fig. 76.

In this problem, it has been shown that the only counter required when the wind acts towards the right is 11′–12′. In some trusses, it might happen that when the wind acts towards the right a counter is required in some panel on the left of the center and also in another panel (not the corresponding one) on the right of the center of the truss. In this case, it would be more convenient and would give less cause for errors in making the combinations for maximum stresses if the dead load stress diagram was drawn for the entire truss. The diagram should also be revised to include both the main diagonal and the counter in each panel in which a counter is required when the wind acts towards the right.

ART. 3. STRESSES IN TRUSSES WITH COUNTERBRACING— COMBINED STRESS DIAGRAM.

130. Determination of Stresses in Trusses With Counterbracing — Combined Stress Diagram. In determining the maximum stresses in a truss with counterbracing by means of the combined stress diagram, it is necessary to first find which diagonal in each panel is stressed by the combined loadings. The stress diagram is then constructed, using only those diagonals which are found to be stressed. Two methods will now be given for finding which diagonals are stressed. The first method may be used to advantage for trusses with parallel chords, and the second, for trusses with non-parallel chords.

(1) *Trusses with Parallel Chords.* If the truss has parallel chords, the simplest method for determining which diagonal in each panel is stressed consists of an application of the condition of equilibrium that $\Sigma V = 0$ to the external forces on one side of a section and the members cut by the section. This method will now be explained by means of a problem.

It is required to find which diagonals are stressed in the truss loaded as shown in Fig. 77. The load line, together with the reactions, is shown in Fig. 77, b. To determine whether 2–3 or 2′–3′ is acting, cut the members X–3, 2–3, 2′–3′, and Y–2 by

the section m–m, and apply the condition that $\Sigma V = 0$ to the forces acting upon the shaded portion of the truss. Since the chords are horizontal members and can resist no vertical force, it is seen that the vertical component of the stress in the diagonal 2–3, or in the diagonal 2′–3′, must be equal and opposite in direction to the resultant of the external forces to the left of the section m–m. Now the resultant of these external forces is represented in the load line (Fig. 77, b) by EB, and acts downward; therefore, the vertical component of the stress in the diagonal must act upward for equilibrium. Referring to Fig. 77, it is seen that if a member is in tension, the force it exerts upon the shaded portion of the truss acts away from that portion; and if it is in compression, the force it exerts acts toward the shaded portion. Therefore, if the diagonals are tension members, as they are in the Pratt truss shown in Fig. 77, the member 2–3 will act in tension. In like manner, it may be shown that the diagonal 4–5 is also in action. From the above discussion, it is seen that by observing whether the resultant of the external forces to the left of any panel acts upward or downward and knowing whether the diagonals are tension or compression members, it may be determined at once which diagonal in any panel is stressed. The method described above is called the method of shears.

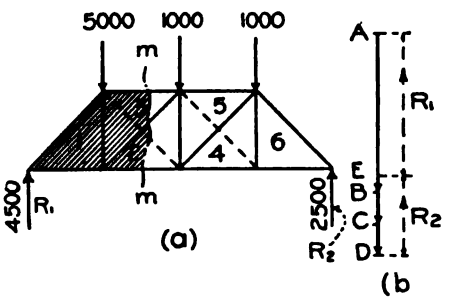


FIG. 77.

The stresses may now be found by constructing the stress diagrams for the combined loads, using only those diagonals which have been found to be stressed. It is seen that this method requires considerable work if several combinations must be made to determine the maximum stresses; as it is necessary to determine for each combination which diagonals are stressed, and then to draw a stress diagram for each combination.

(*2*) *Trusses with Non-parallel Chords.* If the chords of the truss are not parallel, the condition of equilibrium that $\Sigma M = 0$ may be used to determine which diagonals are stressed. The application of this method will now be shown by means of a problem.

It is required to determine which diagonals are stressed in the truss loaded with dead and wind loads, as shown in Fig. 78.

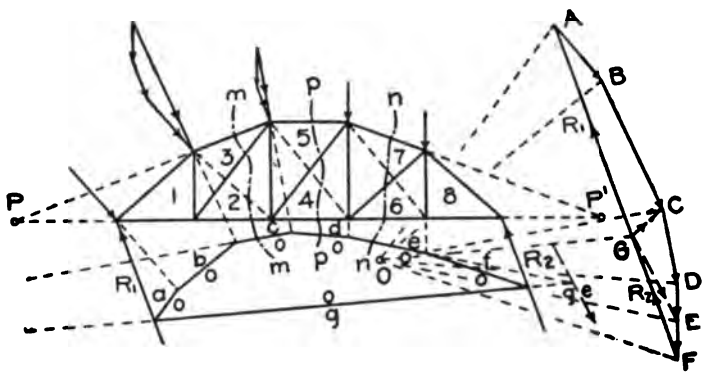


FIG. 78.

The reactions (assumed to be parallel) are first determined by means of the force and funicular polygons, as shown in Fig. 78. To determine whether 2–3 or 2′–3′ is in action due to the combined loads, cut these members, together with the chord members in the same panel, by the section m–m; and consider the members cut and the external forces to the left of the section. Prolong the chord members X–3 and Y–2 until they intersect at P, and take this point as the center of moments. Now from the condition that $\Sigma M = 0$, the moment of the stresses in the members cut by the section must balance the moment of the external forces to the left of the section m–m. But the moment of each chord stress is zero, since its line of action passes through the center of moments; therefore, the moment of the stress in the diagonal 2–3, or the diagonal 2′–3′, must balance the moment of the external forces to the left of the section m–m. The next step is to determine

in which direction the moment of the external forces to the left of m–m tends to produce rotation. Now the resultant of these external forces is represented in the force polygon by GC, acting in the direction from G towards C; and its line of action is through the intersection of the strings oc and og. Although the intersection of these strings falls outside of the limits of the drawing, it is evident that the moment of the resultant of the external forces is clockwise; therefore, the moment of the stress in the diagonal acting at this time must be counter-clockwise. In the truss shown in Fig. 78, the diagonals are tension members; therefore, the diagonal 2–3 is stressed.

In like manner, by using the section n–n and taking the center of moments at P′, it is found that the moment of the resultant GE of the external forces to the left of the section is counter-clockwise; hence the moment of the stresses in the diagonal 6–7, or 6′–7′, is clockwise. The diagonal 6–7 is therefore stressed.

To determine whether 4–5 or 4′–5′ is stressed, cut the members by the section p–p. Instead of using the method of moments for this case, which would necessitate first finding the kind of stress in X–5 or Y–4, the method of shears, described in § 130 (1), will be used. From the force polygon, it is seen that the vertical component of the resultant GD of the external forces to the left of the section p–p acts downward; therefore, the vertical component of the stress in the diagonal must act upward. The diagonal 4–5 is therefore stressed.

The stresses may now be found by constructing the stress diagrams for the combined loads, using only those diagonals which have been found to be in action due to the combined loading.

It is seen that this method may be used to advantage if only a single combination is required to determine the maximum stresses.

CHAPTER XIV.

THREE-HINGED ARCH.

131. Definition. An arch is a structure which has inclined reactions for vertical loads. The only type of arch which will be considered here, and the one commonly used for long span roof trusses, is the three-hinged arch. A three-hinged arch is composed of two simple beams or trusses, hinged together at the crown, and also hinged at their points of support. This is the only form of arch construction which is statically determinate. An example of a three-hinged arch is shown in Fig. 79, a. This structure is composed of two simple trusses, hinged together at the crown C and also hinged at the supports A and B.

The reactions may be obtained by applying the principles which have already been explained. The method of finding these reactions may best be explained by first determining the reactions due to a single load.

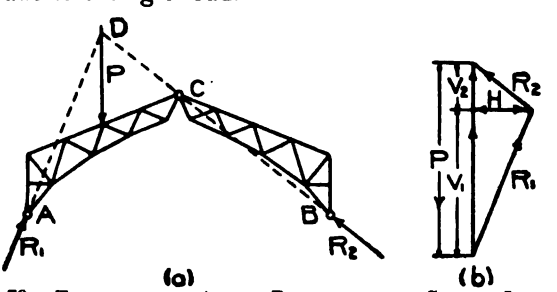


FIG. 79. THREE-HINGED ARCH—REACTIONS FOR SINGLE LOAD.

132. Reactions Due to a Single Load. It is required to find the reactions in the three-hinged arch shown in Fig. 79, a, due to the single load P.

This arch is composed of the two segments AC and BC, and is hinged at the points A, B, and C. The load P is supported by the segment AC, the other segment being unloaded. There are only two forces acting upon the segment BC, the force exerted by the segment AC against the segment BC (acting downward), and the reaction R_2 of the segment BC against its support. Since this segment is held in equilibrium by these two forces, they must have the same line of action and must act in opposite directions.

Now consider the segment AC. This segment is held in equilibrium by three forces, viz.: the reaction R_1 at A, the load P, and the force (acting upward) exerted by the segment BC against the segment AC. This last force is equal in magnitude, but acts in an opposite direction to the force exerted by the segment AC against the segment BC; and is also equal to the reaction R_2. Since the segment is held in equilibrium by these forces, they must intersect at a common point. This point is determined by prolonging the line of action of the reaction, which acts through the points B and C, until it intersects the line of action of the force P at D (Fig. 79, a). The reactions at C will neutralize each other, and the three-hinged arch, taken as a whole, is in equilibrium under the action of the three forces, R_1, P, and R_2. The lines of action of the two reactions R_1 and R_2 being known, the magnitudes of these reactions may be determined by drawing their force polygon. This polygon is shown in Fig. 79, b, the reactions being represented by R_1 and R_2.

The reaction at either support due to any number of loads may be found by first determining that due to each load separately, and then combining these separate reactions.

The method for finding the reactions and stresses due to a number of vertical loads will now be shown.

133. **Reactions and Stresses for Dead Load.** It is required to find the dead load reactions and stresses for the three-hinged arch, loaded as shown in Fig. 80, a. The span of the arch is 125 ft., and its rise is 50 ft. Both the segments AC and BC are alike, and are symmetrically loaded.

To find the reactions, lay off the load line, as shown in Fig. 80, b, choose any pole O, and draw the funicular polygons, one for each segment, as shown in the figure. The vertical reactions at supports A and B and at the hinge C are

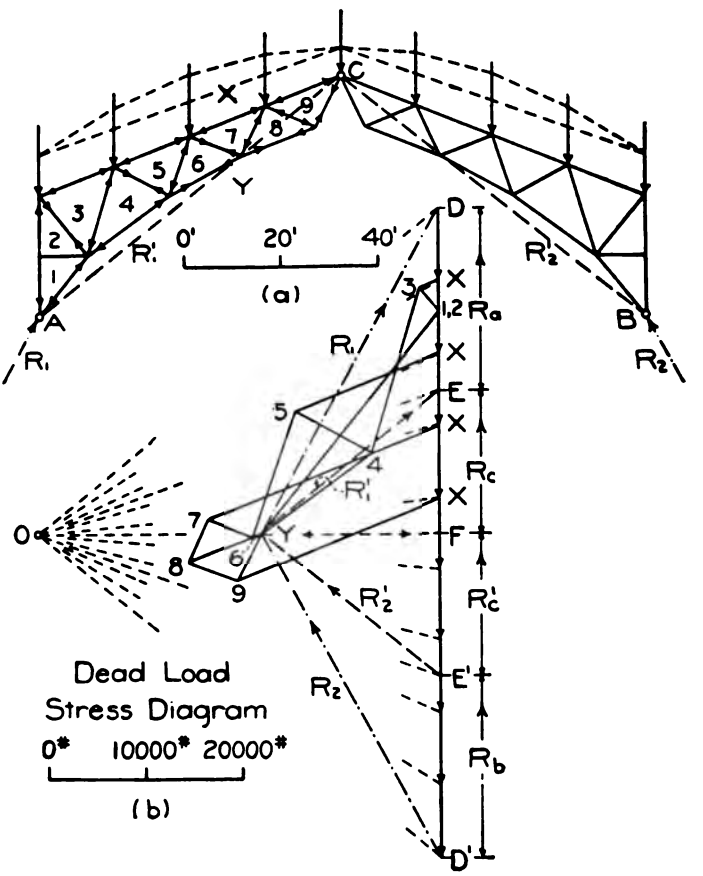


FIG. 80. STRESS DIAGRAM FOR A THREE-HINGED ARCH.

found by drawing the rays from the pole O parallel to the closing strings of these funicular polygons. These reactions are represented by R_a, R_b, and $(R_c + R'_c)$, respectively. Since the truss and the loads are symmetrical about the center line, half of the load at the crown, or R_c, will be transferred

to the left support, and the other half, or R_c, will be transferred to the right support. To determine the reaction at A due to all the loads, consider first the reaction at this point due to the single load R_c acting at C. Since there can be no resisting moment at the hinge C, it follows that the reaction at A caused by this load must also pass through the hinge C, which gives its line of action. Now the vertical component of this reaction is equal to R_c, and its line of action is AC; therefore, its magnitude is given by the line R_1' (Fig. 80, b), drawn from the point E parallel to the line joining the points A and C. The total reaction at A due to all the loads is now determined by combining the vertical reaction R_a with the reaction R_1'. This total reaction is represented by R_1 (Fig. 80, a and Fig. 80, b). The reaction at B is determined in like manner, and is represented by R_2. Taking the equilibrant of the reaction R_1 and of the resultant of the loads acting upon the segment AC, it is seen that the reaction at C is horizontal and acts towards the left. This reaction is represented by FY (Fig. 80, b). In like manner, it may be shown that the reaction at C, due to the loads on the segment BC, is horizontal and acts toward the right, and is represented by YF.

The reactions having been determined, the stresses in the members of the three-hinged arch may be found by drawing the stress diagram, starting the diagram with the forces acting at A. Since the truss and the loads are symmetrical about the center hinge, it is only necessary to draw the stress diagram for one segment. The stress diagram for the segment AC is shown in Fig. 80, b. The magnitudes of the stresses may be determined from the stress diagram, and the kind of stress is indicated by the arrows placed on the members of the truss, as shown in Fig. 80, a.

134. Wind Load Stresses for Windward Segment of Truss. It is required to find the wind load stresses in the windward segment of the three-hinged arch, loaded as shown in Fig. 81, a.

To find the reactions for this segment, consider it as a simple truss supported at the hinges. Construct the force polygon

(Fig. 81, b) for the wind loads, choose any pole O, and draw the funicular polygon, as shown in Fig. 81, a. The reactions are then determined by drawing the ray through the pole O parallel to the closing string of the funicular polygon. These

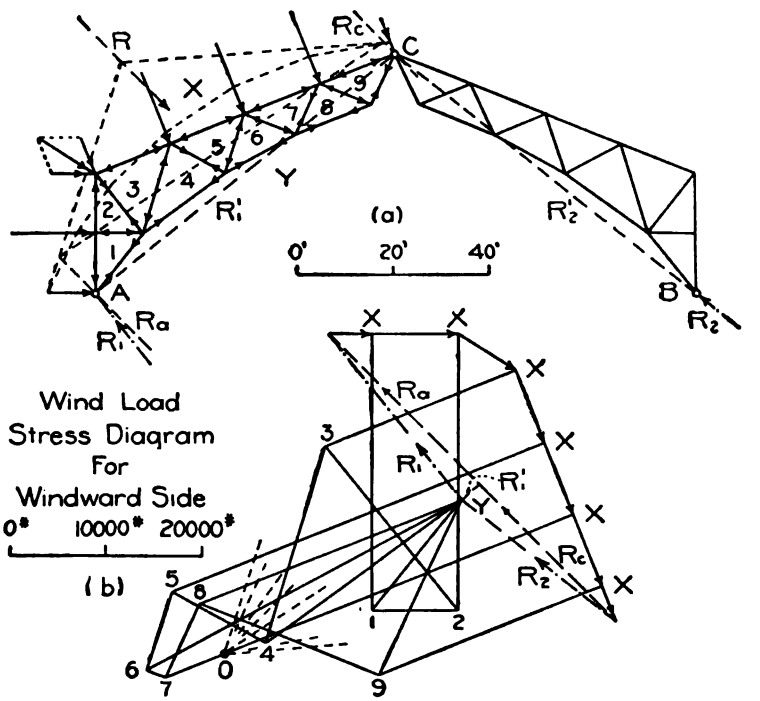


FIG. 81. STRESS DIAGRAM FOR A THREE-HINGED ARCH.

reactions are parallel to the resultant of the wind loads, and are represented by R_a and R_c (Fig. 81, b). Now consider the three-hinged arch as a whole. Since there are no loads on the segment BC, for equilibrium, the right reaction R_2, acting at B, must also pass through the center hinge C. The reaction R_c, which was found by considering the segment AC as a simple truss, will now be resolved into the two reactions R_2 and R_1' (Fig. 81, b), drawn parallel to R_2 and R_1' (Fig. 81, a), respectively. The reaction R_1 is found by drawing the closing line of the force polygon. The arch is held in equilibrium by

the three following forces, viz.: the resultant R of all the wind loads, the reaction R_1 at A, and the reaction R_2 at B. Since the line of action of R_2 is known, that of R_1 may be determined by prolonging R and R_2 until they intersect, and connecting this point of intersection with A. In this problem, the point of intersection of R and R_2 falls outside the limits of the drawing.

The reactions may also be determined, as follows: Since there are no loads on the right segment, the line of action of R_2 must pass through the hinges B and C. Therefore, choose any pole O, and, starting at A, the only known point on the left reaction R_1, draw the funicular polygon for the given truss and loads, closing on the line of action of R_2. The dividing ray, drawn parallel to the closing string, will then determine the two reactions.

The latter method is somewhat simpler than the one shown in Fig. 81.

The reactions having been determined, the stresses in the windward segment may be found by drawing the stress diagram, starting the diagram with the forces acting at A. This diagram is shown in Fig. 81, b. The magnitudes of the stresses may be determined from the stress diagram, and the kind of stress is indicated by the arrows placed on the members of the segment AC.

To determine the maximum and minimum stresses, it is also necessary to find the stresses in the members of the leeward segment BC, and these stresses will be determined in the following section.

135. Wind Load Stresses for Leeward Segment of Truss. It is required to find the wind load stresses in the leeward segment of the three-hinged arch shown in Fig 82, a, the arch and the loads being the same shown as in Fig. 81, a.

To facilitate a comparison of stresses in the windward and leeward segments, the stresses will be found in the same segment AC as in the preceding section.

The wind load reactions are the same as those found in § 134, and are represented in magnitude in Fig. 82, b and in line of action in Fig. 82, a. The wind load stresses are found

by drawing the stress diagram, starting the diagram with the forces acting at A. This diagram is shown in Fig. 82, b. The magnitude of the stresses may be determined from the stress diagram, and the kind of stress is indicated by the arrows placed on the segment AC.

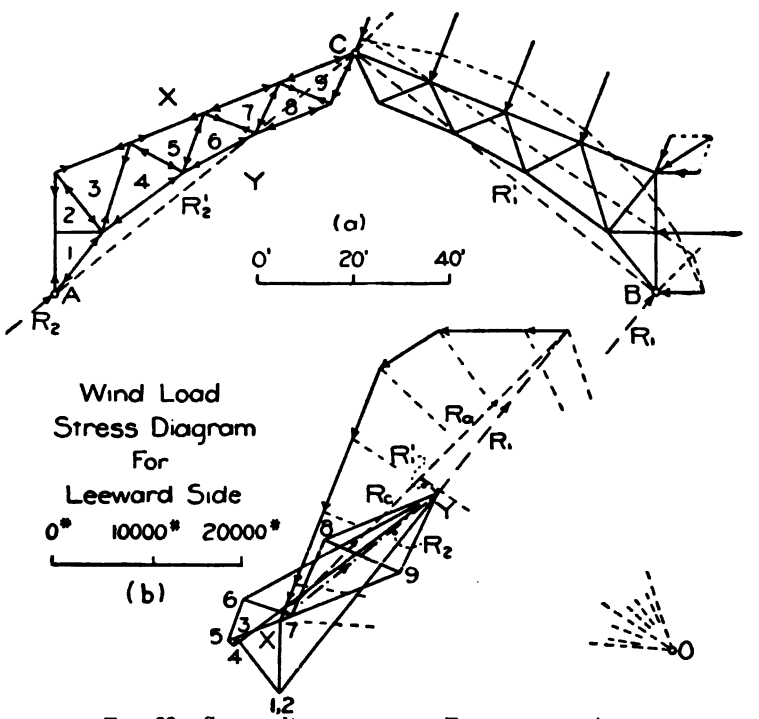


FIG. 82. STRESS DIAGRAM FOR A THREE-HINGED ARCH.

By comparing the wind load stress diagrams and also the kind of stress indicated by the arrows (see Fig. 81 and Fig. 82), it is seen that there are many reversals of stress.

The maximum stresses may be determined by making the combinations indicated in § 127.

CHAPTER XV.

STRESSES IN A TRANSVERSE BENT OF A BUILDING.

136. Construction of a Transverse Bent. It has been assumed in the preceding discussion of roof trusses that the trusses were supported upon walls or pilasters. However, in many types of mill building construction, the trusses are supported by columns, to which they are rigidly connected, thus forming a transverse bent. The columns carry not only the roof trusses and the loads on them, but may also support the side covering and resist the pressure of the wind against the side of the building. In such buildings, the trusses are usually riveted to the columns and are braced by members called knee-braces. The side covering is fastened to longitudinal girts, which are connected to the columns. Additional rigidity is secured by means of a system of wind bracing. This bracing may be placed in the planes of the sides of the building and in the planes of the upper and lower chords of the trusses.

The intermediate transverse bents support a full panel load; while those at the end carry but half a panel load, and are sometimes made of lighter construction. The end bent may be built by running end posts up to the rafters, or the same construction may be used as for the intermediate transverse bents. The latter method is preferable when an extension in the length of the building is contemplated.

137. Condition of Ends of Columns. The stresses in a transverse bent depend to a considerable extent upon the condition of the ends of the columns. Several assumptions may be made, although it is difficult, if not impossible, to exactly realize any of the assumed conditions. The columns may be taken as (1) hinged at the top and base, (2) hinged at the top and fixed at the base, or (3) rigidly fixed at the top and base.

(1) *Columns Hinged at Top and Base.* If the columns merely rest upon masonry piers, or if no effectual attempt is made to fix them at the base by embedding the columns in concrete or by fastening them with anchor bolts, the columns should be taken as hinged at the top and base. The common assumption, and the one which will be made in this text, is that the horizontal components of the reactions due to the wind are each equal to one-half of the horizontal component of the total external wind load acting upon the structure. The vertical components may be found by the method of moments, or by means of the graphic construction shown in § 139.

The maximum bending moment in the column is at the foot of the knee-brace of the leeward column, and is equal to the horizontal component of the reaction multiplied by the distance from the foot of the knee-brace to the foot of the column, i. e., = $H_2 d$ (see Fig. 83).

(2) *Columns Hinged at Top and Fixed at Base.* If the deflections at B, the foot of the knee-brace (Fig. 83), and at C, the top of the column, are assumed to be equal, it may be shown that the vertical components of the reactions are the same as if the columns were hinged at D, the point of contra-flexure. The distance γ from the base of the column to the point of contra-flexure depends upon the relative lengths of h and d. It may be shown that γ varies from $\frac{5}{8}$d when d = $\frac{1}{2}$h, to $\frac{1}{2}$d when d = h. As soon as the position of the point of contra-flexure has been found, the column may be taken as hinged at that point, the wind acting upon the portion below the point of contra-flexure being neglected. The vertical components of the reactions may now be found, and, since the horizontal components are assumed to be equal, the reactions themselves may be determined.

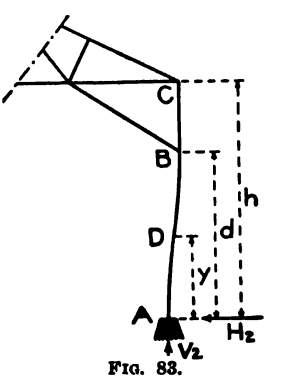


FIG. 83.

The maximum positive bending moment in the column is at the foot of the knee-brace of the leeward column, and is equal

to $H_2\ (d-\gamma)$. The maximum negative moment is at the foot of the column, and is equal to $H_2\gamma$ (see Fig. 83).

(3) *Columns Fixed at Top and Base.* If the columns are fixed at the top and base, the point of contra-flexure is at a distance $\gamma = \frac{d}{2}$ from the base (see Fig. 83). In this case the column may be taken as hinged at the point of contra-flexure, the external wind below this point being neglected. The maximum positive moment is at the foot of the knee-brace, and equals $+\frac{H_2 d}{2}$; and the maximum negative moment is at the base of the column, and equals $-\frac{H_2 d}{2}$.

When an attempt is made to fix the columns, the resulting condition probably lays between that shown in Case 2 and that shown in Case 3. It is seen from Case 2 that γ varies but slightly with a considerable difference in the ratio of d to h, and that it has its minimum value of $\frac{d}{2}$ when $h = d$. In Case 3 it is seen that $\gamma = \frac{d}{2}$. The assumption commonly made when some effective means are used to fix the columns at the base is that $\gamma = \frac{d}{2}$, and this assumption will be made in this text. The horizontal components of the reactions will be taken equal, and the vertical components may be found by moments, or by the method shown in § 139.

138. Dead and Snow Load Stresses. The dead and snow load stresses in the truss of a transverse bent are the same as for a truss supported upon masonry walls. If the columns are hinged at the top, the stresses in them are direct compressive stresses, caused by the loads upon the truss and by the weight of the sides of the building supported by the columns. If the columns are fixed at the top, the deflection of the truss will produce bending moments in the columns and corresponding stresses in the knee-braces. Since the deflection of the truss is usually quite small,

the bending moments in the columns and the stresses in the knee-braces due to vertical loads will be neglected.

The dead load stress diagram for a transverse bent is shown in Fig. 84. The general dimensions of the building and of the transverse bent, together with the loads used, are shown in the figure.

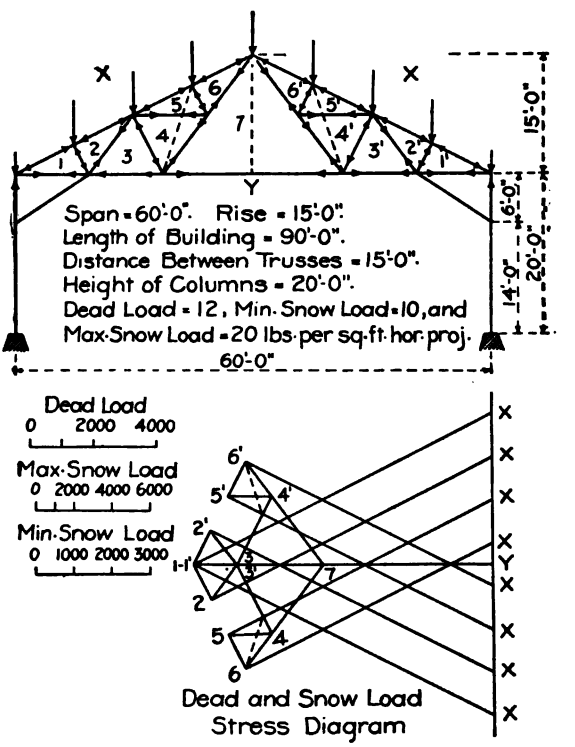


FIG. 84. STRESSES IN A TRANSVERSE BENT.

The snow load stresses may be determined by proportion from the dead load stresses. In this problem the maximum snow, which is used when the wind load is not considered, is taken at 20 lbs. per sq. ft. of horizontal projection; and the minimum snow load, which is used in connection with the wind load, is taken at 10 lbs. per sq. ft. of horizontal projection.

Since the deflection of the truss is neglected, there are no stresses in the knee-braces due to vertical loads.

139. Graphic Method for Determining Wind Load Reactions.* The following is a very convenient graphic method for determining the wind load reactions for a transverse bent.

Lay off ΣW_N (Fig. 85) equal to the total normal wind load acting upon the roof area supported by the transverse bent, its point of application being at the center of the length RN. Also, lay off ΣW_H equal to the total horizontal wind acting upon the side area supported by the column of the bent, its point of appli-

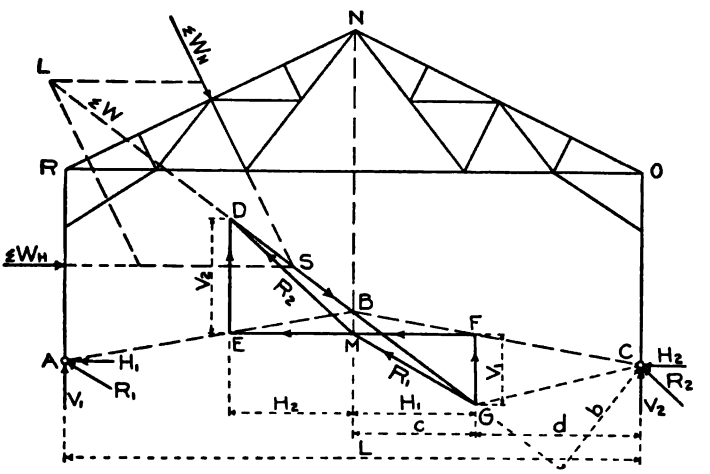


FIG. 85. GRAPHIC METHOD FOR DETERMINING REACTIONS.

cation being at the center of the column length AR. Find the resultant LS = ΣW of the normal and horizontal wind forces, and produce the line of action of this resultant until it intersects at B a vertical line through N, the apex of the truss. Lay off DB = BG = $\frac{1}{2}$LS (since the horizontal components of the reactions are equal, by hypothesis). Join the points A and B, also the points B and C, and from the points D and G, draw the ver-

*This graphic method of determining the reactions is that given in Ketchum's "Steel Mill Buildings."

tical lines DE and GF, respectively. Then ED represents the vertical component V_2 of the right reaction R_2, and GF represents the vertical component V_1 of the left reaction R_1.

This construction may be proved, as follows: Taking the moments of the external forces about the point C, and solving for V_1, we have

$$V_1 = \frac{\Sigma W \times b}{L}, \qquad (1)$$

or, since $BG = \frac{\Sigma W}{2}$ (by construction),

$$V_1 = \frac{4 \text{ (area triangle BGC)}}{L}. \qquad (2)$$

But area triangle $BGC = \frac{FG \times L}{4}$.

For, area triangle BGC = area triangle BGF + area triangle CGF

$$= \frac{FG \times c}{2} + \frac{FG \times d}{2} = \frac{FG \times L}{4}. \qquad (3)$$

Substituting this value of the area of the triangle BGC in equation (2), we have

$V_1 = GF$, which proves the construction.

In like manner, it may be shown that $ED = V_2$.

The left reaction $R_1 = GM$ is now determined by finding the resultant of H_1 and V_1, as shown in the figure. The right reaction $R_2 = MD$ is equal to the resultant of H_2 and V_2.

In the case shown, the columns are taken as hinged at the base. If the columns are fixed at the base, the above construction should be modified by taking the columns as hinged at the point of contra-flexure and neglecting the wind load below this point.

140. Wind Load Stresses—Columns Hinged at Base. The wind load stress diagram for a transverse bent having the columns hinged at the base is shown in Fig. 86. The dimensions of the bent are the same as those shown in Fig. 84. The wind load on the roof is taken at 22 lbs. per sq. ft., which is the normal component of a horizontal wind load of 30 lbs. per sq. ft. (see Fig.

47, Duchemin's formula). The horizontal wind load on the side of the building is taken at 20 lbs. per sq. ft. The distribution of the loads is shown on the truss diagram, and the reactions are determined by the method shown in § 139. Since the reactions act at the bases of the columns, they produce a bending moment at

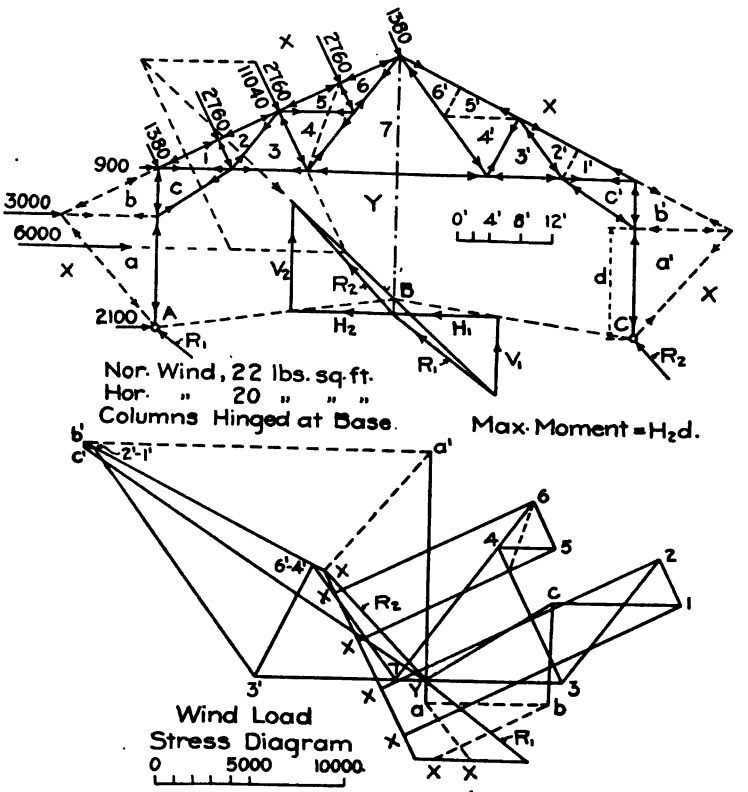


FIG. 86. STRESSES IN A TRANSVERSE BENT—COLUMNS HINGED.

the foot of the knee-brace. The difficulty of drawing the stress diagram due to this moment is overcome by trussing the columns as shown in the figure. The stresses obtained for the members of the auxiliary trusses are not used, and the resulting stresses in the columns are not true stresses. The stresses in all the members of the truss and in the knee-braces are, however, true stresses.

The complete stress diagram is shown in Fig. 86, the kind of stress in each member being indicated by arrows in the truss diagram. The members shown by dotted lines in the truss diagram are not stressed when the wind acts upon the left side of the truss. The true direct stresses in the windward and leeward columns are respectively equal to V_1 and V_2.

The maximum bending moment occurs at the foot of the knee-brace of the leeward column, and is equal to H_2d.

The unit stress in the extreme fiber of the column due to the wind moment may be found from the formula,

$$S = \frac{M\gamma}{I \pm \frac{PL^2}{cE}};^*$$

where

M = maximum bending moment in inch-pounds;

γ = distance from neutral axis to extreme fiber, the axis being perpendicular to the external force causing the bending moment;

I = moment of inertia of the section of the number about an axis perpendicular to the direction of the force causing moment;

P = total direct loading in the member in pounds;

L = length of member in inches.

c = constant depending upon condition of ends of member. For a member hinged at both ends, use c = 10; for member fixed at one end and hinged at the other, use c = 24; and for member fixed at both ends, use c = 32;

Correction, page 157:

"The sign is to be plus if P causes *tension*, and minus if P causes *compression*."

are the same as shown in Fig. 84. The wind load on the roof is taken at 22 lbs. per sq. ft., and on the sides, at 20 lbs. per sq. ft.

It should be noted that in this case the columns are considered as fixed at the points of contra-flexure, and that the wind below

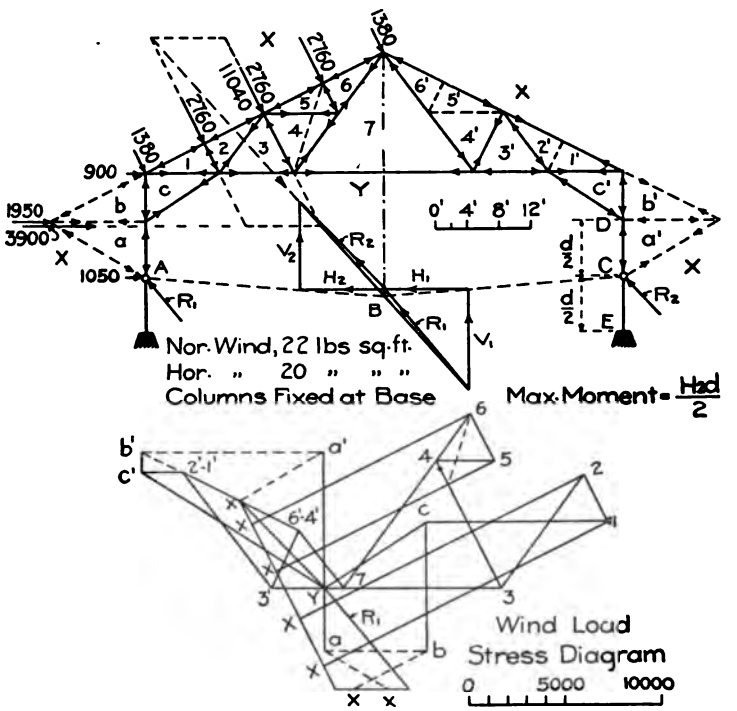


FIG. 87. STRESSES IN A TRANSVERSE BENT—COLUMNS FIXED.

these points is neglected. The remaining solution is similar to that shown in § 140.

The maximum positive bending moment is at the foot of the knee-brace of the leeward column, and is $+\frac{H_2 d}{2}$. The maximum negative bending moment is at the foot of the leeward column, and is $-\frac{H_2 d}{2}$.

The advantages of fixing the columns are shown by a comparison of the stress diagrams given in Fig. 86 and in Fig. 87 and of the maximum bending moments for the two cases. Both stress diagrams are drawn to the same scale.

The unit stress in the column caused by the bending moment may be found by applying the formula shown in § 140.

Since the wind may act from either side, it is seen that many of the members are subjected to reversals of stress.

The maximum and minimum stresses caused by the different loadings may be found by making the combinations indicated in § 127.

CHAPTER XVI.

MISCELLANEOUS PROBLEMS.

142. Stresses in a Grand Stand Truss. It is required to determine the stresses in the grand stand truss shown in Fig. 88, a. In this structure, the knee-brace and the seat beam meet at the point M. Since the left column is braced by the seat beam, all the horizontal component of the wind will be taken by this beam. The right reaction due to the wind loads will therefore be vertical, and there will be no bending moment in this column due to the wind. The wind on the vertical side of the building will not be considered, as it will be resisted by the seat beam and will cause no stresses in the truss. The general dimensions of the truss, together with the loads used, are shown in Fig. 88, a.

The dead load stress diagram is shown in Fig. 88, b. The reactions R_3 and R_3' may be obtained by drawing the force and funicular polygons, or by the method of moments. The kind of stress is indicated by the arrows in the stress diagram, compression being denoted by arrows acting away from each other, and tension by arrows acting toward each other. In this problem, the arrows are placed on the stress diagram, instead of the truss diagram, to avoid confusion; since the same truss diagram is used for all the stress diagrams.

The snow load stress, if considered, may be determined from the dead load stress diagram.

The wind load stress diagram, considering the wind as acting towards the left, is shown in Fig. 88, c. The reactions R_2 and R_2' are determined by means of the force and funicular polygons. In finding these reactions, the resultant ΣW_R of the wind load is used instead of the separate loads to simplify the solution. It is seen that the left reaction has a downward vertical component

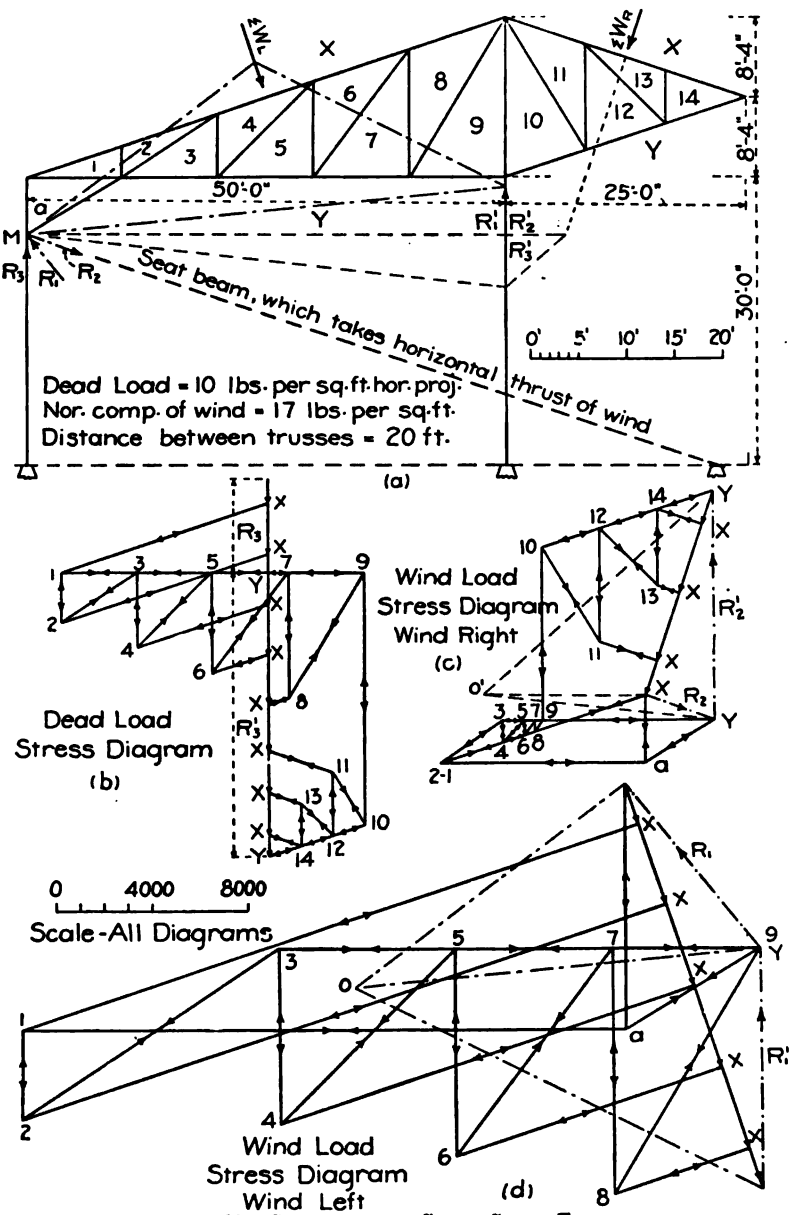


FIG. 88. STRESSES IN A GRAND STAND TRUSS.

due to the wind on the cantilever side of the truss. The stresses may be obtained from the stress diagram shown in Fig. 88, c.

The wind load stress diagram, considering the wind as acting towards the right, is shown in Fig. 88, d. The reactions R_1 and R_1' are determined by means of the force and funicular polygons as shown, the resultant of the wind loads on the left being used instead of the separate loads. The stresses may be obtained from the stress diagram shown in Fig. 88, d.

If the maximum and minimum stresses are required, they may

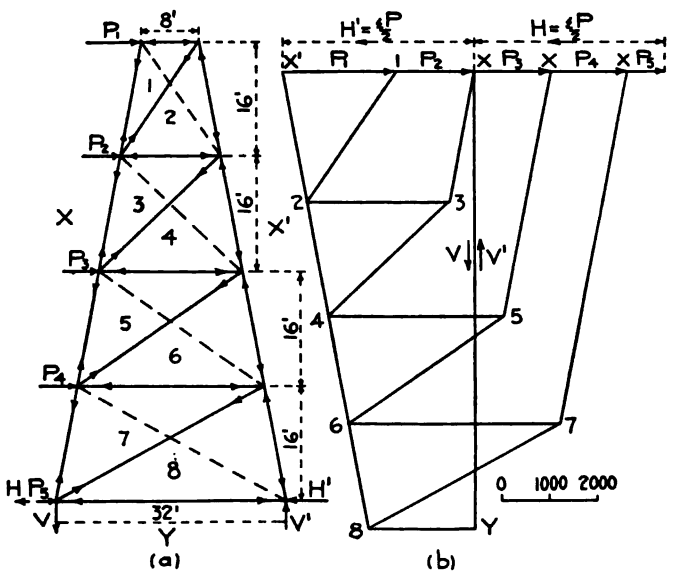


FIG. 89. STRESSES IN A TRESTLE BENT.

be determined by combining the stresses due to the different loadings.

143. Stresses in a Trestle Bent. It is required to find the stresses in the trestle bent, loaded as shown in Fig. 89, a. Since the same detail is used at the bases of the columns, it will be assumed that the horizontal components of the reactions are equal.

The stress diagram for the bent, shown in Fig. 89, b, is drawn

by starting with the force P_1 acting at the top of the bent. The diagram is completed by taking each of the horizontal components of the reactions at the base equal to one-half of the total wind load upon the structure. The stresses may be obtained from the stress diagram, the kind of stress being indicated by the arrows.

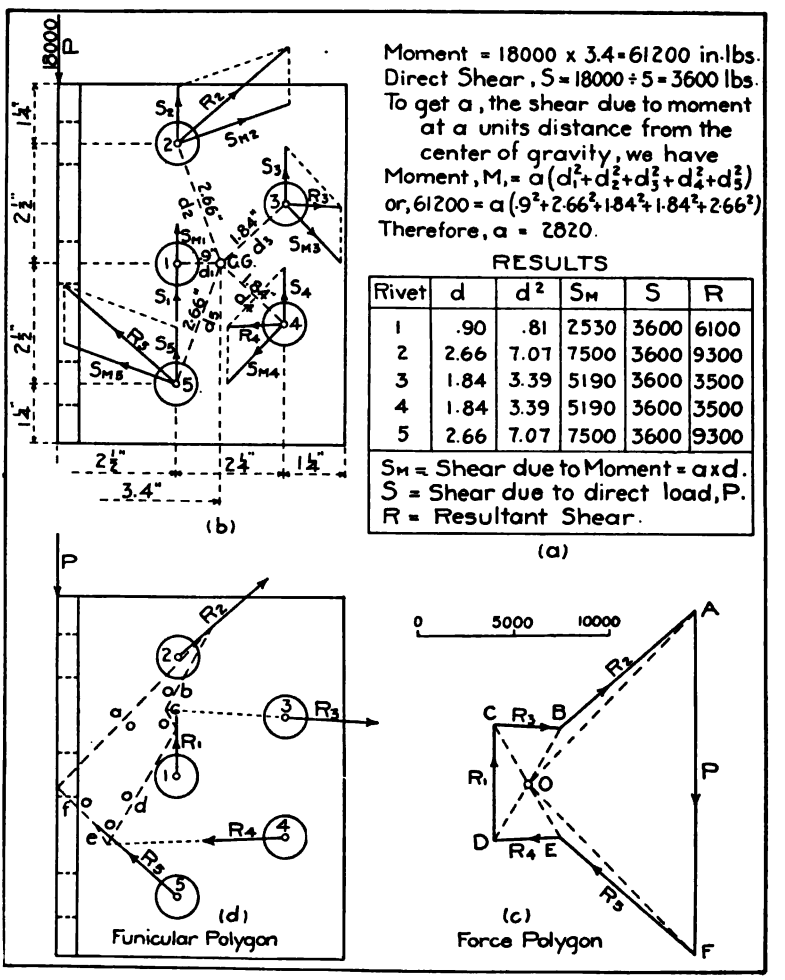


FIG. 90. ECCENTRIC RIVETED CONNECTION.

The vertical components of the reactions are equal, but act in opposite directions.

144. Eccentric Riveted Connection. It is required to find the shearing stress in each of the five rivets in the connection shown in Fig. 90, b. The spacing of the rivets is that used in a six-inch angle for a standard channel connection. The load P is transferred to its connecting member at the left edge of the angle. Since the load is not transferred at the center of gravity of the connecting rivets, it is seen that there is a tendency for rotation, which causes additional shearing stresses in the rivets. The total shearing stress in each rivet may be found by the following method.

Replace the force P by an equal force acting through the center of gravity of all the rivets and a couple whose moment M is equal to P multiplied by the distance from its line of action to the center of gravity of the rivets. In this example, $M = 18\,000 \times 3.4 = 61\,200$ in. lbs. Now the force P acting through the center of gravity will cause a direct shearing stress in each rivet equal to P divided by the number of rivets. In this case, the direct shearing stress is $18\,000 \div 5 = 3\,600$ lbs. The moment M of the couple will cause a shearing stress in each rivet, which may be computed as follows: Since the shear in each rivet due to the moment will vary as the distance of that rivet from the center of gravity of all the rivets, it follows that the resisting moment of each rivet (which is equal to the shear in the rivet multiplied by its moment arm) will vary as the square of its distance from the center of gravity of all the rivets. Now if a represents the shear at a unit's distance from the center of gravity due to the moment M; and d_1, d_2, etc., represent the distances of the respective rivets from the center of gravity, the following relation is true:

$$M = a\,(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2),$$

and

$$a = \frac{M}{\Sigma d^2}.$$

Since a is the shear at a unit's distance due to the moment, the shear on any rivet is equal to a multiplied by its distance from the center of gravity of all the rivets. The total shear in each rivet may now be determined by finding the resultant of the shear due

to the direct load P and that due to the moment of the couple. This resultant may be found by drawing the parallelogram of forces, as shown in Fig. 90, b.

A table showing the important data for this problem, together with the resultant shear on each rivet, is shown in Fig. 90, a. This table shows a convenient form for recording results.

The force polygon for the shearing forces is shown in Fig. 90, c, and the funicular polygon in Fig. 90, d. Since both polygons close, the forces are shown to be in equilibrium.

Referring to the resultant shears given in the last column of the table shown in Fig. 90, a, it is seen that the eccentric connection causes large shearing stresses in some of the rivets. It therefore follows that all eccentric connections should be carefully investigated.

PART III.

BEAMS.

CHAPTER XVII.

BENDING MOMENTS, SHEARS, AND DEFLECTIONS IN BEAMS FOR FIXED LOADS.

This chapter will treat of graphic methods for determining bending moments, shears, and deflections in beams for fixed loads. It will be divided into three articles, as follows: Art. 1, Bending Moments and Shears in Cantilever, Simple, and Overhanging Beams; Art. 2, Graphic Method for Determining Deflections in Beams; Art. 3, Bending Moments, Shears, and Deflections in Restrained Beams.

Graphic methods may be readily applied to the determination of bending moments, shears, and deflections, and in many cases afford a simpler and more comprehensive solution than algebraic processes, especially when these functions are required at several points along the beam.

ART. 1. BENDING MOMENTS AND SHEARS IN CANTILEVER, SIMPLE AND OVERHANGING BEAMS.

145. Definitions. Vertical forces, only, will be considered in this article; as the forces acting upon a beam are usually vertical loads.

Bending Moment. The bending moment at any point, or at

any section, of a beam is the algebraic summation of the moments of all the forces on one side of the point, or section. The bending moment will be considered positive if there is a tendency for the beam to bend convexly downward, and will be considered negative of there is a tendency for the beam to bend convexly upward.

A bending moment diagram is a diagram representing the bending moments at points along the beam due to the given loading.

Shear. The shear at any section of a beam is the algebraic summation of all the vertical forces on one side of the section. The shear is positive if the portion of the beam to the left of the section tends to move upward with reference to the portion to the right of the section. The shear is negative if the portion to the left tends to move downward with reference to the portion to the right of the section.

A shear diagram is a diagram representing the shears at points along the beam due to the given loading.

146. Bending Moment and Shear Diagrams for a Cantilever Beam. Two conditions of loading will be considered for the cantilever beam, viz.: (a) beam loaded with concentrated loads, and (b) beam loaded with a uniform load.

(a) *Cantilever Beam with Concentrated Loads.* It is

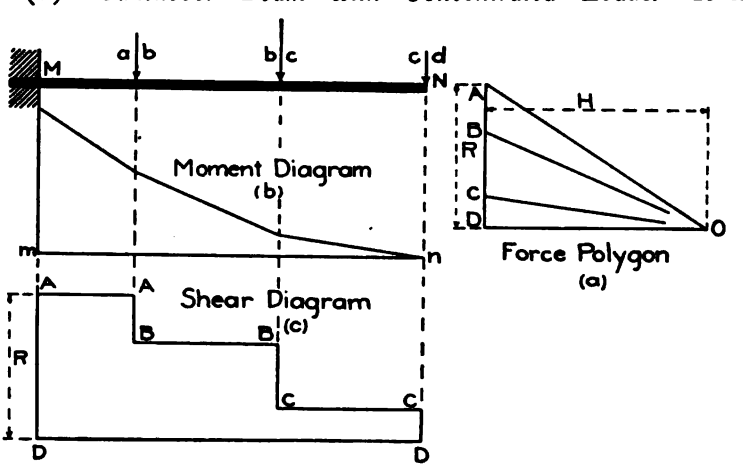


FIG. 91. CANTILEVER BEAM—CONCENTRATED LOADS.

required to draw the bending moment and shear diagrams for the beam MN (Fig. 91), loaded as shown with the three concentrated loads AB, BC, and CD.

Bending Moment Diagram. To construct the bending moment diagram, draw the force polygon (Fig. 91, a), assume any pole O, and draw the funicular polygon (Fig. 91, b) for the given loads. The diagram shown in Fig. 91, b is the bending moment diagram for the beam loaded as shown. For the bending moment at any point along the beam is equal to the intercept under that point, cut off by the funicular polygon and the horizontal line m–n, multiplied by the pole distance H. (See § 64.)

Referring to Fig. 91, it is seen that the maximum bending moment occurs at the fixed end of the beam.

Shear Diagram. To construct the shear diagram, lay off the reaction R = AD (Fig. 91, c), and draw the horizontal line DD. Since there are no loads between the left reaction and the load AB, the shear is constant between the points of application of these forces, and is represented by the intercept between the lines DD and AA. At the point of application of the load BC, the shear is reduced by the amount of that load. The shear is constant between the points of application of the loads AB and BC, and is represented by the intercept between DD and BB. Likewise, the shear between the loads BC and CD is constant, and is represented by the intercept between DD and CC.

Referring to the cantilever beam shown in Fig. 91, it is seen that the maximum bending moment and the maximum shear both occur at the same point—the fixed end of the beam.

(b) *Cantilever Beam with Uniform Load.* It is required to draw the bending moment and shear diagrams for the beam MN (Fig. 93), loaded as shown with a uniform load.

Bending Moment Diagram. Referring to Fig. 92, it is seen

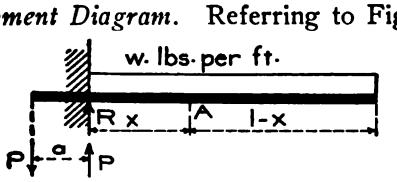


FIG. 92.

that a couple is required to fix the beam at the left end. The magnitude of this couple may be found as follows:

$$\text{Moment of forces to right of A} = \frac{w\,(l-x)^2}{2}, \qquad (1)$$

$$\text{and moment of forces to left of A} = -Pa + Rx - \frac{wx^2}{2}, \qquad (2)$$

which are the equations of a parabola.

For equilibrium, the sum of the moments of the forces on both sides of A must be equal to zero, and the magnitude of the couple, which is Pa, may be found by equating (1) and (2) to zero and solving for Pa, noting that $R = wl$. Then

$$-Pa + Rx - \frac{wx^2}{2} + \frac{w\,(l-x)^2}{2} = 0,$$

or $Pa = \frac{wl^2}{2}$, which is the magnitude of the couple required to fix the beam at the left end. It is seen that the value of P depends upon the arm of the couple.

$$\text{If } x = 0, \quad M = -Pa = -\frac{wl^2}{2},$$

and

$$\text{if } x = l, \quad M = -Pa + \frac{wl^2}{2} = 0.$$

To construct the moment diagram for the beam MN (Fig. 93), divide the load area into any number of equal parts (in this case eight) by verticals, and take the weight of each part as a load acting through its center of gravity. Draw the force polygon (Fig. 93, a) and the funicular polygon (Fig. 93, b) for these loads; and trace a curve (not shown) tangent to the funicular polygon at its ends and at the middle points of its sides. The bending moment at any point along the beam is equal to the intercept, measured between the horizontal line m–n of the funicular polygon and the curve, multiplied by the pole distance H. The greater the number of parts into which the load area is divided, the more nearly will the funicular polygon approach the bending moment parabola.

Shear Diagram. The shear at any point whose distance from the left end of the cantilever beam is x may be represented by the equation, $S = R - wx$. If $x = 0$, $S = R$; and if $x = l$, $S = 0$. Since the load is uniform, the shear decreases by a con-

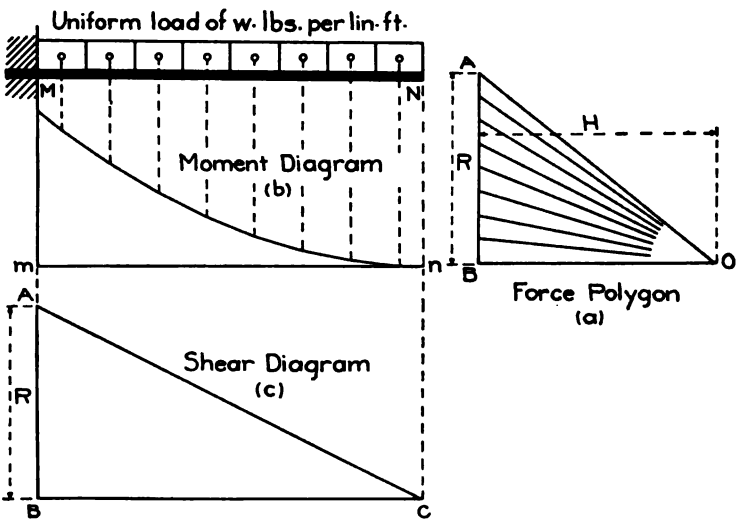


FIG. 93. CANTILEVER BEAM—UNIFORM LOAD.

stant amount towards the right end of the beam. Therefore, to draw the shear diagram, lay off AB (Fig. 93, c) = R, and draw the horizontal line BC. Join the points A and C, completing the triangle ABC, which is the required shear diagram.

Referring to the bending moment and shear diagrams, it is seen that neither the bending moment nor the shear changes sign throughout the length of the beam.

147. **Bending Moment and Shear Diagrams for a Simple Beam.** The bending moment and shear diagram for a simple beam will be constructed for two conditions of loading, viz.: (a) beam loaded with concentrated loads, and (b) beam loaded with a uniform load.

(a) *Simple Beam with Concentrated Loads.* It is required to draw the bending moment diagram and the shear diagram

for the beam MN (Fig. 94), supported at its ends, and loaded with concentrated loads as shown.

Bending Moment Diagram. To construct the bending moment diagram, draw the force polygon (Fig. 94, a), assume any pole O, and draw the funicular polygon (Fig. 94, b), which is the required bending moment diagram. For, the bending moment at any

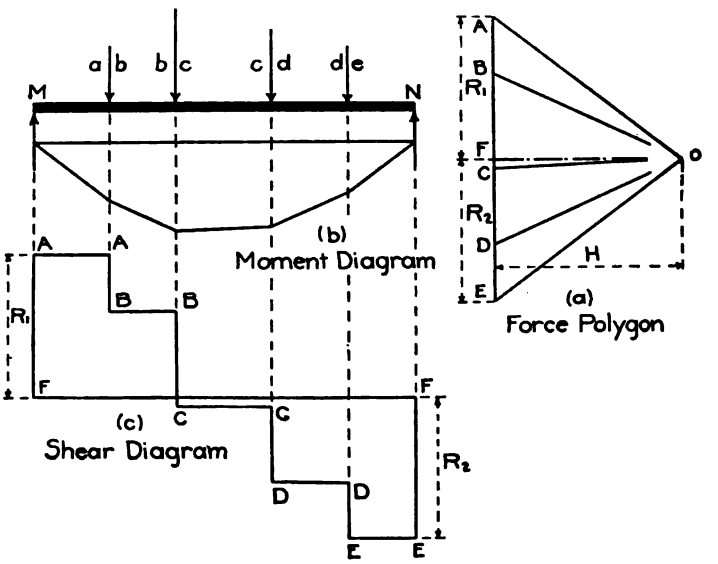


FIG. 94. SIMPLE BEAM—CONCENTRATED LOADS.

point along the beam is equal to the intercept under the point of moments multiplied by the pole distance H. Referring to the bending moment diagram (Fig. 94, b), it is seen that the moment of the forces to the left of any point along a simple beam is positive, and does not change its sign throughout the length of the beam. In this particular example, it is seen that the maximum bending moment occurs at the point of application of the load BC.

Shear Diagram. To construct the shear diagram, lay off $FA = R_1$ (Fig. 94, c), and draw the horizontal line FF. Between the left reaction and the load AB, the shear is equal to R_1; therefore from A, draw the horizontal line AA. Then the intercept measured between FF and AA represents to scale the shear

between the left end of the beam and the load AB. Between the loads AB and BC, the shear is equal to $R_1 - AB$; therefore from A, draw the vertical line AB, representing to scale the force AB, and from B, draw the horizontal line BB. Then the intercept between BB and FF represents to scale the shear between the loads AB and BC. It is seen, by referring to Fig. 94, c, that the shear between the left end of the beam and the load BC is positive. Between the loads CD and DE, the shear equals $R_1 - AB - BC - CD$, and is represented by the intercept between FF and DD. Between the load DE and the reaction R_2, the shear equals $R_1 - AB - BC - CD - DE$, and is represented by the intercept between FF and EE. The shear between the load BC and the right end of the beam is negative.

Referring to Fig. 94, b and Fig. 94, c, it is seen that the maximum bending moment occurs at the point of zero shear, i. e., at the load BC.

(b) *Simple Beam with Uniform Load.* It is required to draw the bending moment and shear diagrams for the beam MN (Fig. 95), supported at its ends, and loaded with the uniform load, as shown.

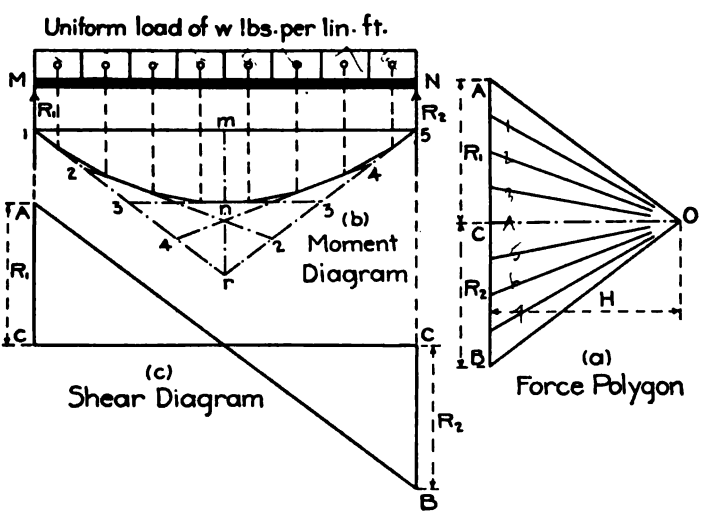


FIG. 95. SIMPLE BEAM—UNIFORM LOAD.

Bending Moment Diagram. To construct the bending moment diagram, divide the load area into any number of equal parts (in this case eight) by verticals, and take the weight of each part as a force acting through its center of gravity. Draw the force polygon (Fig. 95, a) for these loads, assume any pole O, and draw the funicular polygon (Fig. 95, b). Trace a curve (not shown in the figure) tangent to the funicular polygon at its ends and at the middle points of its sides. Then the bending moment at any point along the beam will be equal to the intercept under that point, between the curve and the closing line of the funicular polygon, multiplied by the pole distance H.

The curve drawn tangent to the middle points of the sides of the funicular polygon is a parabola, and the greater number of parts into which the load area is divided, the more nearly will the funicular polygon approaching the bending moment parabola. It will now be shown that the bending moment curve for a uniform load is a parabola.

Let L = span of beam, w = weight of uniform load per linear foot, and x = distance from the left support to the point of moments. Then the bending moment at any point whose distance from the left end is x is, $M = R_1x - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2)$, which is the equation of a parabola. The moment is a maximum when $x = \frac{L}{2}$, and is equal to $\frac{1}{8}wL^2$.

The parabola may be drawn without constructing the force and funicular polygons, as follows: In Fig. 95, b, lay off the ordinate $mn = nr = \frac{1}{8}wL^2$ = moment at the center of the beam; and connect the point r with the points 1 and 5. Divide the lines 1r and r5 into the same number of equal parts, and number them as shown in Fig. 95, b. Join like numbered points by lines, which will be tangents to the required parabola.

Shear Diagram. To construct the shear diagram, lay off $AC = R_1$ (Fig. 95, c), and from C, draw the horizontal line CC. Also, lay off $CB = R_2$ downward from C, and connect the points A and B, which gives the required shear diagram.

It is seen from Fig. 95, b and Fig. 95 c that the shear is zero

at the center of the beam, and that the moment is a maximum at the point of zero shear, i. e., at the center of the beam.

The equation expressing the shear at any point is $S = R_1 - wx = \frac{1}{2}wL - wx = w\left(\frac{L}{2} - x\right)$, which is the equation of the inclined line AB (Fig. 95, c).

It will now be shown that the bending moment at any point along a simple beam is the definite integral of the shear between the point in question and either point of support. For,

$$\int_0^x S = \int_0^x w\left(\frac{L}{2} - x\right) = \frac{w}{2}(Lx - x^2) = M.$$

The above equation shows that the bending moment at any point in a simple beam uniformly loaded is equal to the area of the shear diagram on either side of the point.

148. Bending Moment and Shear Diagrams for an Overhanging Beam. Two conditions of loading will be considered for the overhanging beam, viz.: (a) beam loaded with concentrated loads, and (b) beam loaded with a uniform load.

(a) *Overhanging Beam with Concentrated Loads.* It is

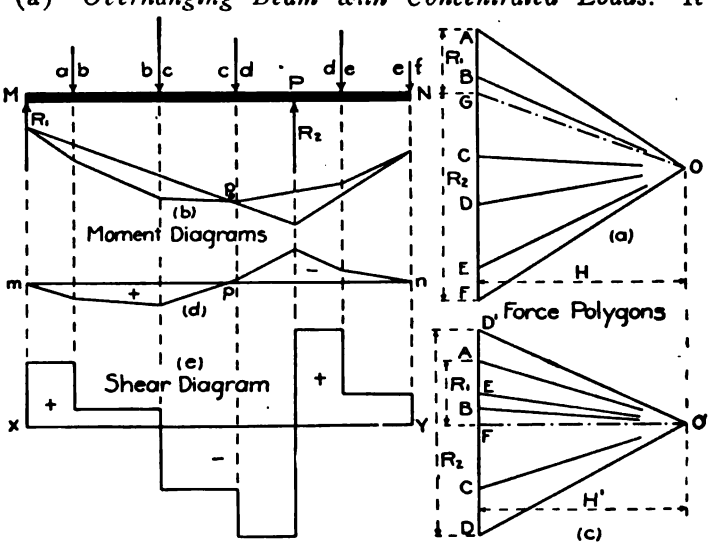


FIG. 96. OVERHANGING BEAM—CONCENTRATED LOADS.

required to draw the bending moment and shear diagrams for the beam MPN (Fig. 96), loaded with concentrated loads.

Bending Moment Diagram. To construct the bending moment diagram, draw the force polygon (Fig. 96, a), assume any pole O, and draw the funicular polygon (Fig. 96, b), which is the required bending moment diagram. The moment at any point along the beam is equal to the intercept under the point multiplied by the pole distance H.

The bending moment diagram shown in Fig. 96, b, is not in a convenient form for comparing moments at different points, and an equivalent diagram will now be drawn which shall have all intercepts measured from a horizontal line. If the line mn (Fig. 96, d), common to the two outside forces R_1 and EF, is made horizontal, it is seen that the intercepts will all be measured from a horizontal line. To draw the bending moment diagram (Fig. 96, d), construct the new force polygon (Fig. 96, c), as follows: Since mn is to be horizontal and common to the forces R_1 and EF, draw the ray O′F (Fig. 96, c) = H (Fig. 96, a), corresponding to the string mn; and from F draw R_1 = FA (already found) upward. From A, draw in succession the loads AB, BC, and CD downward from D; and from D, draw DD′ = R_2 upward. Then from D′, draw the loads D′E = DE and EF downward, closing the force polygon at F. Construct the funicular polygon shown in Fig. 96, d. It is readily seen, since R_1 and EF have a common point F in the force polygon and since FO′ is horizontal, that the funicular polygon, or bending moment diagram (Fig. 96, d), will have intercepts measured from a horizontal line equal to those in Fig. 96, b.

Referring to Fig. 96, b and Fig. 96, d, it is seen that the bending moment for this problem has a maximum positive value at the load BC, that it passes through zero at the point p, and that it has its maximum negative value at P, the point of application of the reaction R_2.

Shear Diagram. The shear diagram for the overhanging beam MN loaded with concentrated loads is shown in Fig. 96, e. Referring to this diagram, it is seen that the shear is positive at the left end of the beam, that it passes through zero at the load

BC, and that it has its maximum negative value between the load CD and the reaction R_2. It is further seen that the shear passes through zero at P, the point of application of the reaction R_2, and that it has its maximum positive value between the reaction R_2 and the load DE.

The maximum moment occurs at the point of zero shear, and since the shear passes through zero at two points, the moment will have maximum values at these points—one of which is the maximum positive moment, and the other, the maximum negative moment.

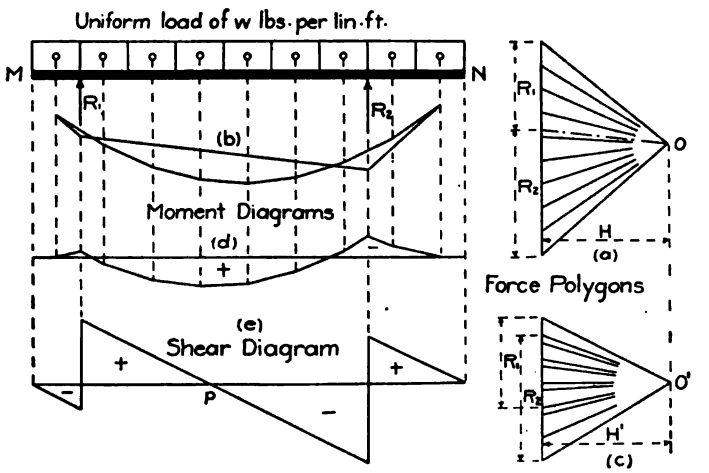


FIG. 97. OVERHANGING BEAM—UNIFORM LOAD.

(b) *Overhanging Beam with Uniform Load.* It is required to draw the bending moment and shear diagrams for the beam MN (Fig. 97). The beam overhangs both supports, and is loaded with a uniform load.

Bending Moment Diagram. To draw the bending moment diagram, divide the load area into any number of equal parts (in this case nine), and assume the weight of each part as a load acting through its center of gravity. Draw the force polygon (Fig. 97, a), and the funicular polygon (Fig. 97, b). Trace a curve (not shown in the figure) tangent to this funicular polygon

at its ends and at the middle points of its sides, which will be the required bending moment diagram.

The equivalent funicular polygon showing the intercepts measured from a horizontal line is shown in Fig. 97, d, and the force polygon is shown in Fig. 97, c. If a curve is drawn tangent to this funicular polygon at its ends and at the middle points of its sides, the diagram will be the required bending moment diagram for the given loads and beam. The bending moment at any point is equal to the intercept under the point of moments multiplied by the pole distance $H = H'$.

Referring to Fig. 97, d, it is seen that the bending moment is negative at both supports, and that it passes through zero between the supports and becomes positive.

Shear Diagram. The shear diagram for the overhanging beam loaded with a uniform load is shown in Fig. 97, e. It is seen that the shear between the left end of the beam and the left support is negative, and that it passes through zero at the left support and becomes positive. The shear passes through zero at p and becomes negative. At the right support it again passes through zero and becomes positive, and is positive between the right support and the right end of the beam.

The shear passes through zero at three points, therefore the bending moment has maxima at these points. The maximum negative moments are at the supports, and the maximum positive moment is between the supports.

Art. 2. Graphic Method for Determining Deflections in Beams.

149. Explanation of Graphic Method —Constant Moment of Inertia. The deflection at any point along a beam may be readily determined graphically, and a graphic method for finding the deflections will now be explained. The graphic method is especially useful when the deflections are required at several points along the beam.

Let MN (Fig. 98, a) be a horizontal beam, supported at its

ends, and let the beam be divided into a sufficient number of parts (in this case four) that the polygon representing its neutral sur-

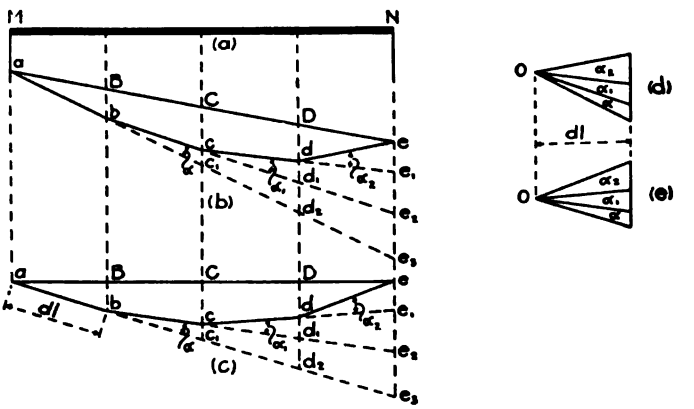


FIG. 98. GRAPHIC DEFLECTIONS.

face may very closely approximate the elastic curve. By the elastic curve is meant the curve assumed by the neutral surface of the beam when the elastic limit of the material is not exceeded. Let ab (Fig. 98, c) be the position taken after flexure by the neutral surface of the segment at the left end of the beam. Also let Fig. 99 represent a portion of a beam which has deflected as shown. The angle α (Fig. 98, c) between bc and bc_1 (the projection of ab) represents the angle of rotation of the section CD (Fig. 99) to C'D', originally parallel to AB. The angle α (Fig. 98, c) also equals the angle α between mn and mo (Fig. 99).

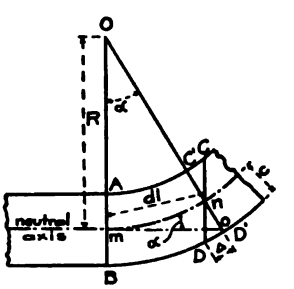


FIG. 99.

Let ab = dl (Fig. 98, c). Then the angle α may be found from the formula $\tan \alpha = \frac{Mdl}{EI}$, where

M = bending moment at b, in inch-pounds,

dl = ab or bc_1,

E = modulus of elasticity of the material, in pounds,

I = moment of inertia of the cross section of the beam, which in this case is assumed to be constant throughout its length.

This formula may be deduced as follows: The stresses at any point in the beam shown in Fig. 99 will vary as the distance of the point from the neutral axis. From the similar triangles Omn and DnD′ (Fig. 99), we have

$$R : c :: dl : \Delta,$$

or,
$$R\,\Delta = c\,dl. \qquad (1)$$

Now let S = stress on extreme fiber, and let E = modulus of elasticity of the material. Then

$$\Delta : S :: dl : E,$$

or,
$$\Delta = \frac{S\,dl}{E}. \qquad (2)$$

Substituting this value of Δ in equation (1), and solving for R, we have

$$R = \frac{E\,c}{S}. \qquad (3)$$

But from the common theory of flexure, we have

$$S = \frac{M\,c}{I}. \qquad (4)$$

Substituting this value of S in equation (3), we have

$$R = \frac{E\,I}{M}. \qquad (5)$$

From Fig. 99, it is seen that $\tan \alpha = \frac{dl}{R}$. (6)

Substituting the value of R found in equation (5) in equation (6), we have

$$\tan \alpha = \frac{M\,dl}{E\,I}. \qquad (7)$$

Calculate the value of the angle α from equation (7), and draw bc. The angles α_1 and α_2 may be found in like manner. Calculate the values of these angles, lay them off at c and d (Fig. 98), respectively, and draw the lines cd and de. Draw the closing line ae, and the vertical ordinates will have their true values in

either Fig. 98, b or Fig. 98, c. For in both Fig. 98, b and Fig. 98, c, we have

at B, $Bb = Bb$,

at C, $Cc = Cc_1 - cc_1 = 2\ Bb - cc_1$,

at D, $Dd = Dd_2 - d_1d_2 - dd_1 = 3\ Bb - 2\ cc_1 - dd_1$,

and at e, $0 = ee_3 - e_2e_3 - e_1e_2 - ee_1 = 4\ Bb - 3\ cc_1 - 2\ dd_1 - ee_1$.

Since cc_1, dd_1, and ee_1 are equal in both Fig. 98, b and Fig. 98, c; therefore Bb, Cc, and Dd must also be equal in both figures. It is thus seen that the closing line ae may be horizontal or inclined without changing the values of the deflection intercepts.

Referring to Fig. 98, it is seen that the successive triangles aBb, bcc_1, cdd_1, and dee_1 have one side equal in each successive pair of triangles. These triangles may therefore be placed in contact, as shown in Fig. 98, d and Fig. 98, e, Fig. 98, d corresponding to Fig. 98, b, and Fig. 98, e to Fig. 98, c. The triangles shown in Fig. 98, d and Fig. 98, e suggest another method of drawing the polygons representing the neutral surface of the beam. Thus with a pole distance dl equal to one division of the span, lay off in succession the distances cc_1, dd_1, and ee_1, computed from the formula $dl \tan \alpha = dl \frac{Mdl}{EI}$. Draw the rays, and construct the funicular polygons shown in Fig. 98, b and Fig. 98, c.

150. Practical Application. The method explained in § 149 will now be applied to a practical problem.

Let MN (Fig. 100) represent a 12 in. × 31.5 lb. × 40 ft. I beam, supported at its ends, and sustaining a load of 4 000 pounds at a point 15 feet from the left end of the beam. It is required to draw the elastic curve representing its neutral surface and find the magnitude of its maximum deflection.

Draw the force polygon shown in Fig. 100, a and the funicular polygon shown in Fig. 100, b. Divide the span into any number of equal parts (in this case eight) by verticals in the moment diagram, and bisect each of these laminæ by a vertical, drawn between the closing line cd and the funicular polygon ced. Consider these verticals as loads, and lay them off successively on the

vertical load line XY (Fig. 100, c). Assume any pole O′, draw rays, and construct a new funicular polygon fgh (Fig. 100, d) with the closing line fg. Draw a curve tangent to the middle

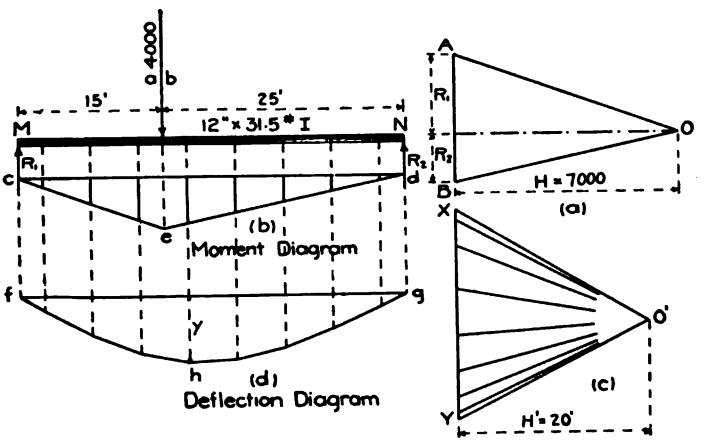


FIG. 100. DEFLECTIONS IN A SIMPLE BEAM—CONCENTRATED LOAD.

points of the sides of this funicular polygon. This curve will then represent the elastic curve of the beam in an exaggerated form, and the intercepts between the funicular polygon and its closing line, which represent the deflections of the beam, are each to be divided by a constant to obtain their true values.

The value of this constant will now be determined. Referring to Fig. 98, b and Fig. 98, c, it is seen that the ordinates cc_1, dd_1, and ee_1 are each equal to dx tan α, which may be written $\frac{Mdx^2}{EI}$; since tan α has been shown equal to $\frac{Mdl}{EI}$, and dl may be taken equal to dx for the elastic curve with a pole distance dx. Now in Fig. 101, a, lay off successive distances on the deflection load line equal to $\frac{M}{H}$, where H = first pole distance; and then construct Fig. 101, b. It is now seen that Fig. 101, a corresponds to Fig. 100, c, and Fig. 101, b to Fig. 100, d. From the similar triangles

bcc_1 and O'12, $cc_1 : \frac{M}{H} :: dx : H'$, from which $cc_1 = \frac{Mdx}{HH'}$. A similar relation is true of the intercepts dd_1 and ee_1. Comparing the values obtained for the verticals cc_1, dd_1, and ee_1 in Fig. 101, a

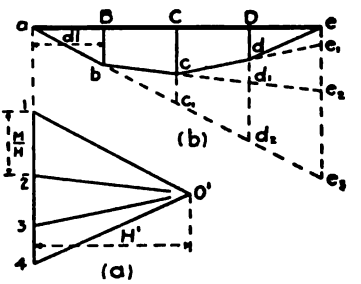


FIG. 101.

with the true verticals cc_1, dd_1, and ee_1 in Fig. 98, b and Fig. 98, c, we find the following relation,

$$\frac{Mdx^2}{EI} \div \frac{Mdx}{HH'} = \frac{HH'dx}{EI}. \qquad (1)$$

Therefore the true verticals may be found from those in Fig. 101 by multiplying the intercepts measured in this figure by

$$\frac{HH'dx}{EI}; \text{ for, } \frac{Mdx^2}{EI} = \frac{Mdx}{HH'} \times \frac{HH'dx}{EI}.$$

The same relation is true for the actual deflection intercepts Bb, Cc, and Dd, which correspond to those in Fig. 100, d; since these are each composed of the elements cc_1, dd_1, and ee_1, as has been previously shown.

In the problem given, $H = 7\,000$ pounds, $H' = 20$ feet $= 240$ inches, $dx = \frac{480}{8} = 60$ inches, the maximum intercept $= 6.8$ feet $= 81.6$ inches, $I = 215.8$, and $E = 29\,000\,000$. Therefore the maximum deflection Δ (in inches) $= \frac{81.6 \times 7\,000 \times 240 \times 60}{215.8 \times 29\,000\,000}$ $= 1.31$.

If the constant by which each measured intercept is to be mul-

tiplied is used in the form $\frac{HH'dx}{EI}$, then the measured intercept, H′, and dx must all be expressed in inches.

If the measured intercept, H′, and dx are expressed in feet, then the constant should be

$$\frac{1728\,HH'dx}{EI}. \qquad (2)$$

The method explained above may be used to find the deflection at any point of a simple beam, a cantilever beam, or a beam overhanging the supports, sustaining either concentrated or uniform loads. If the length of the parts into which the beam is divided is small, this method gives results sufficiently accurate for practical purposes.

151. **Deflection Diagram — Variable Moment of Inertia.** The method of determining deflections, described in § 149 and § 150, is applicable to beams having a constant moment of inertia. For a simple beam, the bending moment increases towards the center, and for an economical design of built-up beams, it is necessary to increase the moment of inertia of the beam correspondingly. When the section of the beam is not constant, the method of determining deflections should be modified; as will now be shown by the solution of a practical problem. The solution which will now be shown gives accurate results.

It is required to draw the deflection diagram for the plate girder shown in Fig. 102. The span of the girder is 62 feet 4 inches, center to center; the distance, back to back of angles, is 6 feet 2 inches; and the girder is loaded with a uniform load of 3 600 pounds per linear foot. The section of the beam at the different points is as shown in the lower part of Fig. 102. The moment of inertia is increased towards the center of the girder by the addition of cover plates.

Draw the force polygon (Fig. 102, a) for the given uniform load. In this case, one-half the total uniform load is assumed to be divided into five parts, and these partial loads are assumed to act through the center of gravity of each part. Assume any pole O and pole distance H, and draw the funicular polygon

(Fig. 102, b). A curve tangent to this polygon will give the moment diagram. Since the girder is symmetrical about its center line, the moment and deflection diagrams need be drawn for only one-half of the span.

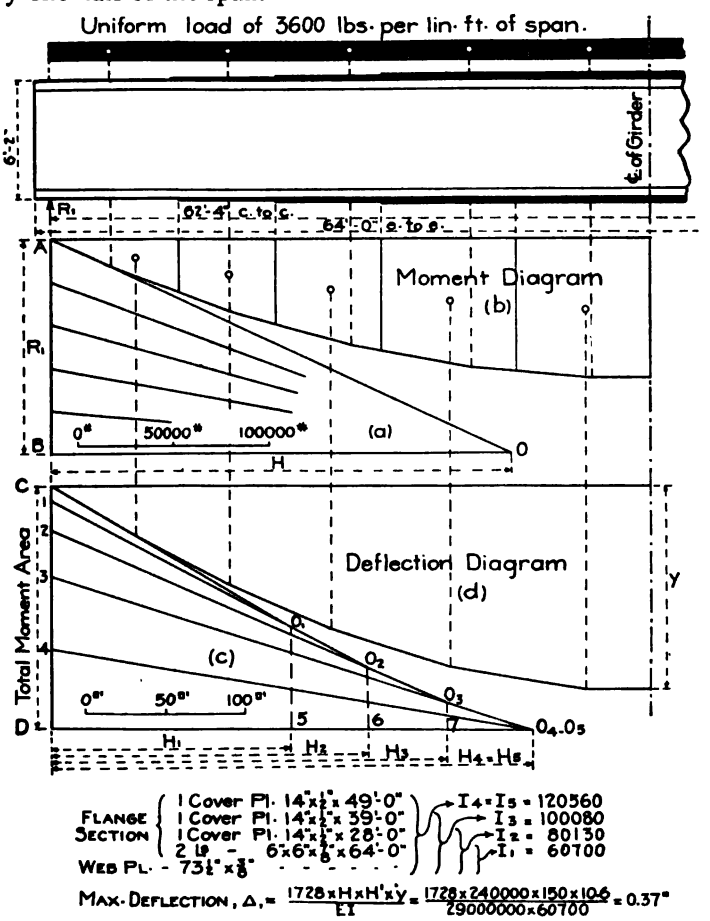


FIG. 102. DEFLECTIONS IN A PLATE GIRDER.

To draw the deflection diagram, instead of dividing the moment area by equidistant verticals as was done in § 150, divide it into laminæ by verticals dropped from the ends of the cover

plates. The moments of inertia between these verticals will then be constant. Compute the area of each lamina in square feet (using scale of beam), locate its center of gravity, and assume a force numerically equal to the area of the lamina to act through its center of gravity. Compute the moments of inertia of the different sections of the beam. The values of these moments of inertia for the given girder are shown in the lower part of Fig. 102. Determine the ratio of the moment of inertia of each section to that at the end of the beam. These ratios (reading from the end of the beam) are equal to 1.00, 1.32, 1.65, and 1.99, respectively. Lay off the moment area to any convenient scale on the load line CD (Fig. 102, c). From D, draw a horizontal line DO_4, and on this horizontal line, lay off the pole distance H_1, using the same scale as for the load line CD. This pole distance may be taken of any convenient length. Now the deflection at any point varies inversely as the moment of inertia of the section at that point. Therefore to draw a deflection diagram whose intercepts shall bear a constant ratio to the true deflections, it is necessary to increase the pole distances in the same ratio that the moments of inertia are increased. Lay off H_2 (Fig. 102, c) $= \frac{I_2}{I_1} \times H_1$; $H_3 = \frac{I_3}{I_1} \times H_1$; and $H_4 = H_5 = \frac{I_4}{I_1} \times H_1$.

Join the point 4 with the point O_4. Also, since the moment of inertia is constant for the two center moment areas, join the point 3 with the same point O_4. From the point 7, draw $7\text{–}O_3$ vertical, to intersect $3\text{–}O_4$ at O_3; and connect the points 2 and O_3. From the point 6, draw $6\text{–}O_2$ vertical, to intersect $2\text{–}O_3$ at O_2; and join the points 1 and O_2. Also, from the point 5, draw $5\text{–}O_1$ vertical, to intersect $1\text{–}O_2$ at O_1; and join the points C and O_1. Using the lines $C\text{–}O_1$, $1\text{–}O_1$, $2\text{–}O_2$, $3\text{–}O_3$, $4\text{–}O_4$, and $D\text{–}O_5$ as rays, draw the funicular polygon (Fig. 102, d). Trace a curve tangent to the polygon, which will give the required deflection diagram.

To get the maximum deflection Δ (at the center), multiply the measured intercept y by $\frac{1728 \times H \times H_1 \times dx}{EI}$. In this case $dx = 1$; since the area was taken instead of the intercept in the

moment diagram. H_1 is always to be taken as the least value for the moment of inertia. Therefore

$$\Delta = \frac{1728 \times 240\,000 \times 150 \times 10.6}{29\,000\,000 \times 60\,700} = 0.37 \text{ inch.}$$

The deflection Δ_1 at any other point along the girder, where the intercept is y_1, may be found by proportion, i. e., if y_1 represents the intercept at the center of the beam, then

$$y_1 : y :: \Delta_1 : 0.37,$$

or $\Delta_1 = \frac{0.37 \times y_1}{y}$. This requires less work than a second substitution in the formula.

Art. 3. Bending Moments, Shears, and Deflections in Restrained Beams.

The following examples of restrained beams will be taken up in this article, viz.: (1) cantilever beam—a beam fixed at one end and free at the other; (2) beam fixed at one end and supported at the other; (3) beam fixed at both ends.

152. Definitions. A restrained beam is a beam fastened at one or more points in such a manner that the beam is not free to deflect at these points.

A beam is fixed at any point if its neutral surface at that point is horizontal.

The bending moment and shear diagrams for a cantilever beam may be drawn without first finding the moment of the fixing couple. In other restrained beams, it is necessary to first find the deflections at the ends, considering the beam as supported at both ends, and then to determine the value of the fixing moment that will make the neutral surface horizontal at the fixed points.

Referring to the overhanging beam shown in Fig. 97, it is seen that the overhanging portions of the beam may be made of such lengths that the beam will be fixed at its supports.

153. (1) Bending Moment, Shear, and Deflection Dia-

grams for a Cantilever Beam. It is required to draw the bending moment, shear, and deflection diagrams for the cantilever beam MN (Fig. 103).

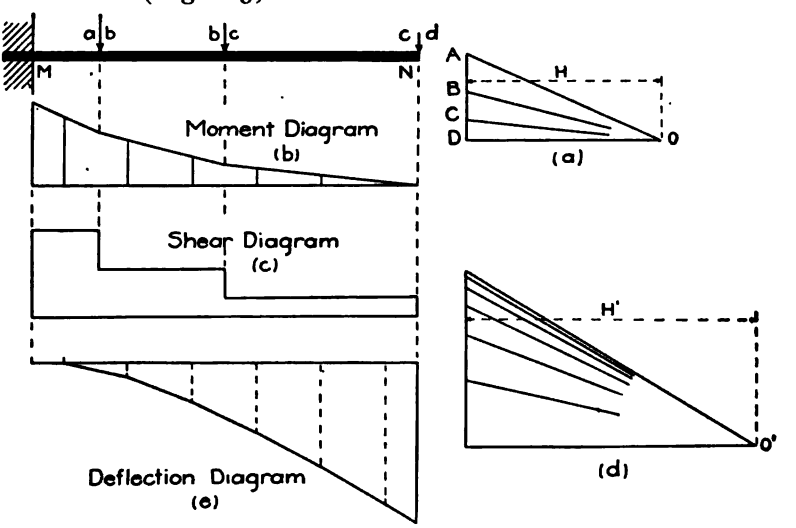


FIG. 103. CANTILEVER BEAM—CONCENTRATED LOADS.

The bending moment and shear diagrams are constructed by the methods explained in Art. 1. The bending moment diagram is shown in Fig. 103, b, and the shear diagram in Fig. 103, c.

To draw the deflection diagram, use the method explained in Art. 2. Divide the moment area (Fig. 103, b) into segments by verticals, and lay off the lengths of these intercepts on a vertical load line (Fig. 103, d). Take any pole O′, and construct the funicular polygon shown in Fig. 103, e. Trace a curve tangent to this funicular polygon, which is the required deflection diagram.

To get the actual deflection at any point, multiply the intercept between the curve and the horizontal line by the constant determined by equation (1), or equation (2), § 150.

154. (2) Bending Moment, Shear, and Deflection Diagrams for Beam Fixed at One End and Supported at the Other. The diagrams will be drawn for the beam loaded (a)

with concentrated loads, and (b) with a uniform load. The construction will first be taken up in detail, and the proof given; after which simplified constructions will be shown.

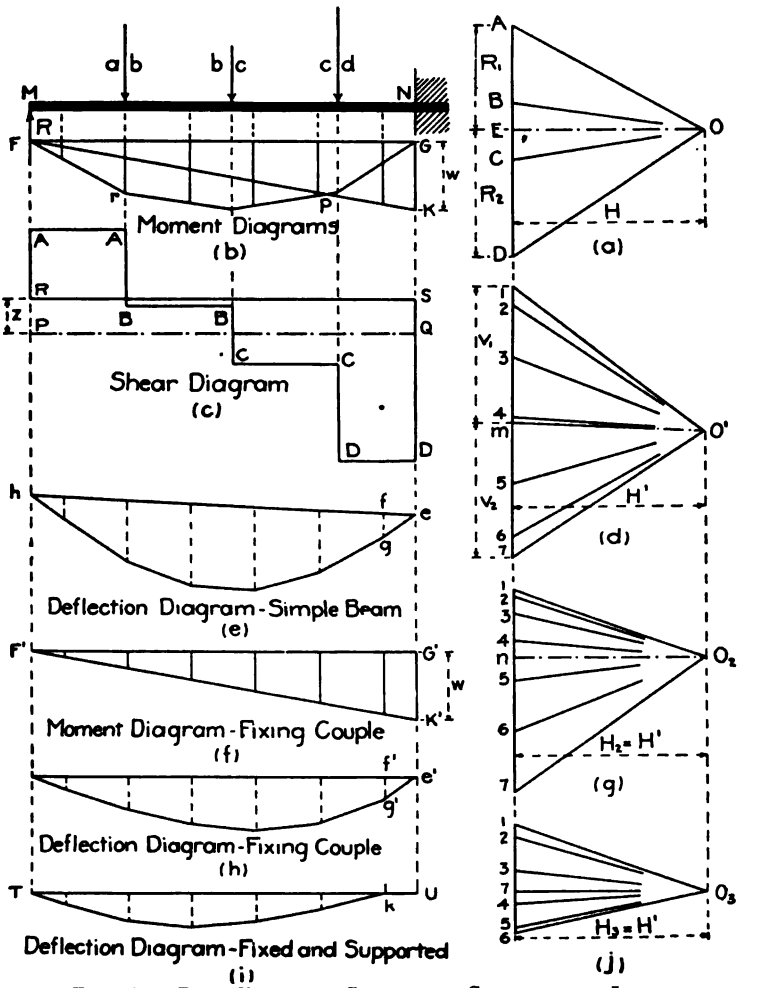


FIG. 104. BEAM FIXED AND SUPPORTED—CONCENTRATED LOADS.

(a) *Beam Fixed and Supported—Concentrated Loads.* It is required to draw the bending moment, shear, and deflection

diagrams for the beam MN (Fig. 104). The beam is fixed at the right end and is supported at the left, and is loaded with the three concentrated loads, as shown.

Assume that the beam is supported at both ends, and draw its bending moment, shear, and deflection diagrams, as in § 150. Also draw the dividing ray O′m (Fig. 104, d) parallel to the closing string eh (Fig. 104, e). The force polygon is shown in Fig. 104, a; the bending moment diagram in Fig. 104, b; the shear diagram in Fig. 104, c; the deflection force polygon in Fig. 104, d; and the deflection diagram in Fig. 104, e.

Since the beam is to be fixed at the right end, assume that a couple whose moment is sufficient to make the neutral surface horizontal is applied at the right end of the beam. Let the triangle F′G′K′ (Fig. 104, f) be the moment area of the couple required to fix the beam at this point. The moment of this fixing couple is equal to wH; where H is the first pole distance, and w is the intercept under the fixed end of the beam, this intercept being the altitude of the moment area for the fixing couple. The value of w may be computed from the formula $w = \frac{3v_2 l'}{S}$; where S = span of beam in feet, l' = length of one division of the moment area in feet, and v_2 = length m–7 (Fig. 104, d). It is seen that w may be found from the above formula as soon as the deflection diagram for the beam treated as supported at both ends has been drawn. Compute w, and construct the bending moment triangle F′G′K′ (Fig. 104, f). Then wH is the moment of the fixing couple at N, and the moment at any point along the beam due to the fixing couple is equal to the intercept under that point multiplied by H.

Construct the deflection diagram for the fixing couple shown in Fig. 104, h. The force polygon, containing the intercepts in the moment triangle (Fig. 104, f), is shown in Fig. 104, g, the pole distance H_2 being taken equal to H′; and the deflection diagram is shown in Fig. 104, h. Since the moment area shown in Fig. 104, f is that for the fixing couple, it is seen that fg (Fig. 104, e) must be equal to f′g′ (Fig. 104, h), i. e., the deflection

caused by the fixing couple must be equal and opposite to that at the end of the beam supported at both ends.

It will now be shown that the altitude G'K' (Fig. 104, f) $= w = \frac{3v_2 l'}{S}$. Assume that the verticals in the triangle F'G'K' were drawn one foot apart. The sum of all the verticals would hen be equal to the area of the triangle. Hence, 1-7 (Fig. 104, g) $= \frac{\text{area of triangle}}{l'} = \frac{Sw}{2l'}$. But $m\text{-}7 = v_2 = \frac{2}{3}(1\text{-}7)$ (Fig. 104, g); as will now be shown. From the similar triangles efg (Fig. 104, e) and O'm7 (Fig. 104, d), ef : O'm :: fg : m-7. Also, from the similar triangles e'f'g' (Fig. 104, h) and O_2n7 (Fig. 104, g), e'f' : O_2n :: f'g' = fg : n-7. But ef = e'f'; therefore m-7 = n-7. Since the moment area from which the intercepts in Fig. 104, g are taken is a triangle, it is seen that m-7 = n-7 = $\frac{2}{3}(1\text{-}7)$. Now $m\text{-}7 = v_2 = \frac{2}{3}(1\text{-}7) = \frac{2}{3}\frac{Sw}{2l'} = \frac{Sw}{3l'}$, from which

$$w = \frac{3v_2 l'}{S}. \qquad (1)$$

To construct the deflection diagram for the beam fixed at the right end and supported at the left, draw TU (Fig. 104, i) horizontal, and from this horizontal line, lay off ordinates equal to the differences between the ordinates in Fig. 104, e and Fig. 104, h. Draw the polygon as shown, and trace a curve tangent to this polygon at the middle points of its sides, which will be the required deflection diagram.

To get the actual deflections, multiply the intercepts in the deflection diagram by the constant given in equation (1), or equation (2), § 150.

Simplified Construction. A simpler construction for the above will now be given. The bending moment diagram, considering the beam supported at both ends, is FrpGF (Fig. 104, b). To draw the diagram for the beam fixed at the right end and supported at the left, from G, lay off GK = w (computed from equation 1), and connect the points F and K, completing the fixing moment triangle FGK. Now the fixing moment tends to

cause rotation in an opposite direction to that caused by the loads on the beam. Therefore the differences between the ordinates in the polygon FrpGF and the triangle GFK will give the ordinates of the bending moment diagram for the beam fixed at one end and supported at the other. This bending moment diagram is FrpGKF. The moment at any point along the beam is equal to the intercept in the diagram under the point multiplied by the pole distance. The point p is the point of contra-flexure, i. e., it is the point where the beam changes curvature and has zero moment. Since the change in curvature of the neutral surface is proportional to the bending moment, it is evident that the deflection diagram for the beam fixed at one end and supported at the other may be drawn directly from its bending moment diagram. To draw the deflection diagram by the method suggested, lay off the intercepts between the line FK (Fig. 104, b) and the broken line FrpG on the deflection load line (Fig. 104, j), noting that intercepts representing positive moments are laid off in one direction, and those representing negative moments, in the opposite direction. From the point 7, draw the horizontal line $7\text{–}O_3$, and with the pole distance $H_3 = H'$, construct the funicular polygon (Fig. 104, i). Trace a curve tangent to this polygon, which will give the required deflection diagram.

The latter method requires fewer constructions, and is more accurate than the former.

It is seen that the fixing moment at N will decrease the reaction at M and increase that at N by an amount z, which is the force represented by the distance between the shear axes PQ (considering the beam supported at both ends) and the shear axis RS (considering the beam fixed at the right end and supported at the left).

To find z, let M_f = the moment of the fixing couple at N. Then $M_f = wH$, and $z = \frac{M_f}{S} = \frac{wH}{S}$, where H = the pole distance and S = the span of the beam. Compute z, and lay it off upward from P. Draw the shear axis RS, which is the shear axis for the beam fixed at the right end and supported at the left.

(b) *Beam Fixed and Supported—Uniform Load.* It is required to draw the bending moment, shear, and deflection diagrams for the beam MN (Fig. 105), fixed at the right end and supported at the left, and loaded with a uniform load.

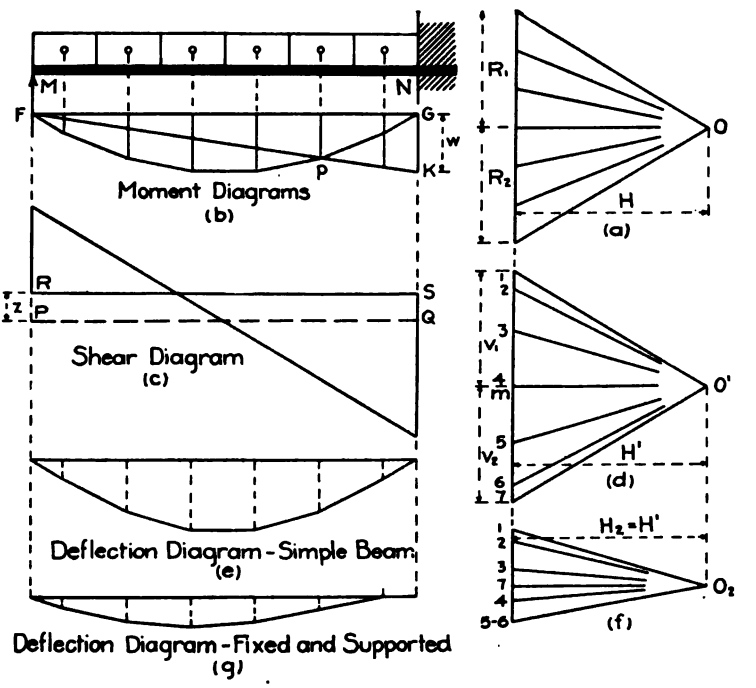


FIG. 105. BEAM FIXED AND SUPPORTED—UNIFORM LOAD.

Divide the uniform load into segments, and assume the weight of each segment as a force acting through its center of gravity. Apply the method explained in the preceding article, and construct the bending moment, shear, and deflection diagrams. Trace curves (not shown in the diagram) tangent to the moment and deflection diagrams, and measure intercepts between the closing lines and these curves. The bending moment diagram is shown in Fig. 105, b; the shear diagram in Fig. 105, c; the deflection diagram, considering the beam as supported at both ends, in Fig. 105, e; and the deflection diagram, for the beam fixed at the right

end and supported at the left, in Fig. 105, g. The intercepts laid off in Fig. 105, f are the distances between the line FK and the

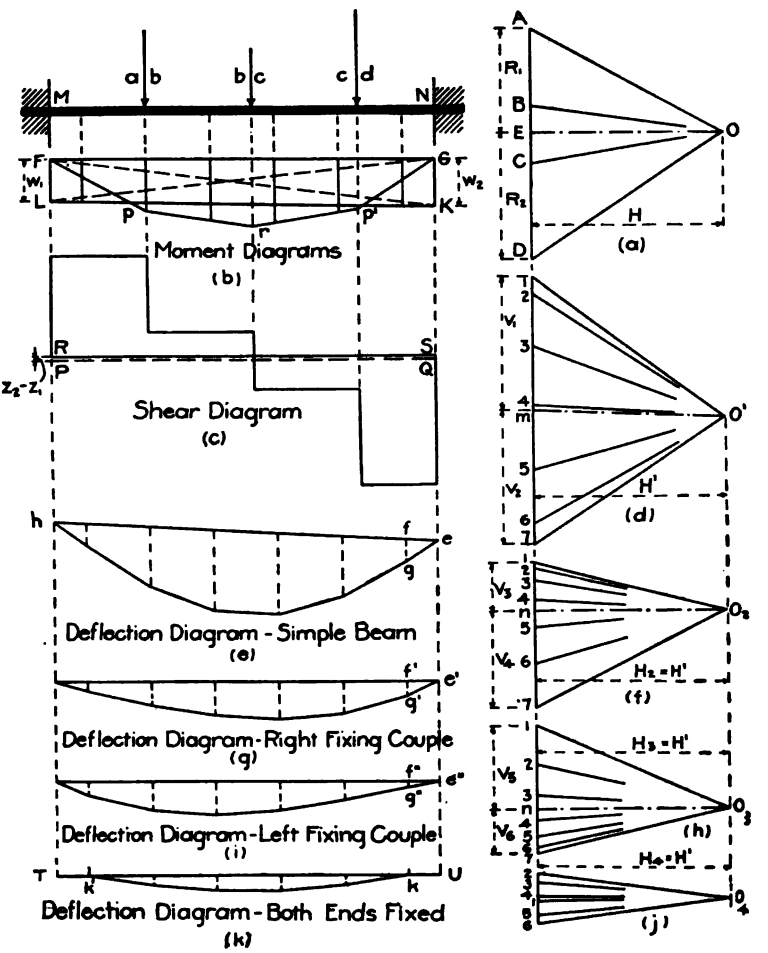


FIG. 106. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOADS.

curve (not shown), drawn tangent to the funicular polygon. The point of contra-flexure is at p (Fig. 105, b).

To determine the actual deflections, multiply the intercepts in

Fig. 105, g by the constant shown in equation (1), or equation (2), § 150.

155. **(3) Bending Moment, Shear, and Deflection Diagrams for a Beam Fixed at Both Ends.** It is required to draw the bending moment, shear, and deflection diagrams for the beam MN (Fig. 106), fixed at both ends, and loaded with concentrated loads, as shown.

Construct the bending moment, shear, and deflection diagrams, considering the beam as supported at both ends, and draw the ray O′m (Fig. 106, d), dividing the deflection load line at m into v_1 and v_2.

Since the beam is fixed at both ends, assume a couple to act at each end whose moment is sufficient to make the neutral surface horizontal at these points. The moment M_1, which fixes the right end of the beam, is represented by the moment area triangle FGK (Fig. 106, b); and the moment M_2, which fixes the left end, is represented by the moment area triangle GFL (see preceding section). Draw the force polygon (Fig. 106, f), for the moment triangle FGK, and construct its deflection diagram (Fig. 106, g). Draw the dividing ray O_2n parallel to the closing string, which divides the load line into v_3 and v_4. Also draw the force polygon (Fig. 106, h) for the moment triangle GFL, and construct its deflection diagram (Fig. 106, i). Draw the dividing ray O_3n, which divides the load line into v_5 and v_6.

The angles 1O′m and mO′7 (Fig. 106, d) and their corresponding angles in the deflection diagram represent the deflections at the ends of the beam, considered as supported at both ends. Likewise, the angles $1O_2n$ and $nO_2\,7$ (Fig. 106, f) and their corresponding angles in the deflection diagram represent the deflections at the ends of the beam for the right fixing couple; and $1O_3n$ and nO_37 and their corresponding angles in the deflection diagram represent the deflections for the left fixing couple. Since the pole distances H′, H_2 and H_3 are all equal, it is seen that v_1, v_2, v_3, v_4, v_5, and v_6 are respectively proportional to these angles. To fix the beam at the ends, i. e., to make the neutral surface horizontal at the ends, v_1 must equal $v_3 + v_5$, and v_2 must equal $v_4 + v_6$.

The values of w_1 and w_2 (Fig. 106, b) may be found, as follows: As has been shown in § 154, the length of the deflection load line 1–7 (Fig. 106, f) for the moment triangle FGK $= \frac{Sw_2}{2l'}$; and this line 1–7 is divided by the ray O_2n into one-third and two-thirds its length. Therefore v_3, which equals $\frac{1}{3}$ (1–7), $= \frac{Sw_2}{6l'}$; and v_4, which equals $\frac{2}{3}$ (1–7), $= \frac{Sw_2}{3l'}$. In like manner, considering the deflection load line 1–7 (Fig. 106, h) for the moment triangle GFL, it may be shown that

$$v_5 = \frac{Sw_1}{3l'}, \text{ and } v_6 = \frac{Sw_1}{6l'}. \qquad (1)$$

Now,
$$v_1 = v_3 + v_5 = \frac{Sw_2}{6l'} + \frac{Sw_1}{3l'}, \qquad (2)$$

and
$$v_2 = v_4 + v_6 = \frac{Sw_2}{3l'} + \frac{Sw_1}{6l'}. \qquad (3)$$

Solving equations (2) and (3) for w_1 and w_2, we have

$$w_1 = \frac{2l'}{S}(2v_1 - v_2), \qquad (4)$$

and
$$w_2 = \frac{2l'}{S}(2v_2 - v_1). \qquad (5)$$

To get the bending moment at any point for the beam fixed at both ends, compute w_1 and w_2 from the above formulæ, and lay them off downward from F and G (Fig. 106, b), respectively. Join the points L and K. Then the moment area FGKL represents the bending moment of the fixing couples at both ends of the beam. Since the polygon Fprp'GF represents the bending moment, considering the beam supported at both ends, it is seen that the bending moment for the beam fixed at both ends is represented by the moment area Fprp'GKLF. The moment at any point is equal to the intercept in this diagram multiplied by the pole distance H.

The points p and p′ are points of contra-flexure.

To draw the deflection diagram (Fig. 106, k) for the beam fixed at both ends, construct the deflection load line (Fig. 106, j) by laying off on this load line distances equal to the intercepts between the line LK (Fig. 106, b) and the broken line Fprp'G. The intercepts 6–1 and 1–2 are laid off in an upward direction, while 2–3, 3–4, 4–5, and 5–6 are laid off in a downward direction. From the point 1, with the pole distance $H_4 = H'$, draw the horizontal line $1\text{–}O_4$. Construct the funicular polygon (Fig. 106, k), and draw a curve tangent to the polygon at the middle points of its sides, which will give the required deflection diagram. It is seen that the neutral surface is horizontal at the ends of the beam. To obtain the actual deflections, multiply the intercepts in Fig. 106, k by the constant shown in equation (1), or equation (2), § 150.

The deflection diagram might also have been drawn by subtracting from the intercepts in Fig. 106, e the sum of the corresponding intercepts in Fig. 106, g and Fig. 106, i; and laying off their differences from the horizontal line TU.

The former method is preferable, and is subject to less error than the latter. When the former described method is used, the diagrams shown in Fig. 106, f, Fig. 106, g, Fig. 106, h, and Fig. 106, i need not be drawn except to prove the constructions.

To construct the shear diagram, it is necessary to find the effect upon the reactions due to the fixing moment at each end.

Let z_1 = decrease in the reaction at M due to the fixing moment at N,

and z_2 = decrease in the reaction at N due to the fixing moment at M.

Then $z_2 - z_1$ = distance (to scale of forces) between shear axes PQ and RS (Fig. 106, c).

Now $z_1 = \frac{w_1 H}{S}$, and $z_2 = \frac{w_2 H}{S}$ (see § 154 for proof).

Therefore, $$z_2 - z_1 = \frac{H}{S}(w_2 - w_1). \qquad (6)$$

If z_2 is greater than z_1 (as in this problem), $z_2 - z_1$ is to be

measured upward from PQ (the shear axis for the beam supported at both ends); and if z_1 is greater than z_2, then $z_2 - z_1$ is to be measured downward from PQ.

Draw the shear diagram and shear axis PQ, considering the beam as supported at both ends, lay off $z_2 - z_1$ upward from PQ, and draw the shear axis RS, which is the shear axis for the beam fixed at both ends.

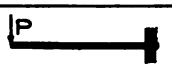
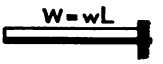
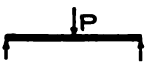
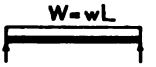
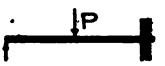
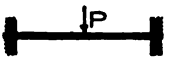
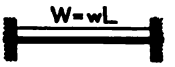
Beam & Loading	Max. Moment	Max. Deflection
	$-PL$	$\frac{PL^3}{3EI}$
	$-\frac{WL}{2}$	$\frac{WL^3}{8EI}$
	$+\frac{PL}{4}$	$\frac{PL^3}{48EI}$
	$+\frac{WL}{8}$	$\frac{5WL^3}{384EI}$
	$+\frac{5PL}{32}, -\frac{3PL}{16}$	$\frac{0.0093\,PL^3}{EI}$
	$+\frac{9WL}{128}, -\frac{WL}{8}$	$\frac{0.0054\,WL^3}{EI}$
	$+\frac{PL}{8}, -\frac{PL}{8}$	$\frac{PL^3}{192EI}$
	$+\frac{WL}{24}, -\frac{WL}{12}$	$\frac{WL^3}{384EI}$

Fig. 107. Formulae for Maximum Moments and Deflections.

156. Algebraic Formulae. In Fig. 107 are given several algebraic formulæ for determining the maximum bending moments and maximum deflections in beams for some of the simpler forms of loading.

CHAPTER XVIII.

MAXIMUM BENDING MOMENTS AND SHEARS IN BEAMS FOR MOVING LOADS.

This chapter will treat of the determination of the maximum bending moment and maximum shear at any point in a simple beam loaded with moving loads; and of the position of the moving loads for a maximum bending moment or maximum shear.

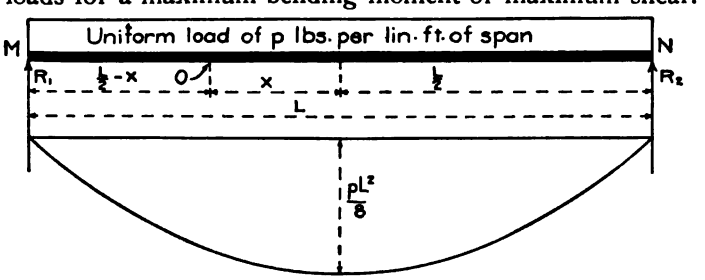


FIG. 108. SIMPLE BEAM—UNIFORM LOAD.

157. Beam Loaded with a Uniform Load. (a) *Maximum Bending Moment.* It is required to find the maximum bending moment at any point O (Fig. 108) of a beam loaded with a uniform load. First assume the beam to be unloaded, and then assume a uniform load to move onto the beam. The bending moment at every point along the beam is increased with each addition of the uniform load until it reaches its maximum value at every point when the beam is fully loaded. It has been shown in § 147, b that the bending moment diagram for a beam fully loaded with a uniform load is a parabola with a maximum ordinate at the center of the beam equal to $\frac{1}{8}pL^2$. The maximum bending moment diagram for the beam MN is shown in Fig. 108.

The bending moment at any point O (Fig. 108), at a distance x from the center of the beam is

$$M = \frac{pL^2}{8} = \frac{px^2}{2}. \qquad (1)$$

The above equation may be written

$$M = \frac{p}{2}\left(\frac{L^2}{4} - x^2\right) = \frac{p}{2}\left(\frac{L}{2} + x\right)\left(\frac{L}{2} - x\right). \qquad (2)$$

This equation expressed in words is: The bending moment at any point in a beam uniformly loaded is equal to one-half the load per foot multiplied by the product of the two segments into which the beam is divided by the given point.

The maximum bending moment at any point along a beam uniformly loaded may be readily found from equation (2).

(b) *Maximum Shear.* It is required to find the maximum shear at any point O of the beam MN (Fig. 109), loaded with a uniform load.

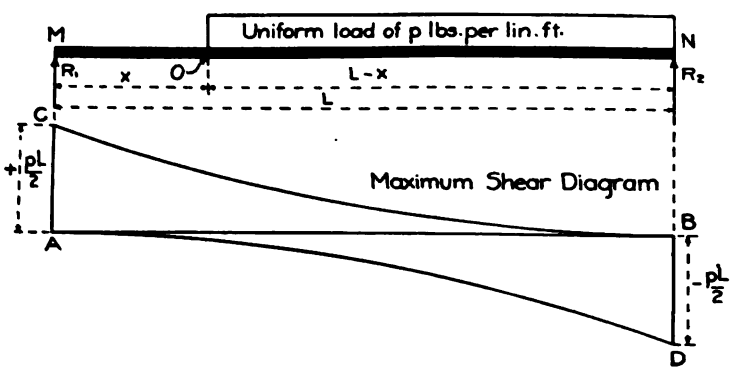


FIG. 109. SIMPLE BEAM—UNIFORM LOAD.

It has been shown in § 147, b that the equation expressing the shear in a beam fully loaded with a uniform load is $S = p\left(\frac{L}{2} - x\right)$, where p is the uniform load per foot, L the length of the beam, and x the distance from the left end of the

beam to the point where the shear is to be determined. It is seen from this equation that when the beam is fully loaded the shear has its maximum positive value $+\frac{pL}{2}$ when $x = 0$; that it has its maximum negative value $-\frac{pL}{2}$ when $x = L$; and is zero when $x = \frac{L}{2}$.

Now it is seen that the load to the left of any point O (Fig. 109) decreases the positive shear at that point by the amount of the load, while it increases the left reaction by a less amount. Since the shear is equal to the reaction minus the load to the left, it will have its maximum value at any point when there is no load to the left of the point. Therefore, to get a maximum positive shear at any point in a beam due to a uniform load, the segment to the left of the point should be unloaded and that to the right fully loaded.

The equation for a maximum positive shear is

$$S = R_1 = \frac{p\,(L-x)\,(L-x)}{2L} = \frac{p}{2L}(L-x)^2. \qquad (3)$$

This is the equation of the parabola CB, which has its vertex at the right end of the beam. The maximum ordinate is at $x = 0$, the left end of the beam, and is equal to $\frac{pL}{2}$.

To get the maximum negative shear at any point, the portion to the left of the point should be fully loaded and that to the right unloaded. The equation expressing the maximum negative shear is

$$S = -R_2 = -\frac{px^2}{2L}, \qquad (4)$$

which is the equation of the parabola AD.

158. Beam Loaded with a Single Concentrated Load. (a) *Maximum Bending Moment.* Let the beam MN (Fig. 110) be

loaded with a single moving load P. The maximum bending moment at any point O occurs when the load is at that point; for a movement of the load to either side of the point decreases the

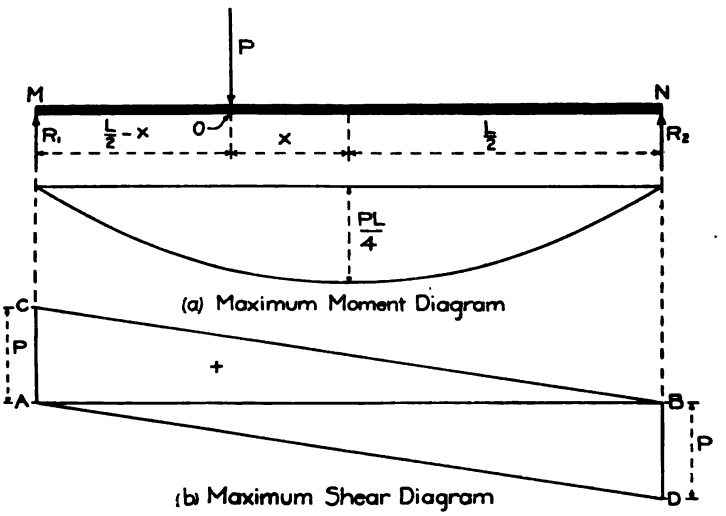


FIG. 110. SIMPLE BEAM—CONCENTRATED MOVING LOAD.

opposite reaction and hence the bending moment. The equation expressing the maximum bending moment at any point due to a single moving load is

$$M = \frac{P\left(\frac{L}{2}+x\right)\left(\frac{L}{2}-x\right)}{L}. \qquad (5)$$

This is the equation of the parabola (Fig. 110, a), which has its maximum ordinate equal to $\frac{PL}{4}$ when $x = 0$.

If $\frac{2P}{L}$ is substituted for P in equation (1), it is seen that we have equation (5). Therefore, the bending moment due to a single concentrated load is the same as for twice that load uniformly distributed over the beam.

(b) *Maximum Shear.* To get the maximum positive shear

at any point O (Fig. 110) due to a single moving load, the load should be placed an infinitesimal distance to the right of the point. (For all practical purposes it may be considered at the point.) The maximum shear may be expressed by the equation

$$S = \frac{P\left(\frac{L}{2} + x\right)}{L}, \tag{6}$$

which is the equation of the straight line CB. The positive shear is a maximum when $x = \frac{L}{2}$, i. e., at the left end of the beam.

The maximum negative shear occurs when the load is just to the left of the point, and may be expressed by the equation

$$S = \frac{-Px}{L}, \tag{7}$$

which is the equation of the straight line AD.

159. **Beam Loaded with Concentrated Moving Loads.** (a) *Position for Maximum Moment.* Let MN (Fig. 111) be a beam loaded with concentrated moving loads at fixed distances apart. (For simplicity, three loads are here taken, although any number

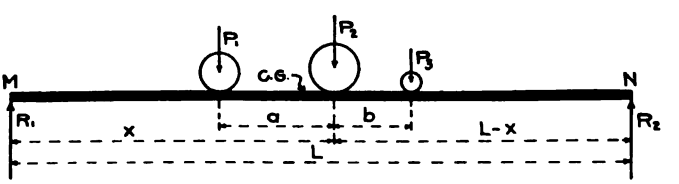


FIG. 111. POSITION FOR MAXIMUM MOMENT—MOVING LOADS.

might have been considered.) It is required to find the position of the loads for a maximum bending moment in the beam, together with the value of this moment.

The determination of the position of any number of moving loads for a maximum bending moment at any point in a beam will be treated in § 208.

Let x be the distance of one of the loads P_2 from the left end

of the beam when the loads are so placed that they produce a maximum moment under P_2. Now if the bending moment is found, and its first derivative placed equal to zero, the position of the loads for a maximum bending moment may be determined. To determine the bending moment, first find the reaction R_1 by taking moments about the right end of the beam. Thus,

$$R_1 = \frac{P_1 (L - x + a) + P_2 (L - x) + P_3 (L - x - b)}{L}$$

$$= \frac{(P_1 + P_2 + P_3) (L - x) + P_1 a - P_3 b}{L}, \qquad (8)$$

and the bending moment under P_2 is

$$M = R_1 x - P_1 a$$

$$= \frac{x (P_1 + P_2 + P_3) (L - x) + x (P_1 a - P_3 b)}{L} - P_1 a. \qquad (9)$$

Differentiating equation (9) and placing it equal to zero, we have

$$\frac{dM}{dx} = \frac{(P_1 + P_2 + P_3) (L - 2x) + P_1 a - P_3 b}{L} = 0. \qquad (10)$$

Solving equation (10) for x, we have

$$x = \frac{L}{2} + \frac{P_1 a - P_3 b}{2 (P_1 + P_2 + P_3)}. \qquad (11)$$

The position of the wheel P_2 for a maximum bending moment in the beam is determined by equation (11). For, $P_1 a - P_3 b =$ the moment of the loads on the beam about P_2, and $\frac{P_1 a - P_3 b}{P_1 + P_2 + P_3} =$ distance from P_2 to the center of gravity of all the loads; hence

$$x = \frac{L}{2} + \frac{1}{2} \text{ (distance from } P_2 \text{ to c. g. of all the loads).}$$

Therefore, the criterion for a maximum bending moment under the load P_2 is: *The load P_2 must be as far from one end of the beam as the center of gravity of all the loads is from the other end.*

Since P_2 is any one of the moving loads, it is seen that theoretically this criterion must be applied to, and the bending moment found for, each of the loads; and the greatest value taken as the maximum bending moment. However, it may often be determined by inspection which load will give a maximum moment. If some of the loads are heavier than others, the maximum moment will occur under one of the heavier loads.

If x had been measured from the center of the beam instead of from the ends, the following equivalent criterion for the maximum bending moment would have been found: For a maximum bending moment under the load P_2, this load must be as far to one side of the center of the beam as the center of gravity of all the loads is to the other side of the center.

The methods employed for determining the bending moment for fixed loads may be applied to moving loads as soon as their position for a maximum bending moment has been found.

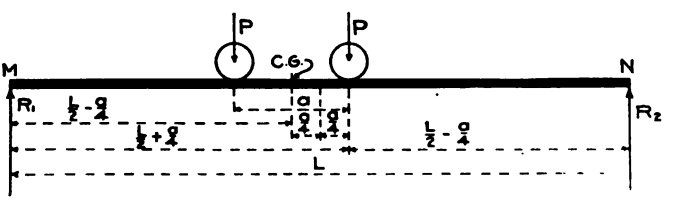


FIG. 112. POSITION FOR MAXIMUM MOMENT—TWO EQUAL LOADS.

A special case of the above, which often occurs, is that of a beam loaded with two equal loads at a fixed distance a apart (Fig. 112). Applying the criterion for a maximum moment to this case, it is seen that x (measured from the end) $= \frac{L}{2} + \frac{a}{4}$,

or if measured from the center $= \frac{a}{4}$.

Placing one of the loads P at a distance $\frac{a}{4}$ from the center of the beam, and taking moments about the left end (noting that the loads are equal), we have

$$R_2 = \frac{P\left(L - \frac{a}{2}\right)}{L},$$

and the maximum moment M is

$$M = R_2\left(\frac{L}{2} - \frac{a}{4}\right) = \frac{P\left(L - \frac{a}{2}\right)\left(\frac{L}{2} - \frac{a}{4}\right)}{L}$$

$$= \frac{\left(L - \frac{a}{2}\right)^2}{2L}. \qquad (12)$$

This equation is true for the ordinary values of a and L, but does not give a maximum bending moment when $a > 0.586L$. When $a > 0.586L$, one of the wheels placed at the center of the beam (the other being then off of the beam) will give a maximum bending moment, as will now be shown.

Assume that one of the two equal loads P is placed at the center of the beam, and that the distance a between the loads is greater than 0.5L. The maximum bending moment is then equal to $\frac{PL}{4}$. Equating this to the value found for the maximum bending moment in equation (12), we have

$$\frac{PL}{4} = \frac{P\left(L - \frac{a}{2}\right)^2}{2L}.$$

Solving for a, we have

$$a = 0.586L. \qquad (13)$$

Therefore, when $a = 0.586L$, the moment due to a single load P at the center of the beam is the same as the moment given by placing the loads according to the criterion for maximum moments. When $a > 0.586L$, the maximum bending moment for two equal loads at a fixed distance apart is given by placing one of the loads at the center of the beam; and the criterion for a maximum bending moment does not apply.

If the two loads are unequal, the maximum moment will always occur under the heavier load.

(b) *Position for Maximum Shear.* For two equal loads, the maximum end shear will occur when both loads are on the span and when one of the loads is at an infinitesimal distance from the end of the beam; since the reaction will be a maximum for this condition. For a maximum shear at any point along a beam due to two equal loads, one of the loads must be at the point.

For two unequal loads, the maximum end shear will occur when both loads are on the span and when the heavier load is at an infinitesimal distance from the end. For a maximum shear at any point along the beam due to two unequal loads, the heavier load must be at the point.

For a maximum shear at any point along a beam due to any number of loads, one of the loads must be at the point. The criterion for determining which load placed at the point will give a maximum shear will now be determined.

Let MN (Fig. 113) be a beam loaded with any number (in this case four) concentrated loads, and let O be any point along the beam whose distance from the left end is x. Also let $\Sigma P =$ the sum of all the loads on the beam when there is a maximum shear. It is required to determine which load placed at O will give a maximum shear at that point.

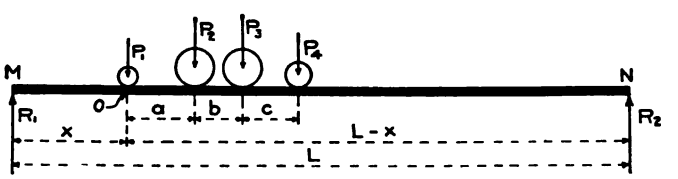


FIG. 113. POSITION FOR MAXIMUM SHEAR—MOVING LOADS.

When P_1 is placed at O, the shear S_1 at O is equal to the left reaction R_1, i. e.,

$$S_1 = R_1 = \frac{P_1(L-x) + P_2(L-x-a)}{L}$$

$$+ \frac{P_3[L-x-(a+b)] + P_4[L-x-(a+b+c)]}{L}$$

$$= \Sigma P(L-x) - P_2a - P_3(a+b) - P_4(a+b+c).$$

When P_2 is at O, the shear S_2 at O is

$$S_2 = R_1 - P_1 = \frac{P_1(L-x+a) + P_2(L-x)}{L}$$

$$+ \frac{P_3(L-x-b) + P_4[L-x-(b+c)]}{L} - P_1$$

$$= \frac{\Sigma P(L-x) + P_1a - P_3b - P_4(b+c) - P_1L}{L}.$$

Subtracting S_2 from S_1, we get the difference in the shear for the two cases, or

$$S_1 - S_2 = \frac{P_1L - \Sigma Pa}{L}. \qquad (14)$$

From the above equation, it is seen that S_1 will be greater than S_2 if $P_1L > \Sigma Pa$, i. e., if

$$\frac{P_1L}{a} > \Sigma P. \qquad (15)$$

The above equation expressed in words is: *The maximum positive shear at any point along a beam occurs when the foremost load is at the point if* $\frac{P_1L}{a}$ *is greater than* ΣP. *If* $\frac{P_1L}{a}$ *is less than* ΣP_1, *the greatest shear will occur when some succeeding load (usually the second) is at the point.*

The methods employed to determine the shear due to fixed loads may be applied to moving loads as soon as their position for a maximum shear has been determined.

PART IV.

BRIDGES.

CHAPTER XIX.

TYPES OF BRIDGE TRUSSES.

Bridge trusses are comparatively recent structures, the ancient bridges being pile trestles or arches. Somewhat later, a combination of arch and truss was used, although the principles governing the design were not understood. It was not until 1847 that the stresses in bridge trusses were fully analyzed, although trusses were constructed according to the judgment of the builder before this date. In 1847, Squire Whipple issued a book upon bridge building, and he was the first to correctly analyze the stresses in a truss. Soon afterward, the solution of stresses became very generally understood, wooden trusses were discarded for iron ones, and still later, steel replaced iron as a bridge-truss material. From this time, the development of bridge building was very rapid, culminating in its present high state of efficiency.

160. Through and Deck Bridges. Bridges may be grouped into two general classes, viz.: through bridges and deck bridges.

A *through* bridge is one in which the floor is supported at, or near, the plane of the lower chords of the trusses (see Fig. 114, e). The traffic moves through the space between the two trusses. Except in the case of a pony truss (one in which there is no overhead bracing), a system of overhead lateral bracing is used.

A *deck* bridge is one in which the floor is supported directly upon the upper chords of the trusses. In this type, the trusses are below the floor (see Fig. 114, d).

161. **Types of Bridge Trusses.** In Fig. 114 are shown several types of bridge trusses that have been very generally used.

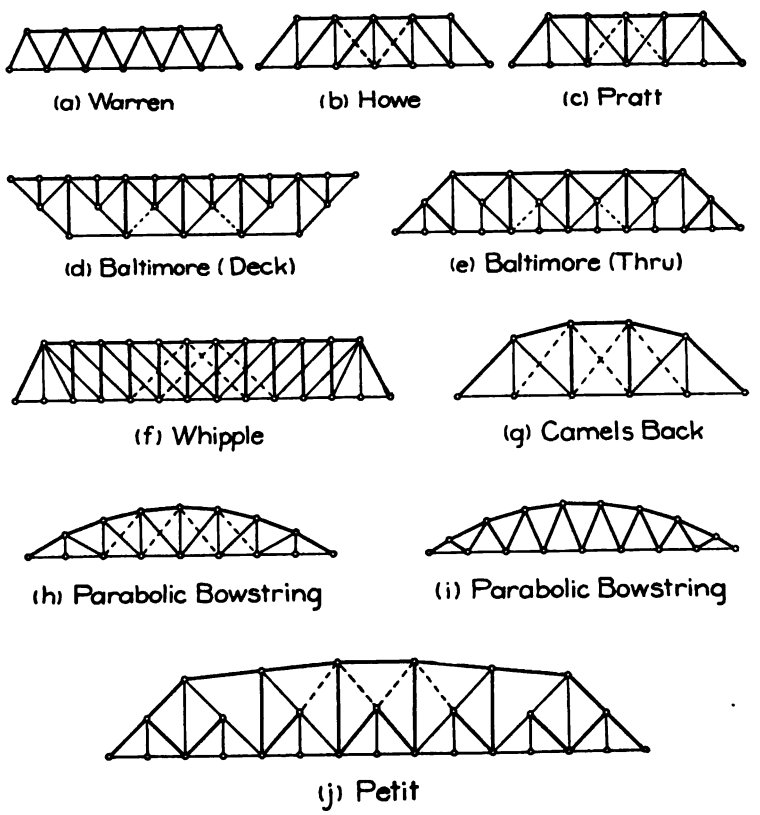


FIG. 114. TYPES OF BRIDGE TRUSSES.

Fig. 114, a shows a Warren truss. This truss is still used for short spans, but has the disadvantage that the intermediate web members are subjected to reversals of stress.

Fig. 114, b shows a Howe truss. This truss was in favor

when wood was extensively used as a building material for trusses, but is little used at present. It has the disadvantage of having long compression web members.

Fig. 114, c shows a Pratt truss. This form of truss is extensively used, both for highway and railroad bridges, up to about a 200-foot span. It is economical and permits of good details.

Fig. 114, d shows a Baltimore deck truss, and Fig. 114, e, a Baltimore through truss. These trusses are used for comparatively long spans, and have short compression members.

Fig. 114, f shows a Whipple truss. This truss was quite extensively used, but is now seldom employed. It is a double intersection truss, and has a redundancy of web members. The stresses are indeterminate by ordinary graphic methods.

Fig. 114, g shows a Camels-Back truss. This truss is used both for short and long spans.

Fig. 114, h and Fig. 114, i show Parabolic Bowstring trusses. The upper chord panel points are on the arc of a parabola. A great disadvantage of these types is that the upper chord changes direction at each panel point and that the web members change both their angle of inclination and length at the panel points.

This type is sometimes modified by placing the panel points on the arc of a circle.

Fig. 114, j shows a Petit truss. This truss is quite extensively used for long spans, and is economical.

162. Members of a Truss. The general arrangement of members is given in the through Pratt railroad truss shown in Fig. 115. The arrangement of members in the various types of trusses is somewhat similar to that shown. In this truss, the tension members are shown by light lines, and the compression members by heavy lines.

Main Trusses. Each truss consists of a top chord, a bottom chord, an end post, and web members. The web members may be further subdivided into hip-verticals, intermediate posts, and diagonals. The diagonals may be divided into main members and counters, the main members being those stressed under a dead load, and the counters those stressed only under a live load. In Fig. 115, L_0U_1 is an end post; U_1U_2, a panel length of the upper

chord; L_0L_1, a panel length of the lower chord; U_1L_1, a hip vertical; U_1L_2 and U_2L_3, main diagonals; and L_2U_3, a counter.

Lateral Bracing. The bracing in the plane of the upper

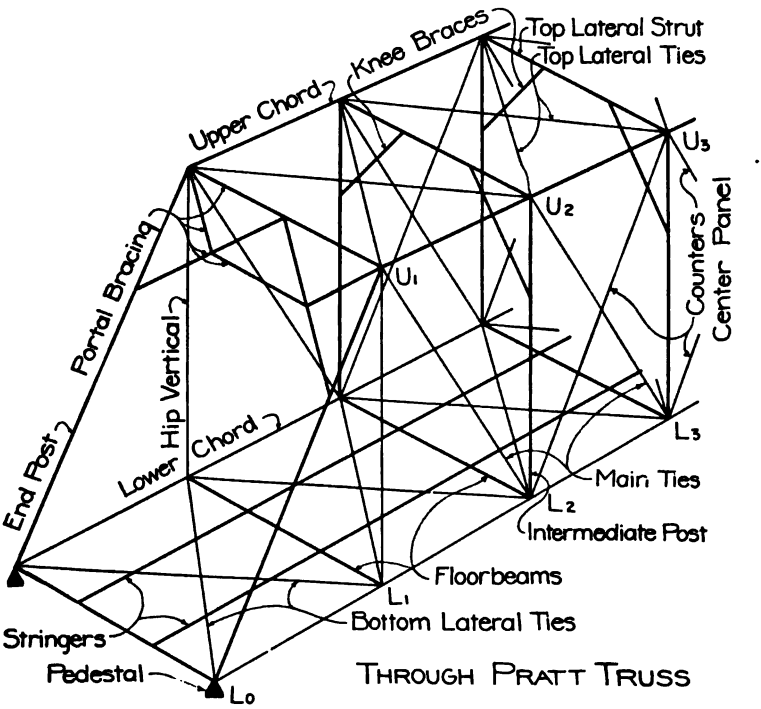


FIG. 115. ARRANGEMENT OF TRUSS MEMBERS.

chord (Fig. 115) is called the *top lateral bracing;* and that in the plane of the lower chord, the *bottom lateral bracing.* The members of the lateral systems are stressed by wind loads and by the vibrations due to live loads. The top lateral system is composed of top lateral struts and ties. The floorbeams act as the struts in the lower lateral system.

Portals. In through bridges, the trusses are held in position and the bridge made rigid by a system of bracing in the planes of the end posts. This system of bracing is called the *portal bracing,* or *portal.*

Knee-braces and Sway Bracing. The braces connecting the top lateral struts and intermediate posts (see Fig. 115), in the plane of the intermediate posts, are called *knee-braces.* When greater rigidity is required, a system of bracing somewhat similar to the portal bracing is used instead of the knee-braces. This bracing is called the *sway bracing.* The top lateral strut is also the top strut of the sway bracing. Knee-braces and sway bracing are often omitted on small span highway bridges.

Floor System. The floor systems of ordinary highway bridges differ considerably from those of railroad bridges. Both types, however, have cross-beams running from one hip vertical or intermediate post to the opposite one. These beams are called *floor-beams.* The beams at the ends of the bridge are called the end floorbeams, and those at the intermediate posts, intermediate floorbeams. The end floorbeams are usually omitted in highway bridges, and an end strut, or joist raiser, is substituted.

In railroad bridges, there are beams which run parallel to the chords and are connected at their ends to the floorbeams. These beams are called *stringers.*

In highway bridges, there are several lines of beams which run parallel to the chords and which rest upon the floorbeams. These beams are called *joists.*

In railroad bridges, the ties, which support the rails, rest directly upon the stringers; and in highway bridges, the floor surface is supported directly by the joists.

Pedestals. The supports for the ends of the trusses are called *pedestals.* For spans over about 70 feet, the pedestals at one end of the bridge are provided with rollers, to allow for expansion and contraction.

Connections. The members of the truss may be either riveted together or connected by pins. In the former case, the truss is said to be a riveted truss, and in the latter, a pin-connected truss. Riveted trusses are often used for short spans and are very rigid. Pin-connected trusses are easy to erect, and are used for both short and long spans.

CHAPTER XX.

LOADS.

The loads for which a bridge must be designed may be classified, as follows: dead load, live load, and wind load; and these loads will be discussed in the following three articles.

ART. 1. DEAD LOAD.

The dead load is the weight of the entire bridge, and includes the weights of the trusses, bracing, floor, etc. It is, of course, necessary to determine the weight of the trusses before the dead load stresses in them may be determined; therefore an assumption must be made as to the dead load. If the weights of similar bridges are available, then these weights may be used to determine the dead load; but it is customary to use formulæ for finding the approximate dead load. It should be borne in mind that the dead load for a single-track bridge is carried by two trusses, each supporting one-half the total load.

163. **Weights of Highway Bridges.** The total dead load per linear foot of span for a highway bridge not carrying interurban cars may be very closely approximated by the formula*

$$w = 140 + 12\,b + 0.2\,bL - 0.4\,L, \qquad (1)$$

where w = weight of bridge in pounds per linear foot,
b = width of bridge in feet (including sidewalks, if any),
L = span of bridge in feet.

*Merriman and Jacoby's "Roofs and Bridges," Part II.

The weight of a heavy interurban riveted bridge may be closely approximated by the formula†

$$w = 600 + 1.8\,L + 27\,b + \frac{1}{12}bL\left(1 + \frac{1}{1000}L\right), \qquad (2)$$

where,

w = weight of bridge in pounds per linear foot,
b = width of roadway (including sidewalks),
L = span of bridge in feet.

164. Weights of Railroad Bridges. The weights of railroad bridge trusses per linear foot of span are given very closely by the following formulæ:‡

$$\text{For E50,}\quad w = (650 + 7L), \qquad (3)$$

$$\text{“ E40,}\quad w = \tfrac{7}{8}(650 + 7L), \qquad (4)$$

$$\text{“ E30,}\quad w = \tfrac{3}{4}(650 + 7L). \qquad (5)$$

The above formulæ do not include the weights of the ties and rails, which may be assumed at 400 pounds per linear foot of track. If solid steel floors are used, the weight of the track should be taken at 700 pounds per linear foot.

E50, E40, and E30 refer to the live load for which the bridge is designed; as given by Theodore Cooper in his "General Specifications for Steel Railroad Bridges and Viaducts." This live loading will be explained in § 167 (b).

These formulæ give the weights of single-track spans. Double-track spans are about 95 per cent heavier.

Art. 2. Live Load.

The live load consists of the traffic moving across the bridge. For highway bridges, the live load consists of vehicles, foot passengers, and interurban cars; and for railroad bridges it consists

†E. S. Shaw.
‡F. E. Turneaure.

of trains. The standard highway bridge specifications of Theodore Cooper and J. A. L. Waddell give the live loads to be used for different classes of highway bridges, and are recommended by the writer for general use.

165. Live Load for Light Highway Bridges. The live load for highway bridges is usually specified in pounds per square foot of floor surface. The weights given in the following table are recommended for use when the bridge does not carry interurban traffic.

LIVE LOADS FOR HIGHWAY BRIDGE TRUSSES.

Spans	up	to	100	feet		100	lbs.	per	sq.	ft.
"	100	to	125	"		95	"	"	"	"
"	125	to	150	"		90	"	"	"	"
"	150	to	200	"		85	"	"	"	"
"	over		200	"		80	"	"	"	"

In some states, the law requires that highway bridges be designed for a live load of 100 pounds per square foot of floor surface, but the law as usually stated is defective in that the allowable unit stresses are not given.

The floor systems of light highway bridges should be designed to carry a live load of 100 pounds per square foot of floor space, and to be of sufficient strength to carry a heavy traction engine. The total load is obtained by multiplying the weight per square foot by the clear width of the roadway and sidewalks; and this load is to be so placed as to give the maximum stress in each truss member.

166. Live Load for Interurban Bridges. For the live load on interurban bridges, the student is referred to the highway bridge specifications of Theodore Cooper and J. A. L. Waddell.

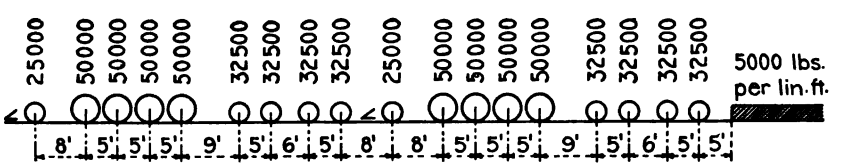
167. Live Load for Railroad Bridges. The live load for railroad bridges varies on account of the great variations in spans and wheel spacings of engines. The bridges for main tracks are usually designed for the heaviest engines now in use, or that may reasonably be expected to be built in the near future.

The live load may be treated either (a) as a uniform load, (b) as a number of concentrated wheel loads followed by a uniform train load, or (c) as an equivalent uniform load. The second loading is preferable and gives more accurate results, but requires more work to determine the stresses.

(a) *Uniform Load.* When train loads were light, it was customary to design the bridges for a uniform load instead of considering actual wheel concentrations; and a uniform load is still often used in practice. The same load is generally taken for both the chord and the web numbers. This method is simpler than that involving wheel loads, and gives fair results if intelligently used. Care should be exercised, however, in determining the load to be used for each truss.

(b) *Concentrated Wheel Loads.* The method of using concentrated wheel loads is complicated by the great variation in the weights and spacings of engine wheel loads. Most railroad companies specify that the stresses shall be computed for two engines and tenders followed by a uniform train load.

The present practice among many railroad companies is to use a conventional wheel loading, one of the best examples of which being that given by Theodore Cooper in his "Specifications for Steel Railroad Bridges and Viaducts". The three common classes



Live Load per Track for Cooper's E-50 Loading

FIG. 116. COOPER'S E50 LOADING.

recommended by Theodore Cooper are his E50, E40, and E30. Fig. 116 shows the wheel loads and spacings for Cooper's E50 loading, which corresponds to the heaviest engines in common use, although a live load corresponding to an E60 loading has been used.

An E40 loading has the same wheel spacings, the weight of

each wheel being four-fifths of that for the E50 loading, followed by a uniform train load of 4000 pounds per linear foot of track.

An E30 loading has the same wheel spacings, the weight of each wheel being three-fifths of that for the E50, followed by a uniform train load of 3000 pounds per linear foot of track.

EQUIVALENT UNIFORM LOADS – COOPER'S E-40.

Span (Ft.)	Equivalent Uniform Load			Span (Ft.)	Equivalent Uniform Load		
	Chords	Webs	Fl. Bms.		Chords	Webs	Fl. Bms.
10	9000	12000	8200	46	6330	7240	5240
11	9310	11640	7960	48	6220	7140	5200
12	9340	11330	7830	50	6110	7060	5140
13	9340	11080	7600	52	6040	6940	5130
14	9210	10860	7460	54	5960	6820	5120
15	9030	10670	7330	56	5880	6720	5110
16	8850	10500	7120	58	5800	6620	5090
17	8650	10350	6940	60	5730	6530	5080
18	8430	10240	6780	62	5690	6490	5080
19	8220	10100	6630	64	5700	6450	5070
20	8000	10000	6500	66	5620	6450	5070
21	8040	9780	6390	68	5560	6380	5060
22	8040	9580	6290	70	5510	6340	5060
23	8010	9400	6200	72	5490	6320	5030
24	7960	9230	6120	74	5460	6300	5010
25	7890	9080	6040	76	5440	6290	4990
26	7780	8930	5970	78	5420	6270	4970
27	7660	8790	5900	80	5400	6250	4950
28	7540	8660	5830	82	5370	6230	4930
29	7420	8540	5770	84	5340	6200	4910
30	7300	8430	5720	86	5310	6180	4890
31	7220	8320	5680	88	5270	6150	4870
32	7140	8190	5650	90	5250	6130	4860
33	7050	8080	5620	92	5250	6110	4850
34	6960	7980	5600	94	5210	6090	4810
35	6870	7890	5570	96	5170	6060	4780
36	6820	7820	5530	98	5150	6040	4760
37	6760	7750	5500	100	5140	6020	4740
38	6700	7690	5460	125	5100	5770	4720
39	6630	7630	5430	150	5010	5570	4700
40	6560	7570	5400	175	4890	5350	4680
42	6530	7450	5340	200	4740	5240	4660
44	6470	7340	5300	250	4510	5030	4640

FIG. 117. EQUIVALENT UNIFORM LOADS.

The great advantage of Cooper's loadings is that the spacing of wheels is the same for all loadings. It is thus seen that the

moments and shears for the different loadings are proportional to the class of loading.

The actual determination of moments, shears, and stresses due to concentrated wheel loads will be given in Chapter XXIII.

(c) *Equivalent Uniform Load.* An equivalent uniform load is one which will give results closely approximating those for actual wheel concentrations. To secure accuracy, it is necessary to use a different uniform load for each span; also a different load for the chords, the webs, and the floorbeams. The table in Fig. 117 gives the equivalent uniform loads for Cooper's E40 loading. For E50 loading use 125 per cent of these values, and for E30 use 75 per cent.

The stresses in trusses due to equivalent uniform loads may be determined by the methods given in Chapter XXI.

ART. 3. WIND LOADS.

The wind load is usually expressed in one of the two following ways: (a) in pounds per square foot of actual truss surface; or (b) in pounds per linear foot, treated as a dead load acting upon the upper and lower chords and as a live load acting upon the traffic as it moves over the bridge.

168. Wind Load. (a) The method of stating the wind load in pounds per square foot of actual truss surface has the disadvantage that an assumption must be made as to the area of the truss surface. If this method is used, the wind load is usually taken at 30 pounds per square foot of truss surface. The load may be considered to act against the windward truss only, or to be equally resisted by the two trusses.

169. (b) The method of specifying the wind load in pounds per linear foot is more logical, and is usually employed.

For highway bridges, the usual practice is to take a wind load of 150 pounds per linear foot, treated as a dead load acting upon both the upper and lower chords; and a load of 150 pounds per linear foot, treated as a live load acting upon the portion of the bridge covered by the traffic.

For railroad bridges, a higher value is used for that portion of the wind which is treated as a live load. This is to provide, not only for the wind loads, but also for stresses caused by the vibrations due to trains. The following wind loads are recommended as conforming to good practice: 150 pounds per linear foot, acting upon both the upper and lower chords; and 450 pounds per linear foot of live load upon the bridge, the latter force being assumed to act six feet above the base of the rail. The portion of the wind load considered as live load will be resisted by the bottom laterals in through bridges, and by the top laterals in deck bridges.

CHAPTER XXI.

STRESSES IN TRUSSES DUE TO UNIFORM LOADS.

In this chapter there will be given several graphic methods for determining the stresses in various types of bridge trusses under uniform loads, together with the algebraic method of coefficients. These methods will be explained by the solution of particular problems; as they may be more easily understood than general ones. Most of the methods are of general application to the various types of bridge trusses; but the problem should first be studied to determine which method may be employed to the best advantage.

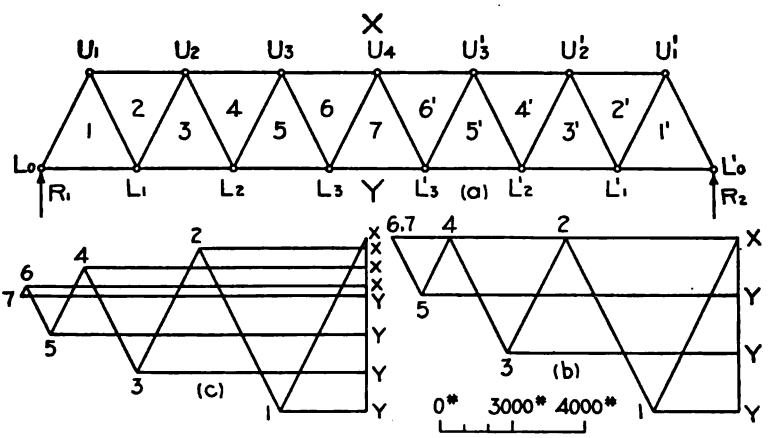
The chapter will be divided in seven articles, as follows: Art. 1, Stresses in a Warren Truss by Graphic Resolution; Art. 2, Stresses in a Pratt Truss by Graphic Resolution; Art. 3, Stresses by Graphic Moments and Shears; Art. 4, Stresses in a Bowstring Truss—Triangular Web Bracing; Art. 5, Stresses in a Parabolic Bowstring Truss; Art. 6, Wind Load Stresses in Lateral Systems; and Art. 7, Stresses in Trusses with Parallel Chords by the Method of Coefficients.

ART. 1. STRESSES IN A WARREN TRUSS BY GRAPHIC RESOLUTION.

The application of the method of graphic resolution to the solution of the stresses in a Warren truss will now be explained.

170. **Problem.** It is required to find the maximum and minimum dead and live load stresses in all the members of the highway Warren truss shown in Fig. 118, a. The truss has a span L of 70 feet; a panel length l of 10 feet; and a depth D of 10 feet. The bridge has a width of 14 feet.

171. **Dead Load Stresses.** The dead load may be obtained by applying equation (1), § 163. This load is found to be 476 lbs. per ft. of span, or say 240 lbs. per ft. of truss. The panel load $W = 240 \times 10 = 2400$, all of which will be assumed to act at the lower chord. The effective reaction $R_1 = 3 \times 2400 = 7200$.



D = 10 ft., l = 10 ft.; L = 70 ft.; w = 240 lbs. per ft.

FIG. 118. DEAD LOAD STRESSES—WARREN TRUSS.

To determine the dead load stress, lay off XY = 7200 (Fig. 118, b), and draw the dead load stress diagram for one-half of the truss. Since the truss and loads are symmetrical and the full dead load is always on the bridge, it is necessary to draw the diagram for only one-half of the truss. The kind of stress is determined by placing the arrows on the truss diagram as the stress diagram is drawn. The dead load stresses for both the chord and web members are shown in Fig. 119, a and Fig. 119, b. By a comparison of these stresses, it is seen that, for this truss, the two web members meeting upon the unloaded chord have the same numerical stress. It is also seen that, for an odd-panel truss, the center web members are not stressed.

The assumption is sometimes made that one-third of the dead load acts at the upper chord and two-thirds at the lower chord.

The dead load stress diagram for this assumption is shown in Fig. 118, c. For highway bridges, it is customary to assume all the dead load on the lower chord.

MAXIMUM AND MINIMUM STRESSES IN CHORD MEMBERS

	Upper Chord			Lower Chord			
Chord Member	X-2	X-4	X-6	Y-1	Y-3	Y-5	Y-7
Dead Load	- 7200	-12000	-14400	+ 3600	+ 9600	+13200	+14400
Live Load	-21000	-35000	-42000	+10500	+28000	+38500	+42000
Maximum	-28200	-47000	-56400	+14100	+37600	+51700	+56400
Minimum	- 7200	-12000	-14400	+ 3600	+ 9600	+13200	+14400

(a)

DEAD LOAD STRESSES IN WEB MEMBERS

Web Member	X-1	1-2	2-3	3-4	4-5	5-6	6-7
Dead Load	- 8060	+ 8060	- 5370	+ 5370	- 2690	+ 2690	0

(b)

LIVE LOAD STRESSES IN WEB MEMBERS

Web Member	X-1	1-2	2-3	3-4	4-5	5-6	6-7
Live Load at L'_1	- 1120	+ 1120	- 1120	+ 1120	- 1120	+ 1120	- 1120
" " " L'_2	- 2240	+ 2240	- 2240	+ 2240	- 2240	+ 2240	- 2240
" " " L'_3	- 3360	+ 3360	- 3360	+ 3360	- 3360	+ 3360	- 3360
" " " L_3	-4480	+4480	-4480	+4480	-4480	+4480	+3360
" " " L_2	- 5600	+ 5600	- 5600	+ 5600	+2240	-2240	+2240
" " " L_1	- 6720	+ 6720	+ 1120	- 1120	+ 1120	- 1120	+ 1120
Max. Live Load	-23520	+23520	-16800	+16800	-11200	+11200	- 6720
Min. " "	0	0	+ 1120	- 1120	+ 3360	- 3360	+ 6720
Uniform " "	-23520	+23520	-15680	+15680	- 7840	+ 7840	0

(c)

MAXIMUM AND MINIMUM STRESSES IN WEB MEMBERS

Web Member	X-1	1-2	2-3	3-4	4-5	5-6	6-7
Dead Load	- 8060	+ 8060	- 5370	+ 5370	- 2690	+ 2690	0
Max. Live Load	-23520	+23520	-16800	+16800	-11200	+11200	-6720
Min. Live Load	0	0	+ 1120	- 1120	+ 3360	- 3360	+ 6720
Max. Stress	-31580	+31580	-22170	-22170	-13890	+13890	-6720
Min. Stress	- 8060	+ 8060	- 4250	+ 4250	+ 670	- 670	+6720

(d)

FIG. 119. TABLE OF STRESSES—WARREN TRUSS.

172. Live Load Stresses. The live load will be taken at 1400 lbs. per linear ft. of bridge, or 700 lbs. per ft. per truss.

Since the upper and lower chords are parallel, the chords will resist the bending moment due to the loads, and the web members will resist the shear.

(a) *Chord Stresses.* It is seen that each increment of live load brought upon the bridge increases the bending moment and therefore increases the stresses in each chord member. The maximum live load stress in each member of the upper and lower chord is therefore given when the bridge is fully loaded with the live load. The minimum stress occurs when there is no live load on the bridge.

Since the maximum live load chord stresses are obtained when the bridge is fully loaded, it is seen that it is unnecessary to draw a new stress diagram; as the live load chord stresses may be obtained from the corresponding dead load stresses by multiplying each stress by the ratio of the live load to the dead load.

The maximum live load chord stresses are shown in Fig. 119, a. The minimum live load chord stresses are zero.

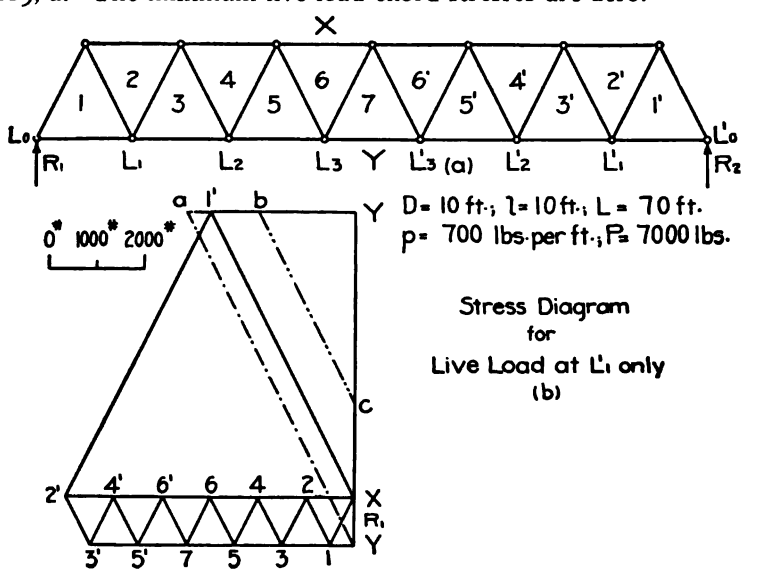


FIG. 120. LIVE LOAD STRESSES—WARREN TRUSS.

(b) *Web Stresses.* By partially loading the truss with live load, it is possible to obtain stresses which are larger than those obtained when the bridge is fully loaded; or it is even possible to get stresses of an opposite kind to those due to the dead load.

The maximum and minimum live load web stresses will now be found by applying one load at a time, and noting the effect of that load upon the stress on each web member.

Fig. 120, b shows the stress diagram for a single live panel load placed at L_1', the reaction at L_0 being $\frac{P}{7}$. Since the truss is symmetrical about the center line, a load placed at L_1' will produce the same stresses in the web members to the left of the center as a load at L_1 will in the web members to the right of the center. It will therefore be necessary to record only one-half of the web members, thus simplifying the table of stresses. From the stress diagram (Fig. 120, b), it is seen that the stresses in all of the members to the left of L_1' due to a load at L_1' are constant.

The stresses in the web members to the left of the center line of the truss due to a live load at L_1' are shown in the first line of Fig. 119, c, these stresses being alternately tension and compression. Now place a live load at L_2', the remainder of the truss being unloaded. The reaction at L_0 equals $\frac{2}{7}$ P, and it is seen without drawing another stress diagram that the stresses in the web members to the right of the center line are twice those due to the live load at L_1'. The stresses due to a load at L_2' are shown in the second line of Fig. 119, c. It is also seen that the stresses for a load at L_3' are three times those for a load at L_1'. The stresses for a load at L_3' are shown in the third line of Fig. 119, c. The stresses for a live load at L_3 are shown in the fourth line; for a live load at L_2 in the fifth line; and for a load at L_1 in the sixth line.

Maximum and Minimum Live Load Web Stresses. By the *maximum* live load stress is meant the greatest live load stress that ever occurs in the member; and by the *minimum* live load stress is meant the smallest live load stress if there is no reversal of stress, or the greatest stress of an opposite kind if there is a reversal of stress in the member.

From Fig. 119, c, it is seen that the addition of each load produces a compressive stress in X–1 and a tensile stress in 1–2, the maximum stress being — 23 520 for X–1 and + 23 520 for 1–2

when the truss is fully loaded. The minimum live load stresses in X–1 and 1–2 are zero when there is no live load on the truss.

It is also seen that loads at L_1', L_2', L_3', L_3, and L_2 all produce compression in 2–3 and tension in 3–4; and that the load at L_1 produces tension in 2–3 and compression in 3–4. The maximum live load stresses in 2–3 and 3–4 are therefore obtained by adding the stresses due to the loads at L_1', L_2', L_3', L_3, and L_2, and are — 16 800 and + 16 800, respectively, (see Fig. 119, c). The minimum live load stresses in 2–3 and 3–4 are obtained when there is a live load at L_1 only, and are + 1120 and — 1120, respectively.

The maximum live load stresses in 4–5 and 5–6 are obtained when the loads at L_1', L_2', L_3', and L_3 are on the bridge, and are — 11 200 and + 11 200, respectively. The minimum live load stresses in 4–5 and 5–6 are obtained when the loads at L_1 and L_2 are on the bridge, and are + 3360 and — 3360, respectively.

The maximum live load stress in 6–7 is obtained when the loads at L_1', L_2', and L_3' are on the bridge, and is — 6720. The minimum live load stress in 6–7 is obtained when the loads at L_1, L_2, and L_3 are on the bridge, and is + 6720.

A comparison of the corresponding stresses in line 7 and line 9 shows the difference in the stresses in each member for the bridge loaded for maximum live load stresses, and fully loaded with live load.

173. Maximum and Minimum Dead and Live Load Stresses. (a) *Chord Stresses.* The maximum and minimum chord stresses due to dead and live loads are shown in Fig. 119, a. The maximum chord stresses are obtained when the bridge is fully loaded with dead and live loads, and are equal to the sum of the dead and live load stresses. The minimum chord stresses are obtained when there is no live load on the bridge, and are the dead load stresses.

(b) *Web Stresses.* The maximum and minimum web stresses due to dead and live loads are shown in Fig. 119, d. The maximum web stresses are obtained by adding the dead and maximum live load stresses; since they have the same signs.

The minimum web stresses are obtained by adding, algebraically, the dead and minimum live load stresses. It is seen that the dead and minimum live load stresses have opposite signs.

By comparing the dead and the minimum web stresses, it is seen that the members 4–5, 5–6, and 6–7 have reversals of stress.

174. Loadings for Maximum and Minimum Stresses. From a comparison of the stresses in Fig. 119, the following conclusions may be drawn for the maximum and minimum dead and live load stresses in a Warren truss.

(a) *For maximum chord stresses, load the bridge fully with dead and live loads.*

(b) *For minimum chord stresses, load the bridge with dead load only.*

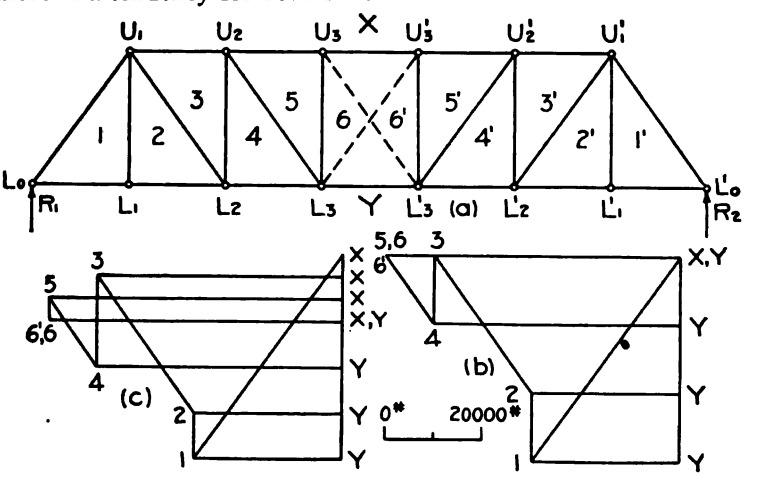
(c) *For maximum web stress in any member, load the longer segment of the bridge with live load, the shorter segment being unloaded.*

(d) *For minimum web stress in any member, load the shorter segment of the bridge with live load, the longer segment being unloaded.*

175. Simplified Construction for Live Load Web Stresses. Referring to Fig. 120, b, it is seen that the numerical stresses in all the web members to the left of any lower chord panel point are constant when the live load extends to the right of that panel point. As the stresses are alternately tension and compression, it is seen that it is only necessary to draw the triangle YaY (Fig. 120, b). This triangle is constructed by making YY equal to one live panel load, and drawing Ya parallel to bc to meet a horizontal line Ya. The inclination of bc is determined by making Yb equal to (or some multiple of) the half panel length, and Yc equal to (or the same multiple of) the depth of the truss. The stresses in the members to the right of L_1' when there is a load at L_1' are equal to $\frac{1}{7}$ Ya. When there is a load at L_2', the stresses to the right of L_2' are equal to $\frac{2}{7}$ Ya, etc. It is therefore unnecessary to draw the entire diagram shown in Fig. 120, b.

ART. 2. STRESSES IN A PRATT TRUSS BY GRAPHIC RESOLUTION.

176. The stresses in a through Pratt truss will be determined in this article. It was shown in the preceding article that some of the web members of the Warren truss were subjected to reversals of stress. In the Pratt truss shown in Fig. 121, a, the web members are so constructed that they can not resist reversals of stress, the intermediate posts taking compression only, and the intermediate diagonals, tension only. This involves the use of another set of diagonals, called counters, in those panels where there is a tendency for reversals.



$\frac{1}{3}$ load on upper chord, $\frac{2}{3}$ on lower. Load all on lower chord.
D = 28 ft.; l = 20 ft.; L = 140 ft.; w = 710 lbs. per ft.

FIG. 121. DEAD LOAD STRESSES—PRATT TRUSS.

The two center diagonals of the seven-panel truss shown in Fig. 121, a are both counters; as there is no dead load shear in the center panel of an odd-panel truss. The counter U_3L_3' acts when the live load extends to the right of L_3', and the counter $U_3'L_3$, when the live load extends to the left of L_3.

Care must be taken in determining the stresses in the verticals adjacent to the counters; as they differ greatly from those which would exist in these members if the main ties could resist compression.

177. **Problem.** It is required to find the maximum and minimum dead and live load stresses in the through Pratt truss

MAXIMUM AND MINIMUM STRESSES IN CHORD MEMBERS.

	Upper Chord			Lower Chord			
Chord Member	X-3	X-5	X-6	Y-1	Y-2	Y-4	Y-6
Dead Load	− 50.7	− 60.8	− 60.8	+ 30.4	+ 30.4	+ 50.7	+ 60.8
Live Load	−182.8	−219.2	−219.2	+109.6	+109.6	+182.8	+219.2
Maximum	−233.5	−280.0	−280.0	+140.0	+140.0	+233.5	+280.0
Minimum	− 50.7	− 60.8	− 60.8	+ 30.4	+ 30.4	+ 50.7	+ 60.8

(a)

DEAD LOAD STRESSES IN WEB MEMBERS

Web Member	X-1	1-2	2-3	3-4	4-5	5-6	6-6'
Dead Load	− 52.4	+ 14.2	+ 34.9	− 14.2	+ 17.5	0.0	0.0

(b)

LIVE LOAD STRESSES IN WEB MEMBERS

Web Member	X-1	1-2	2-3	3-4	4-5	5-6	6-6'
Live Load at L'_1	− 9.0	0.0	+ 9.0	− 7.3	+ 9.0	− 7.3	+ 9.0
" " " L'_2	− 18.0	0.0	+ 18.0	− 14.6	+ 18.0	− 14.6	+ 18.0
" " " L'_3	− 27.0	0.0	+ 27.0	− 21.9	+ 27.0	− 21.9	+ 27.0
" " " L_3	− 36.0	0.0	+ 36.0	− 29.2	+ 36.0	+ 21.9	− 27.0
" " " L_2	− 45.0	0.0	+ 45.0	+ 14.6	− 18.0	+ 14.6	− 18.0
" " " L_1	− 54.0	+ 51.2	− 9.0	+ 7.3	− 9.0	+ 7.3	− 9.0
Max. Live Load	−189.0	+ 51.2	+135.0	− 73.0	+ 90.0	−43.8	+54.0
Min. " "	0.0	0.0	− 9.0	+ 21.9	− 27.0	+43.8	−54.0
Uniform " "	−189.0	+ 51.2	+126.0	− 51.2	+ 63.0	0.0	0.0

(c)

MAXIMUM AND MINIMUM STRESSES IN WEB MEMBERS

Web Member	X-1	1-2	2-3	3-4	4-5	4-5 Counter	5-6	6-6' Counter
Dead Load	− 52.4	+ 14.2	+ 34.9	− 14.2	+ 17.5		0.0	0.0
Max. Live Load	−189.0	+ 51.2	+135.0	− 73.0	+ 90.0		−43.8	+54.0
Min. Live Load	0.0	0.0	− 9.0	+ 21.9	− 27.0		+43.8	−54.0
Max. Stress	−241.4	+ 65.4	+169.9	− 87.2	+107.5	+ 9.5	−43.8	+54.0
Min. Stress	− 52.4	+ 14.2	+ 25.9	0.0	0.0	0.0	0.0	0.0

All stresses are in thousands of pounds

(d)

FIG. 122. TABLE OF STRESSES—PRATT TRUSS.

shown in Fig. 121, a. The truss has a span of 140 feet, a panel length of 20 feet, and a depth of 28 feet.

178. **Dead Load Stresses.** The dead load will be taken at 710 lbs. per ft. per truss. The panel load $W = 710 \times 20 = 14\,200$.

The effective reaction $R_1 = 14\,200 \times 3 = 42\,600$. In this problem, all the dead load will be taken on the lower chord of the truss. The stresses will be expressed in thousands of pounds, being carried to the nearest 100 pounds; thus, 14 200 will be written 14.2.

The dead load stress diagram for the left half of the truss is shown in Fig. 121, b. Since the truss and loads are symmetrical with respect to the center line, it is only necessary to draw the diagram for one-half of the truss.

The dead load chord stresses are shown in Fig. 122, a, and the dead load web stresses in Fig. 122, b. The stresses for the left half of the truss only are shown; as those for the right half are equal to the stresses in the corresponding members shown. It is seen that the upper and lower chord stresses increase from the end toward the center of the truss, and that the web stresses decrease from the end toward the center. It is further seen that the stresses in the web members in the center panel of this truss are zero, and that the stresses in the three center panels of the upper chord are equal.

Fig. 121, c shows the dead load stress diagram for the left half of the truss, assuming that one-third of the dead load is on the upper chord and two-thirds on the lower chord. By comparing the stress diagrams shown in Fig. 121, b and Fig. 121, c, it is seen that the chord and inclined web stresses are the same for both cases, and that the stresses in the vertical posts, only, are changed. When one-third of the dead load is taken on the upper chord, the stresses in the intermediate posts are greater by the amount of the upper chord panel load than when all the load is taken on the lower chord; while the stress in the hip vertical is smaller by the amount of the upper chord panel load.

179. **Live Load Stresses.** The live load will be taken at 2560 lbs. per ft. per truss. The panel load $P = 2560 \times 20 =$ 51 200, or say 51.2.

(a) *Chord Stresses.* Since the addition of each increment of live load increases the bending moment, and since the upper and lower chords in trusses with parallel chords must resist this bending moment, it is seen that the maximum stresses in all the chord

members are obtained when the bridge is fully loaded with live load.

The minimum live load chord stresses are zero, when there is no live load on the bridge.

The chord stresses may be obtained either by loading the bridge fully with live load and drawing a stress diagram similar to that shown in Fig. 121, b, or they may be obtained from the dead load chord stresses by direct proportion. Each chord stress is equal to the corresponding dead load stress multiplied by the

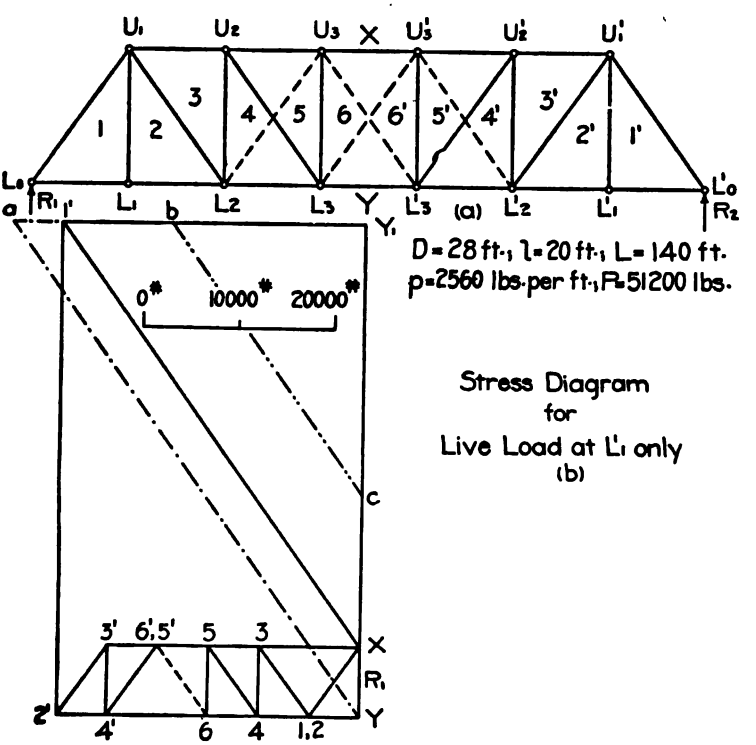


FIG. 123. LIVE LOAD STRESSES—PRATT TRUSS.

ratio of the live load to the dead load. The latter method requires less work and was used in this problem. The maximum live load chord stresses are shown in Fig. 122, a.

(b) *Web Stresses.* The web stresses, and the positions of the loads for maximum and minimum live load web stresses, will now be found by applying one load at a time and obtaining the stress in each member due to that load.

The stress diagram for a single live load at L_1' is shown in Fig. 123, b. Since there are seven panels in this truss, the reaction at L_0 will be equal to $\frac{1}{7}$ P, and the reaction at L_0', to $\frac{6}{7}$P. This diagram is constructed by laying off $R_1 = \frac{1}{7}$ P, and drawing the stress diagram for the entire truss (see Fig. 123, b). In drawing this diagram, it is assumed that the diagonals which are stressed by the dead load, and the diagonal 6–6′ (U_3L_3') are acting. This set of diagonals will act unless the dead load stresses in some of the members are reduced to zero by the live load, thus throwing some of the counters into action.

Since the stresses in 3′–4′, 5–6, and 3–4 are numerically equal, as are also those in 2′–3′, 4′–5′, 5′–6, 5–4, 3–2, and X–1, for a single load at L_1', it is unnecessary to draw the entire diagram shown in Fig. 123, b. The stresses in these members may be obtained from the triangle YaY_1. This triangle is constructed by laying off YY_1 equal to the live panel load P, and drawing Ya parallel to bc to meet a horizontal line through Y_1. The inclination of bc is determined by laying off Y_1b equal to some multiple (in this case the multiple is 2) of the panel length, and Y_1c equal to the same multiple of the depth of the truss. For a single live load at L_1', the reaction at L_0 is equal to $\frac{1}{7}$ P, and the stresses in the inclined diagonals are each equal to $\frac{1}{7}$Ya; while those in the verticals, with the exception of the hip vertical, are each equal to $\frac{1}{7}YY_1$. The stress in the hip vertical 1′–2′ is not influenced by any of the loads on the truss except that at L_1', and therefore the stress in this member is either 0 or P.

The stresses in the web members to the left of the center of the

truss when there is a live load at L_1' are shown in the first line of Fig. 122, c. It is seen that the stress in 1–2 is zero.

When there is a single live load at L_2', the reaction at L_0 is equal to $\frac{2}{7}$ P, and the stresses in the inclined and vertical web members (1–2 excepted) are each equal to $\frac{2}{7}$ Ya and $\frac{2}{7}$ YY_1, respectively. The stresses in the web members to the left of the center when there is a live load at L_2' are shown in the second line of Fig. 122, c.

When there is a single live load at L_3', the reaction at L_0 is equal to $\frac{3}{7}$ P, and the stresses in the web members to the left of the center for this loading are shown in the third line of Fig. 122, c.

The stresses in the web members to the left of the center when there is a single live load at L_3 are shown in the fourth line of Fig. 122, c; when there is a single live load at L_2, in the fifth line; and when there is a single live load at L_1, in the sixth line.

Maximum and Minimum Live Load Web Stresses. By comparing the web stresses shown in Fig. 122, b with those shown in Fig. 122, c, it is seen that the live load stresses in the members to the left of the center due to loads at L_1', L_2', and L_3' have the same signs as the corresponding dead load stresses. It is also seen that loads placed at L_3, L_2, and L_1, successively, produce the same kinds of stress in the members to the left of these points as does the dead load.

A live load placed at L_1 tends to produce an opposite kind of stress in the members 2–3, 3–4, and 4–5 to that caused by the dead load; and a load placed at L_2 tends to produce an opposite kind of stress 3–4 and 4–5.

Since in this truss the intermediate diagonals are tension members, the loads at and to the right of L_3' cause the counter U_3L_3' to act; and loads at and to the left of L_3 cause the counter $U_3'L_3$ to act.

From Fig. 122, c, it is seen that the addition of each live load

produces a compressive stress in X–1, the maximum stress being — 189.0, which is the sum of the stresses caused by the separate loads. The minimum live load stress in X–1 is 0, when there is no live load on the bridge.

It is also seen from Fig. 122, c that the maximum live load stress in the hip vertical 1–2 is obtained when there is a live load at L_1, and is equal to the load at that point. This member simply transfers the load to the joint U_1, and the stress in it is not influenced by any other loads on the truss. The minimum live load stress is 0, when there is no live load on the bridge.

The maximum live load stress in the member 2–3 is obtained when the loads at and to the right of L_2 are on the truss, and is + 135.0, the sum of the separate stresses caused by these loads. The minimum live load stress in 2–3 is obtained when the single live load at L_1 is on the truss, and is — 9.0 (provided there is already that much tension in the member due to the dead load).

The maximum live load stress in the vertical 3–4 is obtained when the loads at and to the right of L_3 are on the truss, and is — 73.0. The minimum live load stress in 3–4 is obtained when the loads at L_1 and L_2 are on the truss, and is + 21.9 (provided the counter in the panel L_2 L_3 does not act for this loading).

The maximum live load stress in 4–5 is obtained when the loads at and to the right of L_3 are on the truss, and is + 90.0. The minimum live load stress in 4–5 is obtained when the loads at L_1 and L_2 are on the truss, and is — 27.0 (provided there is that much dead load tension already in it).

The maximum live load stress in 5–6 is obtained when the loads at L_1', L_2', and L_3' are on the truss, and is — 43.8. The minimum live load stress in 5–6 is obtained when the loads at L_1, L_2, and L_3 are on the truss, also when the truss is fully loaded.

The maximum live load stress in 6–6′ (U_3L_3') is obtained when the loads at L_1', L_2', and L_3' are on the truss, and is + 54.0. The minimum live load stress in 6–6′ is obtained when the loads at L_1, L_2 and L_3 are on the truss, also when the truss is fully loaded.

By comparing the above stresses, the following conclusions may be drawn: (a) there is no stress in the hip vertical 1–2

unless there is a load at L_1, and when there is a load at this point, the stress in the hip vertical is equal to that load; (b) the web members meeting on the unloaded chord have their maximum and minimum live load stresses under the same loading; (c) the maximum live load web stresses are obtained when the longer segment of the truss is loaded with live load; (d) the minimum live load web stresses are obtained when the shorter segment of the truss is loaded.

A comparison of the stresses shown in line 7 and line 9 (Fig. 122, c) shows the differences in the corresponding stresses for the truss loaded for maximum live load stresses and fully loaded with live load.

180. Maximum and Minimum Dead and Live Load Stresses. (a) *Chord Stresses.* The maximum and minimum chord stresses due to dead and live loads are shown in Fig. 122, a. The maximum chord stresses are obtained when the truss is fully loaded with dead and live loads, and are equal to the sums of the corresponding dead and live load stresses. The minimum chord stresses are obtained when there is no live load on the bridge, and are the dead load stresses.

(b) *Web Stresses.* The maximum and minimum web stresses due to live and dead loads are shown in Fig. 122, d. Since the intermediate posts take compression only, and the intermediate diagonals, tension only, care must be used in making the combinations for maximum and minimum stresses. It should be borne in mind that the counter and main tie in any panel cannot act at the same time, and that the counter does not act until the dead load tension in the main tie in that panel has been reduced to zero by the live load. It is thus seen that a main tie can resist as much live load compression as there is dead load tension already in it, and no more, the remainder of the live load stress, if any, being taken by the counter. Since the dead load always acts, it must be considered in all combinations.

Referring to Fig. 122, c and Fig. 122, d, it is seen that the maximum stress in X–1 is obtained when the bridge is fully loaded with live and dead loads, and is —241.4. The minimum

stress in X–1 is obtained when there is no live load on the bridge, and is — 52.4.

The maximum stress in the hip vertical 1–2 is obtained when there is a live load at L_1, and is + 65.4, the sum of the dead and live panel loads. The minimum stress in 1-2 is obtained when there is no live load on the bridge, and is + 14.2.

The maximum stress in 2–3 (see Fig. 122, c and Fig. 122, d) is obtained when there are live loads at and to the right of L_2, and is + 169.9. The minimum stress in 2–3 is obtained when there is a live load at L_1 only, and is + 25.9. In this case, it is seen that the minimum live load stress has an opposite sign from that of the dead load, and subtracts from it. No counter is required in this panel, as the live load compression is less than the dead load tension.

The stress in the member 4–5 will be determined before that in 3–4, as it is necessary to determine which diagonal is acting before the stress in the post can be found.

The maximum stress in 4–5 is obtained when the loads at and to the right of L_3 are on the bridge, and is + 107.5. The minimum stress in 4–5 is obtained when the live loads at L_1 and L_2 are on the bridge, and is 0; as will now be shown. Referring to Fig. 122, c, it is seen that the live load tends to cause a compression of 27.0 in 4–5 when the loads at L_1 and L_2 are on the bridge. The member can only resist a compression of 17.5 (this being the dead load tension already in it), the resulting stress in 4–5 being 0. The remainder of the live load stress tends to distort the member 4–5, and throws the counter in this panel into action. The stress in the counter is then + 9.5.

The maximum stress in 3–4 is obtained when the live loads at and to the right of L_3 are on the bridge, and is — 87.2. The minimum stress in 3–4 is obtained when the live loads at L_1 and L_2 are on the bridge, and is 0. It is seen that the stress in 3–4 must be 0; as the stress in 4–5 has been shown to be 0 for this loading, and for equilibrium at U_2, the stress in 3–4 must also be 0.

The maximum stress in the counter 6–6′ (U_3L_3') is obtained when there are live loads at L_3', L_2', and L_1', and is + 54.0.

The minimum stress in 6–6′ is obtained when there is no live load on the bridge, when the bridge is fully loaded, or when the live loads at L_1, L_2, and L_3 are on the span, and is 0. There are no dead load stresses in the center diagonals of this truss.

The maximum stress in 5–6 is obtained when there are live loads at L_3', L_2', and L_1', and is -43.8. The minimum stress in 5–6 is obtained when there is no live load on the bridge, also when the live loads at L_1, L_2, and L_3 are on the bridge, and is 0.

The maximum stress in the other counter L_3U_3' is obtained when the loads at L_1, L_2, and L_3 are on the bridge, and is $+ 54.0$. The minimum stress is obtained when there are no live loads on the bridge, when the bridge is fully loaded, or when there are live loads at L_3', L_2', and L_1'.

It is unnecessary to determine the stresses in the members beyond the center line; as the truss is symmetrical.

181. Loadings for Maximum and Minimum Stresses. *Conclusions.* From a comparison of the stresses shown in Fig. 122, the following conclusions may be drawn for maximum and minimum dead and live load stresses in a Pratt truss.

(a) *For maximum chord stresses, load the bridge fully with live and dead loads.*

(b) *For minimum chord stresses, load the bridge with dead load only.*

(c) *For maximum web stress in any member (except the hip vertical), load the longer segment of the truss with live load. For maximum stress in the hip vertical, load the bridge so that there will be a live load at L_1.*

(d) *For minimum web stress in any member (except the hip vertical), load the shorter segment of the bridge with live load. For minimum stress in the hip vertical, load the bridge with dead load only.*

In making the combinations for maximum and minimum stresses, it should be borne in mind that the counter in any panel does not act until the dead load stress in the main diagonal in that panel has been reduced to zero by the live load.

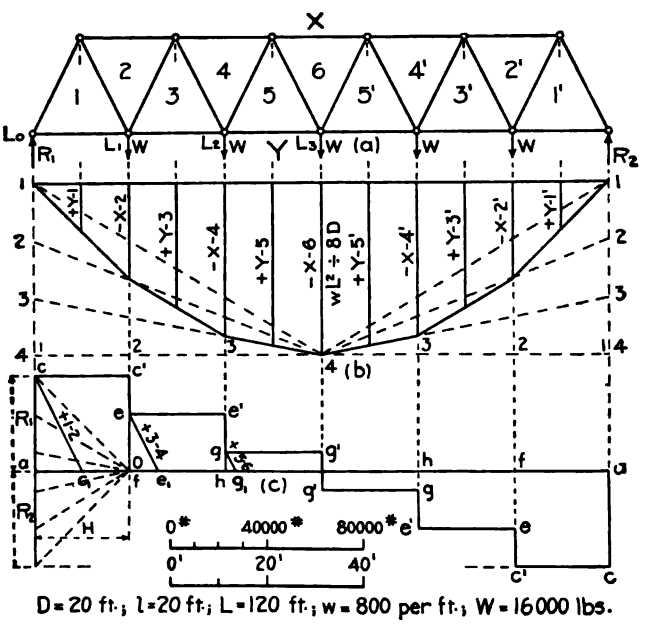
182. The stresses in the members of a Howe truss may be determined in a similar manner to that used in finding those in a

Pratt truss. In the Howe truss, the vertical members take tension only, and the diagonals, compression only.

ART. 3. STRESSES BY GRAPHIC MOMENTS AND SHEARS.

The method of graphic moments and shears, as applied to the solution of the stresses in a bridge truss, will be explained in this article.

183. Problem. It is required to find the stresses in the six-panel Warren truss shown in Fig. 124, a. The truss has a span of 120 feet, a panel length of 20 feet, and a depth of 20 feet. The dead load will be taken at 800 lbs. per ft. per truss, and the live load at 1600 lbs. per ft. per truss.



D = 20 ft.; l = 20 ft.; L = 120 ft.; w = 800 per ft.; W = 16000 lbs.

FIG. 124. DEAD LOAD STRESSES BY GRAPHIC MOMENTS AND SHEARS.

184. Dead Load Chord Stresses. Draw the truss diagram (Fig. 124, a) to scale, and load it fully with dead load. The dead

panel load is equal to $800 \times 20 = 16\,000$. Draw the load line shown in Fig. 124, c for the dead load. Take the pole O with a pole distance H, draw the rays, and construct the funicular polygon shown in Fig. 124, b. The pole distance H, expressed in thousands of pounds, should be numerically equal to the depth of the truss, or to some multiple of the depth, i. e., if $D = 20$, then H should be equal to 20 000, or to some multiple of 20 000. The vertices of the funicular polygon shown in Fig. 124, b lay on the arc of the bending moment polygon for the given truss and loads. The bending moment in any upper chord member is equal to the intercept under the center of moments multiplied by the pole distance H; and the stress in the member is equal to the bending moment divided by the depth of the truss. Now if the pole distance in thousands of pounds is taken equal to the depth of the truss or to some multiple of the depth, then the intercept will represent to scale the stress in the member; provided a scale, which may be determined in the following manner, is used in measuring the intercept. In this problem, $D = 20$; and suppose that $H = 40\,000$, and that the linear scale of the truss is 1 inch = 40 feet. Then the intercepts should be measured to a scale of $40 \times \frac{40\,000}{20} = 80\,000$ pounds to the inch. If H had been taken equal to 20 000 pounds, then the scale to be used in measuring the intercepts should have been 1 inch = 40 000 pounds.

The stresses in the lower chord members are mean proportionals between those in the adjacent upper chord members, and the intercepts are to be taken as shown in Fig. 124, b. It is thus seen that the stress in any chord member may be found directly by scaling the intercept under the center of moments.

The ordinates to the stress polygon shown in Fig. 124, b may be obtained without drawing the force and funicular polygons, in the following manner. The stress in the center member X–6 of the upper chord is equal to the bending moment at the center of the truss due to the uniform load of w lbs. per linear ft. of truss divided by the depth of the truss, i. e., to $\frac{wL^2}{8D}$. Lay off the

middle ordinate X–6 equal to $\frac{800 \times 120 \times 12}{8 \times 20} = 72\,000$, to any given scale. Draw the horizontal line 4–4 equal to one-half the span of the truss; also draw the line 1–4 parallel to the middle ordinate X–6. Divide the horizontal line 4–4 into three equal parts (one-half the number of panels in the truss); also divide the line 1–4 into the same number of equal parts, and number as shown. Draw the radial lines 1–4, 2–4, and 3–4 to meet verticals through the lower chord panel points. Then these points of intersection will give points on the required stress polygon.

If the truss has an odd number of panels, the above method should be modified as follows: Divide the horizontal line 4–4 (one-half the span of the truss) into as many parts as there are panels in the entire truss; and use only the alternate points of division.

It is seen that the method of graphic moments and shears does not give the kind of stress. If the kind of stress is not evident by inspection, then it may be found by algebraic methods.

185. Live Load Chord Stresses. The live load is equal to 1600 lbs. per ft. per truss, and the live panel load is $1600 \times 20 = 32\,000$. The live load chord stresses may be obtained by constructing a diagram similar to that used for the dead load chord stresses, or they may be more easily obtained from the dead load stresses by direct proportion.

186. Dead Load Web Stresses. The reactions for the truss loaded with the dead load are represented by R_1 and R_2 (Fig. 124, c). In a bridge with parallel chords, the web members resist the shear. The shear in panel L_0L_1 is equal to $+ R_1$; in L_1L_2, to $+ (R_1 - W)$; and in L_2L_3, to $+ (R_1 - 2W)$. At L_3, the shear passes through 0. The dead load shear line is shown by the stepped line in Fig. 124, c. Now the stresses in the web members are equal to the shears in the members multiplied by the secant of the angle that the members make with the vertical. The shears are graphically multiplied by the secant of the angle by drawing the lines cc_1, ee_1, and gg_1 parallel to 1–2, 3–4, and 5–6, respectively. The stresses in 1–2, 3–4, and 5–6 are tension. The

stresses in X–1, 2–3, and 4–5 are numerically equal to those in 1–2, 3–4, and 5–6, respectively, and are compression. Since the truss and loads are symmetrical about the center, it is only necessary to draw the diagram for one-half of the truss.

187. Live Load Web Stresses. Another diagram must be drawn for the live load web stresses. This diagram may be constructed in the following manner:

Assume that the Warren truss shown in Fig. 125, a instead of being a simple span is a cantiliver truss, the right end being

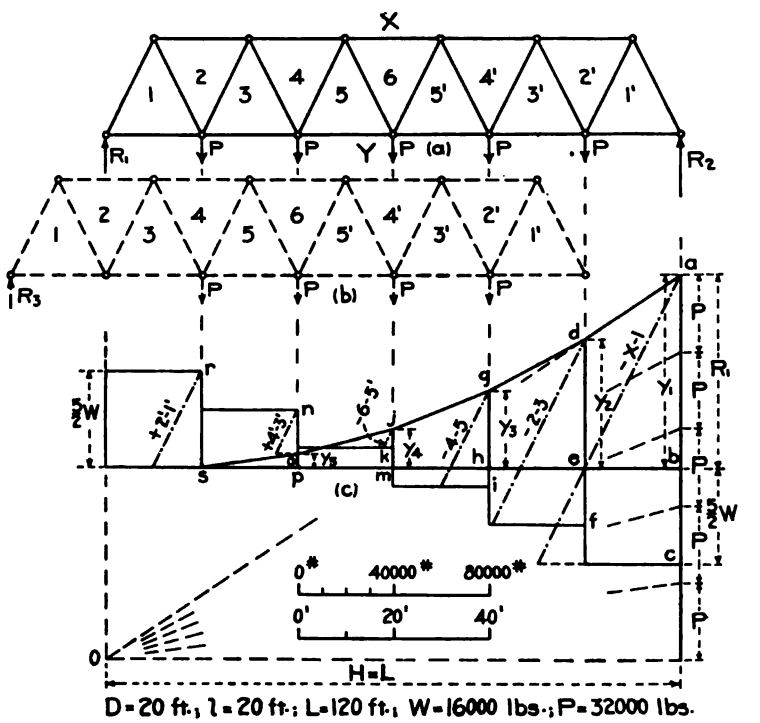


FIG. 125. LIVE LOAD WEB STRESSES—MAXIMUM WEB STRESSES.

fixed and the left end free. Then with the cantilever truss fully loaded with live load, lay off the load line (Fig. 125, c), assume a pole O, and with a pole distance H = span L, draw the funicular polygon, starting the polygon at *a*, the top of the load line. Now

the bending moment at the right support is equal to the intercept y_1 multiplied by the pole distance H. But the given truss instead of being a cantilever truss is a simple span; and is held in position by the reaction R_1 at the left end, instead of being fixed at the right end. Therefore the moment at the right end, which is equal to Hy_1, must be equal to the moment of the left reaction. Now the moment of the left reaction is equal to R_1L, where L is the span of the truss; therefore $Hy_1 = R_1L$, and since $H = L$ by construction, $y_1 = R_1$.

The maximum live load shear in X–1 and 1–2 is obtained when the truss is fully loaded with live load, and is $y_1 = R_1$. The stress in X–1, which is numerically equal to that in 1–2, is obtained by drawing a line through a point *a* parallel to the member X–1.

Now move the truss one panel to the left, but let the loads remain stationary. The new position of the truss, together with the new loading, is shown in Fig. 125, b. With the same pole O and pole distance H, draw a new funicular polygon. This funicular polygon will coincide with a part of the first polygon, the part to the left of the point d (Fig. 125, c), being identical in both polygons. As above, the bending moment at the right end for this loading is equal to the moment of the left reaction, i. e., $Hy_2 = R_3L$, or $y_2 = R_3$. Now the loading shown in Fig. 125, c is that for a maximum shear in the members 2–3 and 3–4; and since $y_2 = R_3$ is the shear in these members, the stresses are obtained by drawing through d a line parallel to the member 2–3.

In like manner, gh may be shown to be the shear in the members 4–5 and 5–6 when there is a maximum stress in 4–5 and 5–6; and the stresses in these members are obtained by drawing a line through g parallel to the member 4–5.

Likewise, jm is equal to the shear in 6–5′ and 5′–4′ when there is a minimum shear in 6–5′ and 5′–4′, and the stresses in these members are obtained by drawing a line through j parallel to the member 6–5′.

Similarly, the minimum stresses in 4′–3′ and 3′–2′ are obtained by drawing a line through o parallel to 4′–3′.

The minimum live load stresses in $2'-1'$ and $X'-1'$ are zero, when there is no live load on the truss.

The kind of stress is not given by this graphic construction, and if not evident by inspection, it may be obtained by algebraic methods.

188. Maximum and Minimum Dead and Live Load Stresses. (a) *Chord Stresses.* The maximum chord stresses may be obtained by adding the dead and live load stresses. The minimum chord stresses are equal to the dead load stresses.

(b) *Web Stresses.* The maximum and minimum web stresses may be obtained by graphically combining the dead and live load shears, provided the dead and live load shear diagrams are placed as shown in Fig. 125, c. In this figure, the dead load shear in X–1 is $bc = \frac{5}{2} W$, the live load shear is $R_1 = y_1 = ab$, and the maximum shear is ac. Likewise, df = maximum shear in 2–3, gi = maximum shear in 4–5, jk = minimum shear in 6–5′, no = minimum shear in $4'-3'$, and rs = minimum shear in $2'-1'$. The stresses in the members are obtained by graphically resolving these shears.

Art. 4. Stresses in a Bowstring Truss—Triangular Web Bracing.

The solution of the maximum and minimum stresses in a bowstring truss will be taken up in this article. The dead load stresses will be determined by graphic resolution, and the live load stresses by a special application of the method of graphic resolution. The method used is general in its application to trusses with triangular web bracing, and the upper chord panel points may lay on the arc of any curve.

189. Problem. It is required to find the maximum and minimum stresses in the bowstring truss shown in Fig. 126, a. The upper chord panel points lay on the arc of a parabola with a middle ordinate of 24 ft. The truss has a span of 160 ft., and a

panel length of 20 ft. The dead load will be taken at 500 lbs. per ft. per truss, and the live load at 1000 lbs. per ft. per truss.

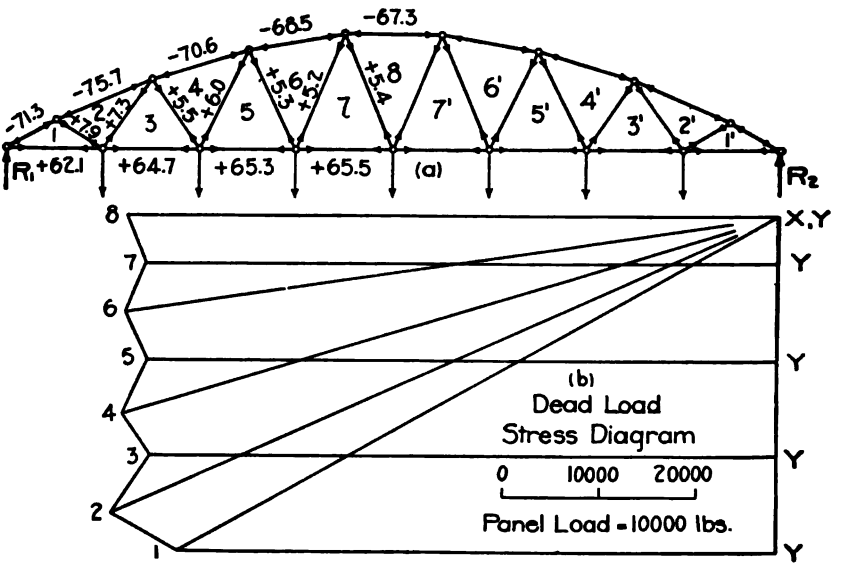


FIG. 126. DEAD LOAD STRESSES—BOWSTRING TRUSS.

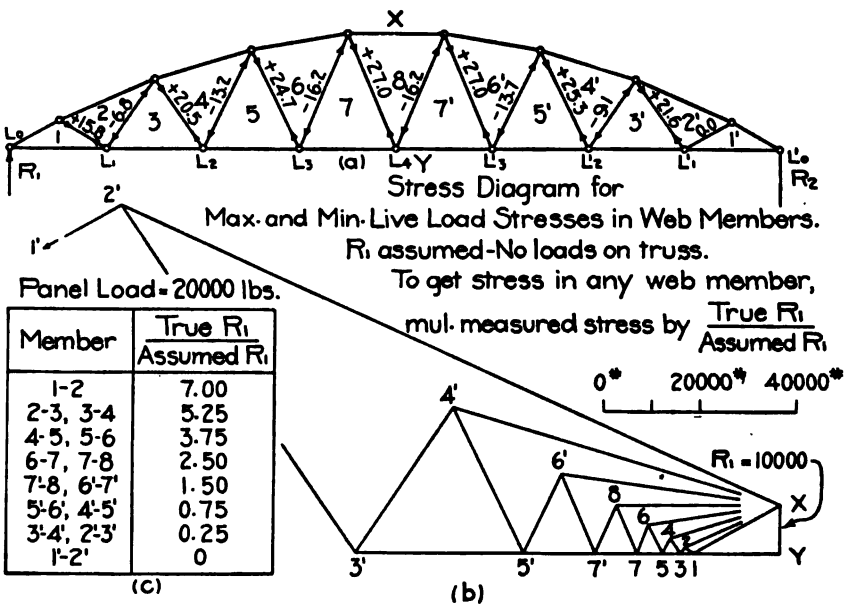
190. Chord Stresses. (a) *Dead Load Chord Stresses.* The dead panel load is equal to $500 \times 20 = 10\,000$ lbs. The dead load chord stresses are obtained by drawing the dead load stress diagram for one-half of the truss, as shown in Fig. 126, b. The stresses, expressed in thousands of pounds, are shown on the members of the truss in Fig. 126, a.

(b) *Live Load Chord Stresses.* The live panel load is equal to $1000 \times 20 = 20\,000$ lbs. The live load chord stresses are obtained by proportion from the dead load chord stresses. These stresses are double the corresponding dead load stresses.

191. Web Stresses. (a) *Dead Load Web Stresses.* The dead load web stresses are obtained from the dead load stress diagram shown in Fig. 126, b. These stresses are shown on the members of the truss in Fig. 126, a. It is seen that the web stresses in this type of truss are small, and that the stresses in

all the web members are tension. These members act as auxiliary members to transfer the load to the upper chord panel points.

(b) *Live Load Web Stresses.* The live load web stresses are obtained from the stress diagram shown in Fig. 127, b. This diagram is constructed by assuming a value (in this case 10 000 lbs.) for the left reaction R_1, and drawing the stress diagram with no loads on the truss.



Member	True R_1 / Assumed R_1
1-2	7.00
2-3, 3-4	5.25
4-5, 5-6	3.75
6-7, 7-8	2.50
7'-8, 6'-7'	1.50
5'-6', 4'-5'	0.75
3'-4', 2'-3'	0.25
1'-2'	0

(c)

FIG. 127. LIVE LOAD WEB STRESSES—BOWSTRING TRUSS.

It has already been shown that, for a maximum live load stress in any web member, the longer segment of the truss should be loaded; and for a minimum live load stress, the shorter segment should be loaded. Therefore, for a maximum stress in any web member to the left of the center of the truss, there should be no loads to the left of that member; and for a minimum stress in the corresponding member on the right of the center, there should be no loads to the left of that member. The shear in any member is therefore equal to the reaction; and the stress in the member may be found by multiplying the stress in that member, obtained

from the stress diagram shown in Fig. 127, b, by the ratio of the actual live load reaction (for a maximum or minimum stress in that member) to the assumed reaction. For example, to get a maximum stress in 1-2, the bridge is fully loaded with live load, and the reaction R_1=70 000. The maximum stress in 1–2 is therefore equal to the stress scaled from Fig. 127, b multiplied by $\frac{70\,000}{10\,000} = +15.8$.

For stresses in 2–3 and 3–4, load all joints except L_1; for stresses in 4–5 and 5–6, load all joints except L_1 and L_2; for stresses in 6–7 and 7–8, load all joints except L_1, L_2, and L_3; for stresses in 8–7′ and 7′–6′, load the joints L_3', L_2', and L_1'; for stresses in 6′–5′ and 5′–4′, load joints L_2' and L_1'; for stresses in 4′–3′ and 3′–2′, load the joint L_1' only; and for stress in 2′–1′, there should be no loads on the truss.

The ratios of the actual reactions to the assumed reactions for the above loadings are shown in Fig. 127, c.

The live load stresses are shown on the members of the truss in Fig. 127, a, and are obtained by scaling the stresses from the diagram in Fig. 127, b and multiplying these stresses by the ratios shown in Fig. 127, c.

192. Maximum and Minimum Dead and Live Load Stresses. (a) *Chord Stresses.* The maximum chord stresses are

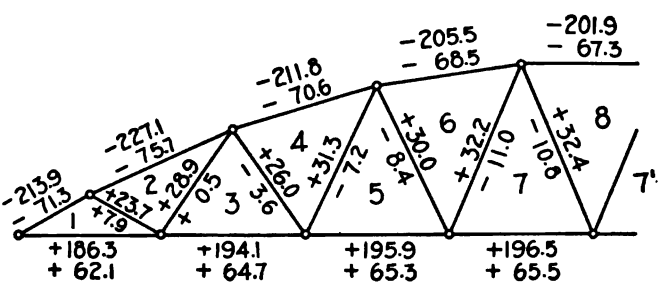


FIG. 128. MAXIMUM AND MINIMUM STRESSES—BOWSTRING TRUSS.

equal to the sums of the corresponding dead and live load stresses, and the minimum chord stresses are the dead load stresses. These stresses are shown on the truss diagram in Fig. 128.

(b) *Web Stresses.* The maximum and minimum web stresses are obtained by combining the dead and live load stresses shown in Fig. 126, a and Fig. 127, a, respectively. In making the combinations, both the stresses in the member and those in the corresponding member on the other side of the center line of the truss must be considered.

The maximum stress in 1–2 is $+7.9+15.8=+23.7$, and the minimum stress is $+7.9$, the dead load stress.

The maximum stress in 2–3 is $+7.3+21.6=+28.9$, and the minimum stress is $+7.3-6.8=+0.5$.

The maximum stress in 3–4 is $+5.5+20.5=+26.0$, and the minimum stress is $+5.5-9.1=-3.6$.

The maximum stress in 4–5 is $+6.0+25.3=+31.3$, and the minimum stress is $+6.0-13.2=-7.2$.

The maximum stress in 5–6 is $+5.3+24.7=+30.0$, and the minimum stress is $+5.3-13.7=-8.4$.

The maximum stress in 6–7 is $+5.2+27.0=+32.2$, and the minimum stress is $+5.2-16.2=-11.0$.

The maximum stress in 7–8 is $+5.4+27.0=+32.4$, and the minimum stress is $+5.4-16.2=-10.8$.

Art. 5. Stresses in a Parabolic Bowstring Truss.

The maximum and minimum stresses in a parabolic bowstring truss will be found in this article by a special graphic method. The method given applies only to trusses whose upper chords panel points lay on the arc of a parabola.

193. Problem. It is required to find the maximum and minimum stresses in the parabolic bowstring truss, half of which is shown in Fig. 129. The truss has a span of 160 ft., a panel length of 20 ft., and a depth at the center of 24 ft. The dead load will be taken at 500 lbs. per ft. per truss, and the live load at 1000 lbs. per ft. per truss.

194. Chord Stresses. (a) *Dead Load Chord Stresses.* The dead panel load $= 500 \times 20 = 10\,000$ lbs., or say 10.0. Now the bending moment at any point in a truss loaded with a uniform load

varies as the ordinates to a parabola; and the stress in any chord member is equal to the bending moment divided by the depth of the truss. Since the moment at any point varies as the ordinates to a parabola, and the moment arm for the stress in any lower

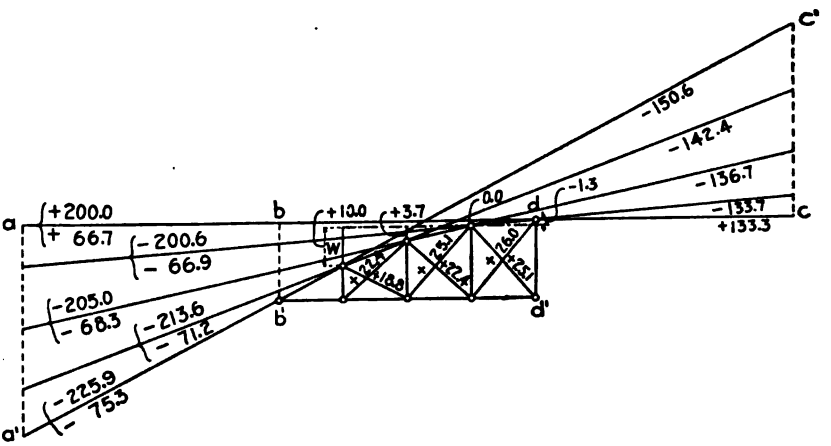


FIG. 129. MAXIMUM AND MINIMUM STRESSES—PARABOLIC BOWSTRING TRUSS.

chord member is an ordinate to the parabola, it is seen that the stress in the lower chord is constant.

For this truss, the moment at the center $= \frac{wL^2}{8}$, and the dead load stress in the lower chord $= \frac{wL^2}{8D} = \frac{500 \times 160 \times 160}{8 \times 24}$ $= 66\,700$. Since the stress in the lower chord is constant when the bridge is fully loaded, it is seen that the diagonals are not stressed, and that the horizontal components of the stresses in the upper and lower chords are equal. The dead load stresses in the upper chord members may be found graphically, as follows: Lay off ab (Fig. 129) equal to the stress in the lower chord, and draw the vertical lines aa′ and bb′. Draw lines parallel to the upper chord members, and the distances intercepted on these lines between the vertical lines aa′ and bb′ will represent the stresses in the corresponding upper chord members.

(b) *Live Load Chord Stresses.* The maximum live load chord stresses are obtained when the bridge is fully loaded with live load, and are found in the same manner as the dead load chord stresses. To find these stresses, lay off bc (Fig. 129) equal to the live load stress in the lower chord $= \frac{1000 \times 160 \times 160}{8 \times 24} = 133\,300$. The upper chord stresses are represented by the distances intercepted between the vertical lines bb′ and cc′ on lines drawn parallel to the chord members.

The minimum live load chord stresses are zero.

195. Web Stresses. (a) *Dead Load Web Stresses.* Since the stress in the lower chord is constant when the bridge is loaded with a uniform load, it is seen that the dead load stresses in the diagonals are zero. The dead load stresses in the verticals are tensile stresses, and are each equal to the dead panel load $= +\ 10.0$.

(b) *Live Load Web Stresses.* The maximum live load stresses in the diagonals and the minimum live load stresses in the verticals may be obtained directly from the diagram shown in Fig. 129. The maximum live load stresses in the verticals, and the minimum live load stresses in the diagonals are obtained when the bridge is fully loaded with live load. The maximum live load stresses in the verticals are + 20.0 (the live panel load), and the minimum live load stresses in the diagonals are 0.

The diagram shown in Fig. 129 is constructed, as follows: Lay off bc to any scale $= \frac{1000 \times 160 \times 160}{8 \times 24} = 133\,300$. Construct the half truss diagram, making the half span equal to $b'd' = \frac{1}{2}bc$. Since the span is 160 ft. and the depth of the truss is 24 ft., make $dd' = \frac{24}{160}bc$. With b′d′ equal to the half span and dd′ equal to the depth at the center, draw the outline of the given truss.

The maximum live load stresses in the diagonals sloping upward to the left are obtained when the longer segment of the bridge is loaded, and the maximum stresses in the diagonals sloping upward

to the right, when the shorter segment of the bridge is loaded with live load. These stresses are obtained directly by scaling the members to the given scale. The minimum live load stress in the first vertical from the left end is zero. The minimum live load stresses in the other verticals are compression, and are represented by the vertical distances between the first upper chord panel point and each succeeding upper chord panel point.

The proof of the construction shown in Fig. 129 is given in § 197.

196. Maximum and Minimum Dead and Live Load Stresses. (a) *Chord Stresses.* The maximum stress in the lower chord is represented by ac (Fig. 129), which represents the sum of the dead and live load chord stresses. The maximum stress in the upper chord members are represented by the distances intercepted between the vertical lines aa′ and cc′ by the upper chord members produced. The minimum chord stresses are the dead load stresses. The maximum and minimum chord stresses are shown on the members produced.

(b) *Web Stresses.* The maximum stresses in the diagonals are represented by the lengths of the diagonals, the dead load stresses in these members being zero. These stresses are shown on the members of the truss diagram. The minimum stresses in the diagonals are zero. The maximum stresses in the verticals are equal to the sums of the dead and live panel loads $= + 10.0 + 20.0 = + 30.0$. The minimum stresses in the verticals are shown in Fig. 129. These stresses are obtained by laying off to scale the dead panel load, $W = 10.0$, above the first upper chord panel point, and drawing the horizontal dot and dash line shown in Fig. 129. The dead and live load stresses are of an opposite kind, and the resulting stress is thus obtained. The minimum stresses in the verticals are represented by the distances between this line and each upper chord panel point. Distances measured below the line represent tensile stresses, and those above, represent compressive stresses.

197. Proof of Construction Shown in Fig. 129. It will now be proved that for the construction shown in Fig. 129, (a) the maximum live load stresses in the diagonals are represented by

the lengths of the diagonals, and (b) that the minimum live load stresses in the verticals, i. e., the maximum live load compressive stresses, are represented by the vertical distances from the first upper chord panel point to each upper chord panel point.

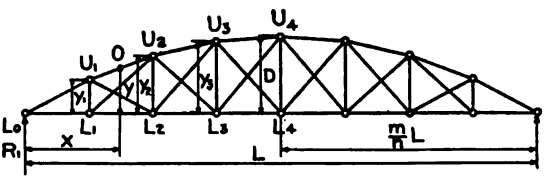


FIG. 130. PARABOLIC BOWSTRING TRUSS.

(a) **Stresses in Diagonals.** Let Fig. 130 represent a parabolic bowstring truss in which the span $= \frac{pL^2}{8D}$, and the depth $= \frac{D}{L} \times \frac{pL^2}{8D} = \frac{pL}{8}$ = panel load P.

Also, let x = abscissa and y = ordinate of any point O on the parabola,
p = live load per foot,
n = number of panels in truss,
m = number of loads on truss,
L = span of truss,
D = depth of truss.

It is required to prove that the maximum live load stress in any diagonal is represented by the length of that diagonal.

The equation of the parabola about the left end L_0 is

$$(\frac{L}{2} - x)^2 = \frac{L^2}{4D}(D - y),$$

from which,

$$y = \frac{4Dx}{L^2}(L - x).$$

Now the horizontal component of the maximum live load stress in any diagonal member is equal to the difference of stress in the two adjacent lower chord members when the truss is loaded for a maximum stress in that diagonal.

Consider any diagonal, say U_2L_3 in panel L_2L_3, whose maximum stress occurs when there are m loads on the truss.

For this loading, $R_1 = \frac{(m+1)m}{2n} \times \frac{pL}{n}$.

Moment of R_1 about the panel point to right of diagonal =

$$\frac{(m+1)m}{2n} \times \frac{pL}{n} \times (n - m)\frac{L}{n},$$

and stress in lower chord member to right of diagonal =

$$S = \frac{\frac{(m+1)m}{2n} \times \frac{pL}{n} \times (n-m)\frac{L}{n}}{y_2}$$

$$= \frac{pL^4(m+1)\ (n-m)m}{8n^2Dx\ (L-x)}.$$

In above equation, $x = \frac{L}{n}(n-m)$, and substituting this value of x, we have

$$S = \frac{pL^4\ (m+1)\ (n-m)m}{8n^2D\ \frac{L}{n}(n-m)\ [L - \frac{L}{n}(n-m)]}$$

$$= \frac{pL^2\ (m+1)}{8nD}.$$

Also moment of R_1 about panel point to left of diagonal =

$$\frac{(m+1)m}{2n} \times \frac{pL}{n} \times (n-m-1)\ \frac{L}{n},$$

and stress in lower chord member in same panel as diagonal =

$$S' = \frac{\frac{(m+1)m}{2n} \times \frac{pL}{n} \times (n-m-1)\ \frac{L}{n}}{y_2}$$

$$= \frac{pL^4\ (m+1)\ (n-m-1)m}{8n^2Dx\ (L-x)}.$$

In above equation, $x = \frac{L}{n}(n-m-1)$, and substituting this value of x, we have

$$S' = \frac{pL^4\ (m+1)\ (n-m-1)m}{8n^2D \cdot \frac{L}{n}(n-m-1)\ [L - \frac{L}{n}(n-m-1)]}$$

$$= \frac{pL^2m}{8nD}.$$

Now $$S - S' = \frac{pL^2\ (m+1)}{8nD} - \frac{pL^2m}{8nD}$$

$$= \frac{pL^2}{8nD} = \text{horizontal component of stress in diagonal.}$$

Now the span of the truss was laid off equal to $\frac{pL^2}{8D}$. Therefore the horizontal component of the maximum stress in the diagonal, which is $\frac{pL^2}{8nD}$, is represented by one panel length of the lower chord. Since one panel length represents the horizontal component of the stress in the

diagonal, the length of the diagonal itself will represent the maximum stress in it.

(b) **Stresses in Verticals.** It is required to prove that the minimum live load stresses in the verticals, i. e., the maximum compressive stresses, are represented by the vertical distances from the first upper chord panel point to each upper chord panel point.

Consider any vertical, say U_2L_2 (Fig. 130), whose maximum compressive stress occurs when there are m loads on the truss. Now the stress in U_2L_2 when there are m loads on the truss is equal to the difference in stress between L_2L_3 and L_1L_2 for this loading multiplied by the tangent of the angle that U_1L_2 makes with L_1L_3.

The stress in the lower chord member to the right of the vertical is

$$S' = \frac{pL^2m}{8nD}.$$

The stress in the lower chord member to the left of the vertical is

$$S'' = \frac{R_1 (n - m - 2) \frac{L}{n}}{y_1}$$

$$= \frac{\frac{(m+1)m}{2n} \times \frac{pL}{n} \times (n - m - 2) \cdot \frac{L}{n}}{\frac{4Dx}{L^2}(L - x)}$$

$$= \frac{pL^4(m+1)(n-m-2)m}{8n^3D\,x\,(L-x)}$$

In above equation $x = \frac{L}{n}(n - m - 2)$, and substituting this value of x, we have

$$S'' = \frac{pL^4(m+1)(n-m-2)m}{8n^3D\frac{L}{n}(n-m-2)[L - \frac{L}{n}(n-m-2)]}$$

$$= \frac{pL^2(m+1)m}{8nD(m+2)}.$$

$$S' - S'' = \frac{pL^2m}{8nD} - \frac{pL^2(m+1)m}{8nD(m+2)}$$

$$= \frac{pL^2m}{8nD(m+2)}.$$

The stress in any vertical member is

$\frac{pL^2m}{8nD(m+2)} \tan \theta$, where θ is the angle that the diagonal which meets the required vertical at the lower chord makes with the lower chord.

An expression will now be found for tan θ.

$$\text{Tan } \theta = \frac{y}{\frac{L}{n}} = \frac{ny}{L}.$$

Now $y = \frac{4Dx}{L^2} \cdot (L - x)$, in which $x = \frac{L}{n} \cdot (n - m - 2)$.

Substituting values of x and y, we have

$$\tan \theta = \frac{\frac{4nD}{L^2} \times \frac{L}{n}(n - m - 2)[L - \frac{L}{n} \cdot (n - m - 2)]}{L}$$

$$= \frac{4D(n - m - 2)(m + 2)}{Ln}.$$

$$\text{Stress in vertical} = \frac{pL^2m}{8nD(m+2)} \times \frac{4D(n - m - 2)(m + 2)}{Ln}$$

$$= \frac{pLm(n - m - 2)}{2n^2} = \frac{pnlm(n - m - 2)}{2n^2}, \text{ where } l = \text{panel length},$$

$$= \frac{Pm(n - m - 2)}{2n}, \text{ where } P = \text{panel load}.$$

For the problem given in § 193, where n = 8;

when m = 6, then stress in vertical = 0;
" m = 5, " " " " = $\frac{5}{16}$P;
" m = 4, " " " " = $\frac{1}{2}$P;
" m = 3, " " " " = $\frac{9}{16}$P;

Since, by construction, $D = \frac{D}{L} \times \frac{pL^2}{8D} = P$, it is seen that the above stresses in the verticals are represented by the ordinates to a parabola whose ordinates are $\frac{7}{16}$P less than those to the first parabola.

The above relation may be proved in a different manner, as follows: Produce each upper chord member of the eight-panel truss (Fig. 130) to intersect the lower chord produced. Load the bridge for the maximum compressive stress in each vertical, and find the stresses in the verticals by moments. If l represents a panel length, the moment arms for the members U_1L_1, U_2L_2, U_3L_3, and U_4L_4 are l, 2 $\frac{2}{5}$ l, 5 l, and 16 l, respectively; the reactions for maximum compressive stresses are $\frac{21}{8}$ P, $\frac{15}{8}$ P, $\frac{10}{8}$P, and $\frac{6}{8}$ P; and the stresses are 0, $\frac{5}{16}$ P, $\frac{1}{2}$ P, and $\frac{9}{16}$P. Now the above ordinates are equal to the ordinates to the given parabola minus the first ordinate $\frac{7}{16}$P.

ART. 6. WIND LOAD STRESSES IN LATERAL SYSTEMS.

The wind load stresses in the upper and lower laterals of a bridge will be determined in this article. All the wind load will be assumed to act on the windward truss; although the assumption is sometimes made that half of the wind acts on each truss. An initial tension is sometimes put into the laterals to give additional rigidity to the truss, but will not be considered in this article. The upper chord members are a part of the top lateral system, and the lower chord members and floorbeams are a part of the bottom lateral system. Most specifications state that the wind load stresses need not be considered unless they are 25, or, in some cases, 30 per cent of the live and dead load stresses.

198. Upper Laterals. It is required to find the stresses in the upper lateral system shown in Fig. 131, a. The panel length is 20 ft., and the width is 16 ft. The wind load will be taken at 150 lbs. per ft., and will be treated as a fixed load. The panel load $W = 150 \times 20 = 3000$ lbs. The end strut of the upper lateral system is also a part of the portal system, and its stress will not be considered at this time.

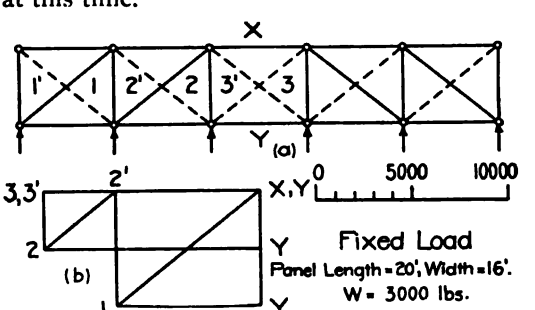


FIG. 131. WIND LOAD STRESSES IN UPPER LATERALS.

(a) *Chord Stresses.* The fixed wind load stress diagram for one-half of the truss is shown in Fig. 131, b. When the wind acts in the direction shown, the diagonals represented by full lines are in action; and the stress diagram was drawn using these diagonals. When the direction of the wind is reversed, the diagonals shown by dotted lines are in action. The stresses in the chord

members for the wind acting in the direction shown in Fig. 131, a are given in the first line of Fig. 132, a. The stresses in these members when the direction of the wind is reversed are given in the second line of Fig. 132, a. It is seen that the chord members are subjected to reversals of stress due to the wind load. Since

STRESSES IN CHORD MEMBERS						
Chord Member	Y-1	Y-2	Y-3	X-1'	X-2'	X-3'
Wind as shown	− 7.50	− 11.25	− 11.25	0.00	+ 7.50	+ 11.25
Opposite direction	0.00	+ 7.50	+ 11.25	− 7.50	− 11.25	− 11.25

(a)

STRESSES IN WEB MEMBERS					
Web Member	1'-1	1-2'	2'-2	2-3'	3'-3
Wind as shown	+ 9.60	− 6.00	+ 4.80	− 3.00	0.00
When wind comes from opposite direction, other set of diagonals acts.					

(b)

FIG. 132. TABLE OF STRESSES—UPPER LATERALS.

the chord members are already stressed, it is seen that there are actually no reversals of stress unless the wind load stresses are of an opposite kind and are larger than the stresses already in the chord members.

(b) *Web Stresses.* The wind load stresses in the web members may be obtained from the stress diagram shown in Fig. 131, b. These stresses are shown in Fig. 132, b. When the wind acts from the opposite direction, the set of diagonals shown by the dotted lines is in action; and the stresses in them are equal to the stresses in the corresponding members when the wind acts in the direction shown.

199. **Lower Laterals.** It is required to find the stresses in the lower lateral system shown in Fig. 133, a. The panel length is 20 ft., and the width is 16 ft. The wind load will be taken at 600 lbs. per ft. Of this load, 150 lbs. per ft. will be treated as a fixed load, and 450 lbs. per ft. as a moving load. The fixed panel load is $150 \times 20 = 3000$ lbs., and the moving panel load is $450 \times 20 = 9000$ lbs.

(a) *Chord Stresses.* (1) *Fixed Load.* When the wind acts in the direction shown in Fig. 133, a, the diagonals shown by full

lines are in action. The fixed wind load stress diagram is shown in Fig. 133, b; and the stresses in the chord members are shown in Fig. 134, a.

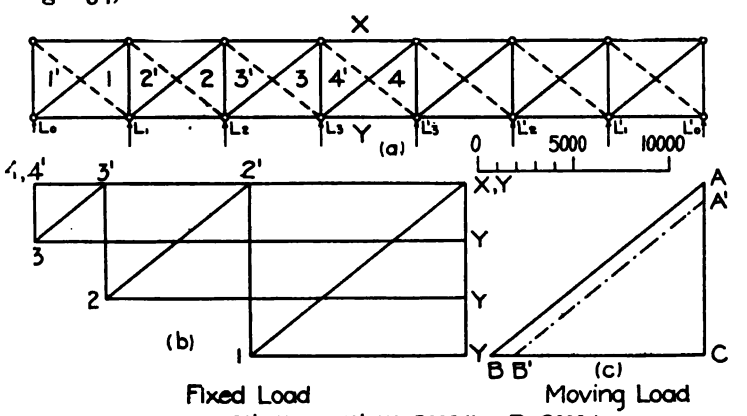


FIG. 133. WIND LOAD STRESSES IN LOWER LATERALS.

(2) *Moving Wind Load.* The maximum stresses in the chord members due to the moving load are obtained when the load covers the entire span. Since the moving panel load is three times that of the fixed load, these stresses are obtained by multiplying the corresponding fixed wind load stresses by three. The stresses due to the moving load are given in the second line of Fig. 134, a.

(b) *Web Stresses.* (1) *Fixed Load.* The stresses in the lower chord web members for the fixed load of 150 lbs. per ft. are obtained from the stress diagram shown in Fig. 133, b. These stresses are given in Fig. 134, b.

(2) *Moving Load.* The stresses in the lower chord web members for the moving load of 450 lbs. per ft. are obtained from the stress triangle shown in Fig. 133, c. This triangle is constructed as follows: Lay off A′C equal to twice the width, and B′C equal to twice the panel length; and draw the line A′B′. Then lay off AC equal to the moving panel load P, and draw AB parallel to A′B′ to meet the horizontal line BC. Then AC and AB represent the stresses in the struts and diagonals, respectively, due to a reaction equal to P. The stresses in these members due to any

reaction are obtained by proportion. The stresses in the web members to the left of the center of the truss for a moving load at each of the lower chord panel points are shown in Fig. 134, c. The maximum stress in each member due to the moving load is shown in the last line of Fig. 134, c.

STRESSES IN CHORD MEMBERS

Chord Member	Y-1	Y-2	Y-3	Y-4	X-1'	X-2'	X-3'	X-4'
Fixed Wind Load	−11.25	−18.75	−22.50	−22.50	0.00	+11.25	+18.75	+22.50
Moving Wind Load	−33.75	−56.25	−67.50	−67.50	0.00	+33.75	+56.25	+67.50
Maximum Stress	−45.00	−75.00	−90.00	−90.00	0.00	+45.00	+75.00	+90.00
Opposite direction	− 0.00	+45.00	+75.00	+90.00	−45.00	−75.00	−90.00	−90.00

(a)

STRESSES IN WEB MEMBERS - FIXED LOAD

Web Member	1'-1	1-2'	2'-2	2-3'	3'-3	3-4'	4'-4
Fixed Wind Load	+14.40	− 9.00	+ 9.60	− 6.00	+ 4.80	− 3.00	0.00

(b)

STRESSES IN WEB MEMBERS - MOVING LOAD

Web Member	1'-1	1-2'	2'-2	2-3'	3'-3	3-4'	4'-4
Moving Load at L'_1	+ 2.06	− 1.29	+ 2.06	− 1.29	+ 2.06	− 1.29	+ 2.06
" " " L'_2	+ 4.12	− 2.58	+ 4.12	− 2.58	+ 4.12	− 2.58	+ 4.12
" " " L'_3	+ 6.18	− 3.87	+ 6.18	− 3.87	+ 6.18	− 3.87	+ 6.18
" " " L_3	+ 8.24	− 5.16	+ 8.24	− 5.16	+ 8.24	− 5.16	
" " " L_2	+10.30	− 6.45	+10.30	− 6.45			
" " " L_1	+12.36	− 7.74					
Max. Moving Load	+43.26	−27.09	+30.90	−19.35	+20.60	−12.90	+12.36

(c)

MAXIMUM STRESSES IN WEB MEMBERS

Web Member	1'-1	1-2'	2'-2	2-3'	3'-3	3-4'	4'-4
Fixed Wind Load	+14.40	− 9.00	+ 9.60	− 6.00	+ 4.80	− 3.00	0.00
Moving Wind Load	+43.26	−27.09	+30.90	−19.35	+20.60	−12.90	+12.36
Maximum Stress	+57.66	−36.09	+40.50	−25.35	+25.40	−15.90	+12.36
When wind comes from opposite direction, other set of diagonals acts							

(d)

FIG. 134. TABLE OF STRESSES—LOWER LATERALS.

200. **Maximum and Minimum Stresses in Upper Laterals.** The upper laterals are loaded with a fixed load only, and the maximum chord stresses are shown in the first line of Fig. 132, a. The

minimum chord stresses are obtained when the wind acts from the opposite direction, and are given in the second line of Fig. 132, a.

The maximum web stresses are shown in Fig. 132, b. The minimum web stresses are zero.

201. Maximum and Minimum Stresses in Lower Laterals. The maximum stresses in the lower chord members for fixed and moving loads, when the wind acts in the direction shown, are given in the third line of Fig. 134, a. The minimum chord stresses are obtained when the wind acts from the opposite direction, and are shown in the fourth line of Fig. 134, a.

The maximum web stresses for fixed and moving loads are given in Fig. 134, d. When the wind acts from the other direction, the other set of diagonals is thrown into action, and the stresses in the first set are zero.

ART. 7. STRESSES IN TRUSSES WITH PARALLEL CHORDS BY THE METHOD OF COEFFICIENTS.

In trusses with parallel chords, the bending moment is resisted by the chords, and the shear by the web members. For such trusses, the method explained in this article may be used to advantage for determining the stresses.

202. Algebraic Resolution—Method of Coefficients. To

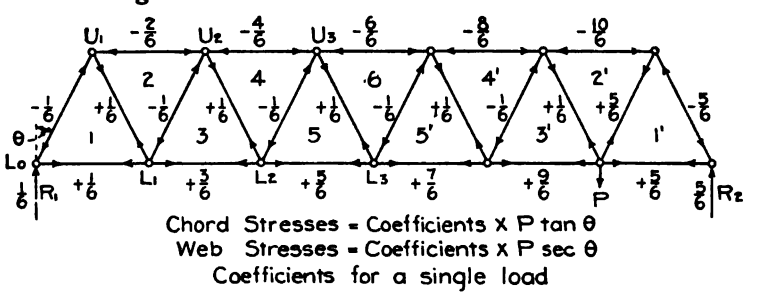


FIG. 135. DEAD LOAD COEFFICIENTS—WARREN TRUSS.

explain this method, let it be required to find the stresses in the members of the truss shown in Fig. 135, the truss being first

loaded with a single load P. For this loading, the left reaction is equal to $\frac{1}{6}$ P, and the right reaction to $\frac{5}{6}$ P. For equilibrium, the summation of both the horizontal and vertical forces at any point must be equal to zero. Resolving the forces at L_0, it is seen that the stress in the member X–1 $= -\frac{1}{6}$ P sec θ, and the stress in Y–1 $= +\frac{1}{6}$ P tan θ; where θ is the angle that the inclined web member makes with the vertical. Using the stress in X–1 already found, and resolving the forces at U_1, the stress in X–2 $= -\frac{2}{6}$P tan θ, and the stress in 1–2 $= +\frac{1}{6}$ P sec θ. Using the stresses in Y–1 and 1–2 already found, and resolving the forces at L_1, the stress in 2–3 $= -\frac{1}{6}$ P sec θ, and the stress in Y–3 $= +\frac{2}{6}$P tan θ. In like manner, the stresses in the remaining members may be determined. Referring to the stresses found above, it is seen that all the chord stresses have the common factor P tan θ, and that all the web members have the common factor P sec θ. It is further seen that these factors are multiplied by coefficients which are expressed in terms of the number of panels in the truss.

These coefficients may be readily found in the following manner: In this discussion, it should be borne in mind that the chord coefficients are to be multiplied by P tan θ, and the web coefficients by P sec θ to get the stresses in the members. Considering the forces at L_0, it is seen that $R_1 = \frac{1}{6}$ and acts upward, therefore, for equilibrium, X–1 $= \frac{1}{6}$ and acts downward; as indicated by the arrow. Since X–1 $= \frac{1}{6}$ and acts toward the left, Y–1 $= \frac{1}{6}$ and acts toward the right. It is seen from the arrows that X–1 is compression (—), and that Y–1 is tension (+).

Now consider the forces at U_1. It has already been shown that X–1 is compression, therefore the arrow on X–1 will act toward the joint. Since X–1 $= \frac{1}{6}$ and acts upward, for equilibrium, 1–2 $= \frac{1}{6}$ and acts downward. Likewise, since both X–1 and 1–2 act toward the right, for equilibrium, X–2 $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$ and acts toward the left.

Next consider the forces at L_1. Since 1–2 $= \frac{1}{6}$ and acts upward, 2–3 $= \frac{1}{6}$ and acts downward. Likewise, since Y–1, 1–2, and 2–3 all act toward the left, for equilibrium, Y–3 $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$ and acts toward the right.

In like manner, the coefficient and the kind of stress may be found for each member (see Fig. 135).

The chord stresses may be obtained by multiplying the coefficients by P tan θ, and the web stresses, by multiplying the coefficients by P sec θ.

The coefficients for any loading may be found in the manner indicated above.

203. Loading for Maximum and Minimum Live Load Stresses. The coefficients for all the members for a load applied at each lower chord joint in turn are shown in Fig. 136. The coefficients shown in the top line are those for a load on the left of the truss. The stresses in the chord and web members may be obtained by multiplying the coefficients by P tan θ and P sec θ, respectively.

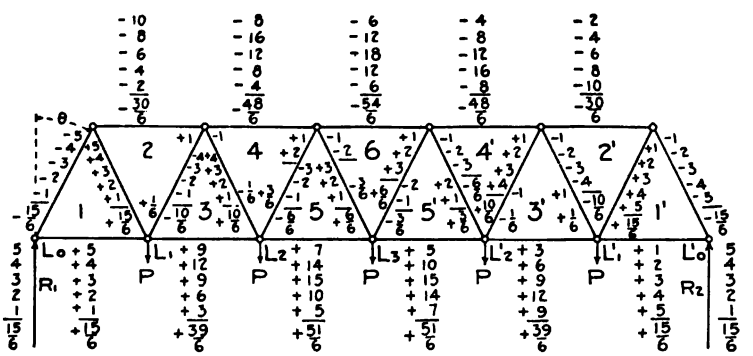


FIG. 136. LIVE LOAD COEFFICIENTS—WARREN TRUSS.

The stress in any number due to any loading may be readily found from Fig. 136. It is seen that each load produces compression in the upper chord and tension in the lower chord. The maximum stresses in the upper and lower chords are therefore obtained when the truss is fully loaded. Referring to Fig. 136, it is seen that the maximum live load stresses in both X–1 and 1–2

are obtained when the truss is fully loaded; since each load produces compression in X–1 and tension in 1–2. The minimum live load stresses are zero, when there are no loads on the truss. The maximum live load stresses in 2–3 and 3–4 are $-\frac{10}{6}$ P sec θ and $+\frac{10}{6}$ P sec θ, respectively, when the loads at L_2, L_3, L_2', and L_1' are on the truss. The minimum live load stresses in 2–3 and 3–4 are $+\frac{1}{6}$ P sec θ and $-\frac{1}{6}$ P sec θ, respectively, when the load at L_1 is on the truss. When the truss is fully loaded, the stress in 2–3 is $(-\frac{10}{6}+\frac{1}{6})$ P sec $\theta = -\frac{9}{6}$ P sec θ, and the stress in 3–4 is $(+\frac{10}{6}-\frac{1}{6})$ P sec $\theta = +\frac{9}{6}$ P sec θ. The maximum stresses in 4–5 and 5–6 are $-\frac{8}{6}$ P sec θ and $+\frac{8}{6}$ P sec θ, respectively, when the loads at L_3, L_2' and L_1' are on the truss. The minimum stresses in 4–5 and 5–6 are $+\frac{8}{6}$ P sec θ and $-\frac{8}{6}$ P sec θ, respectively, when the loads at L_1 and L_2 are on the truss. When the truss is fully loaded, the stress in 4–5 is $(-\frac{8}{6}+\frac{8}{6})$ P sec $\theta = -\frac{8}{6}$ P sec θ, and the stress in 5–6 is $(+\frac{8}{6}-\frac{8}{6})$ P sec $\theta = +\frac{8}{6}$ P sec θ.

Conclusions. The following conclusions may be drawn from the live load coefficicnts shown in Fig. 136.

(1) *For maximum live load chord stresses, load the truss fully with live load. For minimum live load chord stresses, there should be no live load on the truss.*

(2) *For maximum live load web stresses, load the longer segment of the truss only.*

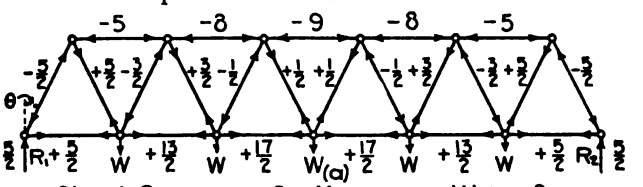
(3) *For minimum live load web stresses, load the shorter segment of the truss only.*

Simplified Method. Instead of considering the coefficients for the member due to each separate load, it is usually more convenient to find the coefficients directly for all the loads. The dead load coefficients for the truss may be readily found by first finding the effective reactions in terms of the panel load, and then determining the coefficients from the end toward the center of the truss (see Fig. 137, a). The live load coefficients for the chords are the same as those for the dead load. The live load coefficient for the maximum or minimum stress in any web member may be readily found from the formula:

$$\text{Live load web coefficient} = \frac{\left(\frac{m+1}{2}\right)m}{n}, \text{ where}$$

m = number of loads on truss for a maximum or minimum stress in any web member, and

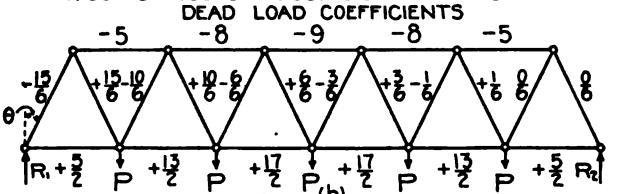
n = number of panels in truss.



Chord Stresses = Coefficients x W tan θ

Web Stresses = Coefficients x W sec θ

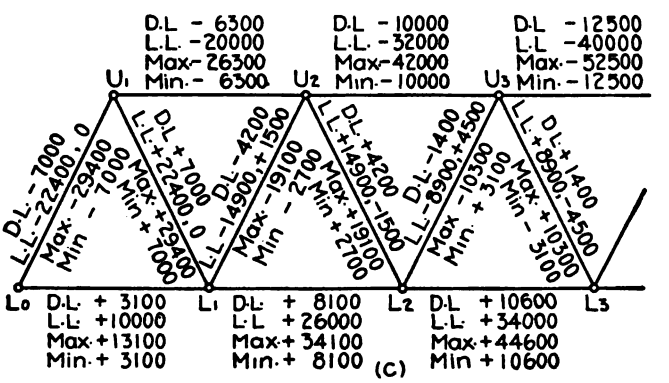
DEAD LOAD COEFFICIENTS



Chord Stresses = Coefficients x P tan θ

Web Stresses = Coefficients x P sec θ

LIVE LOAD COEFFICIENTS



Length = 60 ft.; Height = 10 ft.; Width = 16 ft.

W = 2500 lbs.; P = 8000 lbs.; Tan θ = 0.500; Sec θ = 1.118

FIG. 137. MAXIMUM AND MINIMUM STRESSES—WARREN TRUSS.

The second differences of the maximum coefficients in the web members are constant, which furnishes a check on the work. The numerators of these coefficients are 1, 3, 6, 10, etc., while the denominator of each is the number of panels (see Fig. 136).

204. Coefficients and Stresses in a Warren Truss. It is required to find the maximum and minimum stresses in all the members of the Warren truss shown in Fig. 137. The truss has a span of 60 ft.; a height of 10 ft.; and a panel length of 10 ft. The dead panel load is 2500 lbs., and the live panel load is 8000 lbs.

The dead load coefficients are shown in Fig. 137, a. The dead load chord stresses are obtained by multiplying the chord coefficients by W tan θ, and the dead load web stresses, by multiplying the web coefficients by W sec θ. These stresses are shown on the truss diagram in Fig. 137, c.

The live load coefficients are shown in Fig. 137, b. The chord and web stresses are obtained by multiplying the coefficients by P tan θ and P sec θ, respectively. These stresses are shown in Fig. 137, c. Instead of placing both the maximum and minimum coefficients on the same member, the maximum coefficients are placed on the member and the minimum coefficients on the corresponding member on the other side of the center line.

The maximum and minimum stresses are obtained by combining the corresponding dead and live load stresses. These stresses are shown in Fig. 137, c.

205. Coefficients for a Pratt Truss. The dead load coefficients for a Pratt truss are shown in Fig. 138, a. The hip vertical acts as a hanger to support a single panel load, and its coefficient is unity. The dead load chord and web stresses may be obtained by multiplying the coefficients by W tan θ and W sec θ, respectively.

The live load coefficients are shown in Fig. 138, b. The coefficients for maximum live load stresses are shown on the left, and those for minimum stresses, on the right of the center line. The live load chord and web stresses may be obtained by multiplying the coefficients by P tan θ and P sec θ, respectively.

206. Coefficients for a Baltimore Truss. The dead load

coefficients for a through Baltimore truss are shown in Fig. 139, a. The signs placed before the coefficients and the arrows on the members indicate the kind of stress. The sub-members in this truss carry the loads to the main members and are not a part of the main truss system. Care must be used in determining the coefficients, as some of the sub-members carry the loads toward the center of the truss, and the loads are then carried back to the abutment by the main members. The dead load chord and web stresses may be obtained by multiplying the coefficients by W tan θ and W sec θ, respectively.

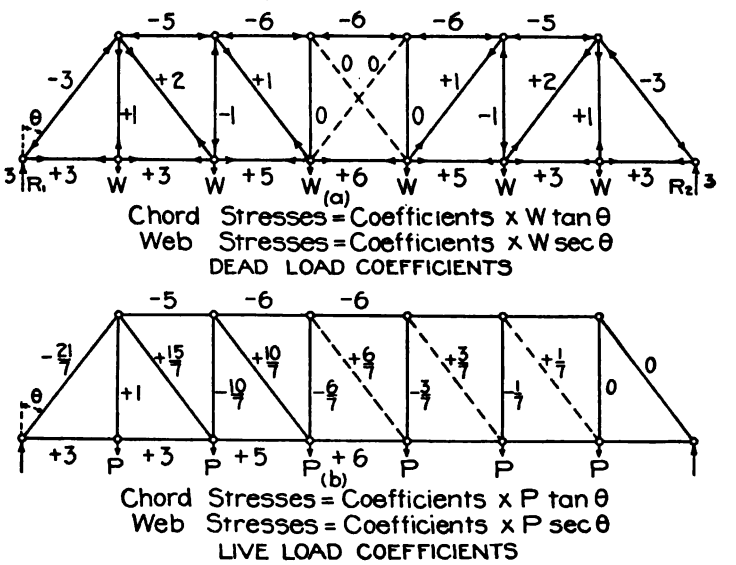


FIG. 138. DEAD AND LIVE LOAD COEFFICIENTS—PRATT TRUSS.

The live load coefficients are shown in Fig. 139, b. The live load chord coefficients are the same as those for the dead load. The coefficients for maximum live load stresses are shown on the left, and those for minimum live load stresses, on the right of the truss. The maximum live load stresses in the diagonal web members whose coefficients are $+\frac{6}{7}\frac{6}{4}$ and $+\frac{5}{7}\frac{2}{4}$ are both obtained when the loads extend to the right of the member whose coefficient is $+\frac{6}{7}\frac{6}{4}$. Likewise, the maximum live load stresses in the

members whose coefficients are $-\frac{4.5}{14}$, $+\frac{4.5}{14}$, and $+\frac{8.8}{14}$ are all obtained when the loads extend to the right of the member whose coefficient is $+\frac{4.5}{14}$. The former loading in this particular case will give the same coefficient. The apparent peculiarity of the

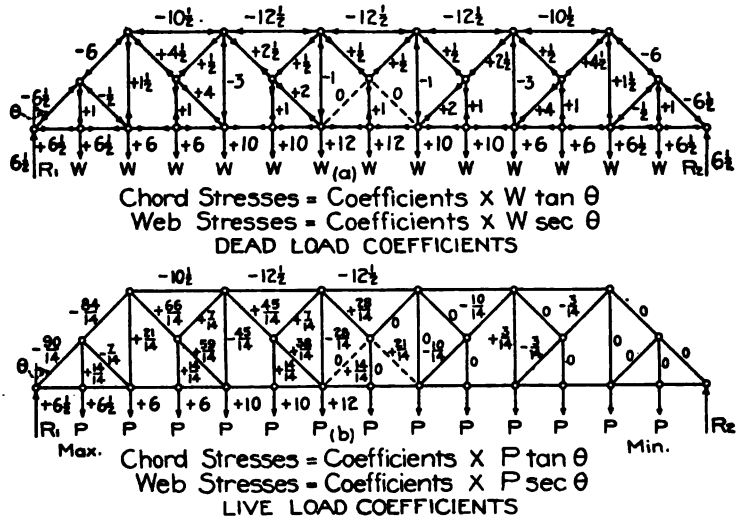


FIG. 139. DEAD AND LIVE LOAD COEFFICIENTS—BALTIMORE TRUSS.

loading for maximum stresses is due to the sub-members. Care should be used in getting the coefficients for minimum web stresses in this type of truss. The live load chord and web stresses may be obtained by multiplying the coefficients by P tan θ and P sec θ, respectively. In making the combinations for maximum and minimum dead and live load web stresses, care must be used in determining whether or not the counters are acting.

CHAPTER XXII.

INFLUENCE DIAGRAMS, AND POSITIONS OF ENGINE AND TRAIN LOADS FOR MAXIMUM MOMENTS, SHEARS, AND STRESSES.

This chapter will treat of the construction of influence diagrams, and of their use in determining the positions of wheel loads for maximum moments, shears, and stresses in girders and trusses. When a series of concentrated loads moves across a girder-bridge, the maximum moments and shears at various points are usually given by different positions of the wheel loads, and in many cases by a different wheel load at each point. Likewise in the case of trusses, the maximum moments in chord members and maximum shears in web members are usually given by different positions of the loads, and often by a different wheel load at the various panel points. It is possible to derive criteria for the positions of wheel loads for maximum moments and shears in girders and trusses, and this chapter will take up the determination of such criteria by means of influence diagrams.

207. Influence Diagrams.* *Definitions.* An influence diagram is a diagram which shows the variation of the effect at any particular point, or in any particular member, of a system of loads moving over the structure. Influence diagrams representing the variations of bending moments and shears in trusses and beams are commonly used, and will be taken up in this chapter. The difference between an influence diagram and a bending moment or shear diagram is that the former shows the variation of bending moments or shears at a particular point, or in a particu-

*For a full discussion of influence diagrams, see a paper by G. F. Swain, Trans. Am. Soc. C. E., July, 1887.

lar member, for a system of moving loads; while the latter shows the bending moments or shears at different points for a system of fixed loads.

The influence diagram is usually drawn for a load unity, and in this chapter only unit influence diagrams will be considered. The moments, shears, or stresses for any system of moving loads may be obtained from the intercepts in the unit influence diagram by multiplying them by the given loads.

The equation representing the influence diagram at any point may be derived by writing the equation for the function due to a load unity at the point.

The principal use of influence diagrams is to find the position of a system of moving loads which will give maximum moments, shears, or stresses; although they may also be used to determine the values of these functions.

208. Position of Loads for a Maximum Moment at Any Point in a Beam, or at Any Joint of the Loaded Chord of a Truss with Parallel or Inclined Chords (a) *Concentrated*

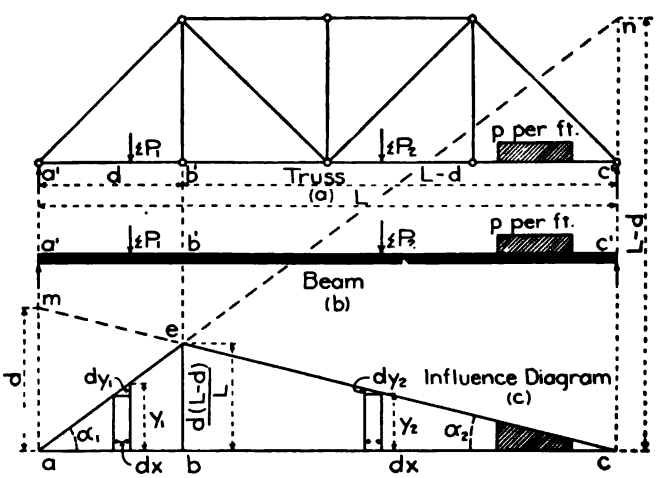


FIG. 140. INFLUENCE DIAGRAM—MOMENT AT LOADED CHORD JOINT b'.

Moving Loads. Let ΣP_1 (Fig. 140, a and Fig. 140, b) be the summation of the moving loads to the left of the point b' of the

truss or beam, and let ΣP_2 be the summation of the moving loads to the right of b'. It is required to draw the influence diagram for the bending moment at b'; also to determine the position of the moving loads for a maximum bending moment at b'.‡

To construct the influence diagram (Fig. 140, c) for the bending moment at b', compute the moment at b' for a load unity at that point. This moment $= \frac{d\,(L-d)}{L} =$ ordinate be. Draw the horizontal line ac, lay off the ordinate be, and draw the lines ae and ce. Now when the load unity is to the left of b', the bending moments at b' are represented by the ordinates to the line ae; and when the load unity is to the right of b', by the ordinates to the line ce.

A convenient method for drawing the influence diagram without actually computing the moment will now be shown. Let y represent the ordinate to the line ae, and x, the distance from the load unity to the left end of the span. Then the equation of the line ae (i. e., for the load unity to the left of b') is

$$y = \frac{(L-d)}{L}\,d - (d-x) = \frac{x\,(L-d)}{L}, \qquad (1)$$

and the equation of the line ce (i. e., for the load unity to the right of b') is

$$y = \frac{d\,(L-x)}{L}. \qquad (2)$$

When $x = d$, these two equations have a common ordinate $= \frac{d\,(L-d)}{L}$. When $x = L$, the ordinate to the line ae is $L - d$; and when $x = 0$, the ordinate to the line ce is d. Therefore, to construct the influence diagram (Fig. 140, c), lay off at a the distance $am = d$, and at c, lay off $cn = L - d$. The intersection of an and cm will then locate the point e of the influence diagram.

‡The criterion determined in this section applies to the unloaded chord joints if they are on the same vertical lines as those in the loaded chords.

The moment at any point along the truss or beam due to any number of moving loads may be found from the unit influence diagram by multiplying the ordinate under each load by the load and taking the sum of these products.

The position of the loads for a maximum bending moment at any point b′ will now be determined. Since ΣP_1 and ΣP_2 represent the summation of all the loads to the left and to the right of b′, respectively, they will have the same effect as the separate loads. The bending moment at b′ due to the loads ΣP_1 and ΣP_2 is

$$M = \Sigma P_1 y_1 + \Sigma P_2 y_2. \qquad (3)$$

Now let the loads be moved a small distance dx to the left, the movement being so small that none of the loads pass a′, b′, or c′. Then the bending moment is

$$M + dM = \Sigma P_1 (y_1 - dy_1) + \Sigma P_2 (y_2 + dy_2). \qquad (4)$$

Subtracting equation (3) from equation (4), we have

$$dM = -\Sigma P_1 dy_1 + \Sigma P_2 dy_2. \qquad (5)$$

Referring to Fig. 140, c, it is seen that $dy_1 = dx \tan \alpha_1$, and that $dy_2 = dx \tan \alpha_2$. But $\tan \alpha_1 = \frac{be}{ab} = \frac{L-d}{L}$, and $\tan \alpha_2 = \frac{be}{bc} = \frac{d}{L}$. Substituting these values of dy_1 and dy_2 in equation (5), dividing through by dx, and placing $\frac{dM}{dx} = 0$ for a maximum, we have

$$\frac{dM}{dx} = \Sigma P_1 \left(\frac{L-d}{L}\right) + \Sigma P_2 \left(\frac{d}{L}\right) = 0. \qquad (6)$$

Solving equation (6), we have

$$\Sigma P_1 L = (\Sigma P_1 + \Sigma P_2)\, d,$$

or

$$\frac{\Sigma P_1}{d} = \frac{\Sigma P_1 + \Sigma P_2}{L}. \qquad (7)$$

Equation (7) is the criterion for the maximum bending moment at b′ in a truss or beam. Since b′ is any point along the truss or beam, this criterion expressed in words is—*the maximum bending moment occurs when the average load to the left of the point is equal to the average load on the entire span.*

For a given system of moving loads, it may happen that two or more positions of the loads will satisfy the criterion. In this case, the moment for each position must be found. As soon as the position of the loads has been determined, the bending moment may be found, either algebraically or graphically, by the methods given for fixed loads.

If the truss has equal panels, the most convenient unit of length to use is a panel length. For a truss with unequal panels, or for a beam, the common unit of length is the foot.

(b) *Uniform Load.* The moment at b′ (Fig. 140) for any length of uniform load of p pounds per linear foot is equal to the area of the influence diagram included between the extreme ordinates of the uniform load multiplied by the load per linear foot. For, the bending moment due to a length dx of the uniform load $= p\,y\,dx$, which is the area under that load. If the length of the uniform load $= x$, the moment at $b' = \int_0^x p\,y\,dx$ $= p \times$ area of influence diagram under the uniform load. For a maximum bending moment at any point, it is seen that the span must be entirely covered with the uniform load. The maximum bending moment at $b' = p \times$ area of influence diagram aec.

209. Position of Loads for a Maximum Moment at Any Joint of the Unloaded Chord of a Truss with Parallel or Inclined Chords. Let ΣP be the total moving load on the truss; let ΣP_1 be the summation of the moving loads to the left of b′ (Fig. 141, a); let ΣP_2 be the summation of the moving loads in the panel b′c′; and let ΣP_3 be the summation of the moving loads to the right of c′ in any truss with horizontal or inclined chords in which the unloaded chord joints are not vertically over those of the loaded chord. Also, let r = the horizontal distance from e′ to the next loaded chord joint toward the left end of the truss; let s = the horizontal distance from e′ to the

left end of the truss; and let L = the span of the truss. It is required to draw the influence diagram for the moment at e′, and to determine the position of the moving loads for a maximum moment at e′.

Now it is seen that the moment at e′ for a load unity moving from b′ to a′ and from f′ to c′ is the same as for a simple truss. The portions of the influence diagram for these parts of the truss are ag and fh, respectively, which are portions of the influence diagram aof. The load in the panel b′c′ is carried to the panel points b′ and c′ by the stringer; and the influence line for this part of the truss is therefore the straight line gh.

The criterion for a maximum moment at e′ may be determined in the following manner: Let the loads be moved a small

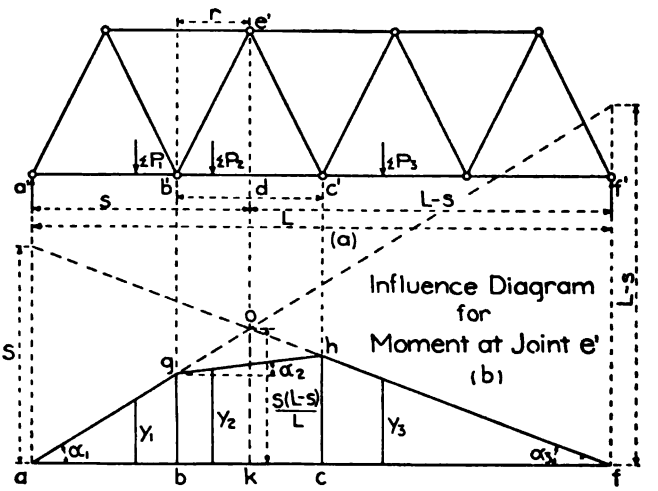


FIG. 141. INFLUENCE DIAGRAM—MOMENT AT UNLOADED CHORD JOINT e′.

distance dx toward the left end of the truss from the position shown in Fig. 141, a. The increase in the moment at e′ will be

$$dM = \Sigma P_3 dx \tan \alpha_3 - \Sigma P_2 dx \tan \alpha_2 - \Sigma P_1 dx \tan \alpha_1.$$

For a maximum,

$$\frac{dM}{dx} = \Sigma P_3 \tan \alpha_3 - \Sigma P_2 \tan \alpha_2 - \Sigma P_1 \tan \alpha_1 = 0. \quad (1)$$

But $\tan \alpha_3 = \frac{s}{L}$; $\tan \alpha_2 = \frac{hc - gb}{d}$; and $\tan \alpha_1 = \frac{L - s}{L}$

Also,

$hc = (L - s + r - d) \tan \alpha_3$, and $gb = (s - r) \tan \alpha_1$.

Substituting these values of hc and gb in the expression for $\tan \alpha_2$, and then substituting the values of $\tan \alpha_3$ and $\tan \alpha_1$, we have

$$\tan \alpha_2 = \frac{rL - sd}{dL}.$$

Substituting the values of $\tan \alpha_3$, $\tan \alpha_2$, and $\tan \alpha_1$ in equation (1), we have

$$\Sigma P_3 \frac{s}{L} - \Sigma P_2 \frac{rL - sd}{dL} - \Sigma P_1 \frac{L - s}{L} = 0.$$

Now substituting ΣP for $\Sigma P_1 + \Sigma P_2 + \Sigma P_3$, and reducing, we have

$$\frac{\Sigma P}{L} - \frac{\Sigma P_2 \frac{r}{d} + \Sigma P_1}{s} = 0, \qquad (2)$$

or

$$\frac{\Sigma P}{L} = \frac{\Sigma P_2 \frac{r}{d} + \Sigma P_1}{s}, \qquad (3)$$

which is the criterion for a maximum moment at any joint of the unloaded chord.

Referring to equation (2), it is seen that $\frac{\Sigma P}{L} - \frac{\Sigma P_2 \frac{r}{d} + \Sigma P_1}{s}$ must become 0 by passing from positive to negative, and that this can only occur when some load passes c′ or b′.

210. Position of Loads for a Maximum Moment at a Panel Point of a Truss with Subordinate Bracing. It is required to determine the position of the wheel loads for a

maximum moment at the panel point g′ of the truss with subordinate bracing shown in Fig. 142, a. A truss with subordinate bracing is one which has points of support for the floor system between the main panel points. It is seen that g′ is the center of moments for determining the stress in the lower chord member b′e′. The portion of the load in the panel c′e′, carried to c′, is on the left of the section p–p; and its moment must therefore be considered in getting the moment at g′.

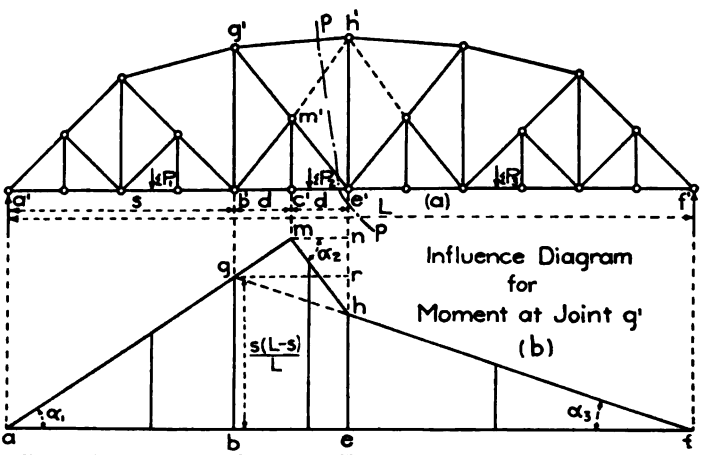


FIG. 142. INFLUENCE DIAGRAM—TRUSS WITH SUBORDINATE BRACING.

The influence diagram for the moment at g′ (Fig. 142, b) is drawn as follows: For a load unity moving from a′ to b′, the moment at g′ increases from o to $\frac{s(L-s)}{L}$; and ag is the influence line for the segment a′b′. Likewise, the influence line for a load unity moving from f′ to e′ is fh. As the load unity moves from b′ to c′, the moment at g′ of the forces to the left of section p–p is increased at the same rate as from a′ to b′; and gm is the influence line for the segment b′c′. The influence line for the segment c′e′ is the straight line mh.

To determine the position of the wheel loads for a maximum moment at g′, let ΣP_1, ΣP_2, ΣP_3, and ΣP be the loads in the

segment a'b', in the panel c'e', in the segment e'f', and on the entire span, respectively. Now if the loads are moved a small distance dx to the left, the moment at g' will be increased by

$$dM = \Sigma P_3 dx \tan \alpha_3 + \Sigma P_2 dx \tan \alpha_2 - \Sigma P_1 dx \tan \alpha_1.$$

For a maximum moment,

$$\frac{dM}{dx} = \Sigma P_3 \tan \alpha_3 + \Sigma P_2 \tan \alpha_2 - \Sigma P_1 \tan \alpha_1 = 0. \qquad (1)$$

But,
$$\tan \alpha_3 = \frac{s}{L}; \quad \tan \alpha_1 = \frac{L-s}{L};$$

and
$$\tan \alpha_2 = \frac{nr + rh}{d} = \frac{d \tan \alpha_1 + 2d \tan \alpha_3}{d} = \frac{L+s}{L}.$$

Substituting these values in equation (1) and reducing, we have

$$\frac{\Sigma P}{L} = \frac{\Sigma P_1 - \Sigma P_2}{s}, \qquad (2)$$

which is the criterion for a maximum moment at g'. This criterion will be satisfied when some wheel load is at the panel point c'.

211. Position of Loads for a Maximum Shear at Any Point in a Beam. (a) *Concentrated Moving Loads.* It is required to find the position of the loads for a maximum shear at any point O (Fig. 143, a).

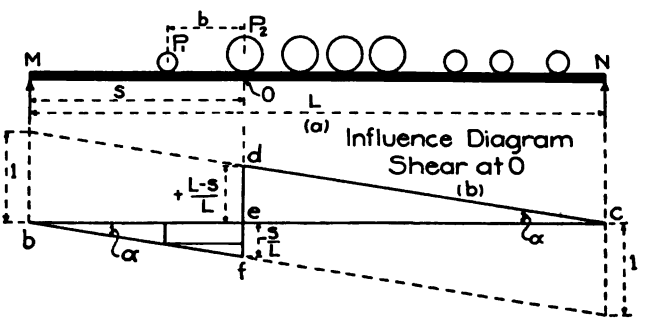


FIG. 143. SHEAR IN A BEAM.

The unit influence diagram (Fig. 143, b) will first be drawn. For a load unity entering the beam at N and moving along the

span, it is seen that the shear increases from zero at N to $+\frac{L-s}{L}$ when the load is just to the right of O. When the load passes O, the shear at that point is suddenly decreased by the amount of the load unity, and is equal to $-\frac{s}{L}$, and then increases to zero at M. For a system of moving loads, it is seen that the positive shear increases as the loads move toward the left until a load passes O, when it is suddenly decreased by the amount of the load. The shear therefore reaches a maximum every time a load reaches O. Now it may happen that, even though the loads are moved to the left and a load passes O, the reaction will be increased sufficiently to increase the shear at O.

A method will now be shown for determining which of the two consecutive loads, P_1 or P_2, when placed just to the right of O, will give a maximum shear.

Let $\Sigma P_1 =$ total load on the beam when P_1 is at O. Now move the loads to the left until P_2 is just to the right of O. If no additional load moves on the beam and no load moves off, the shear will at first be suddenly decreased by P_1, and then gradually increased by an amount equal to $\Sigma P_1 b \tan \alpha = \frac{\Sigma P_1 b}{L}$ (see Fig. 143, b). The total increase in shear will be $\frac{\Sigma P_1 b}{L} - P_1$. If this expression is negative, i. e., if $\frac{P_1}{b} > \frac{\Sigma P_1}{L}$, then P_1 placed at O will give the maximum shear; and if it is positive, i. e., if $\frac{P_1}{b} < \frac{\Sigma P_1}{L}$, then P_2 will give the maximum shear. If $\frac{P_1}{b} = \frac{\Sigma P_1}{L}$, then both wheels give equal shears.

If an additional load moves on the beam and no load moves off when P_2 moves up to O, the expression representing the increase in shear will be $\frac{\Sigma P_1 b}{L} + \frac{P_n x}{L} - P_1$; where P_n is the

load which enters the span, and x is the distance from the right end of the beam to the load P_n. If ΣP_2 is the total load on the beam when wheel P_2 is at O, then the increase in shear will lay between $\frac{\Sigma P_1 b}{L} - P_1$ and $\frac{\Sigma P_2 b}{L} - P_1$. When the former expression is negative and the latter positive, then both positions of the loads should be tried. This condition will occur for only a short distance, to the right of which both expressions are negative, and to the left, both are positive. Wheel 1 at the point will give a maximum shear when $\frac{P_1}{b} > \frac{\Sigma P_2}{L}$, and wheel 2 will give a maximum shear when $\frac{P_1}{b} < \frac{\Sigma P_1}{L}$.

(b) *Uniform Load.* From Fig. 143, b, it is seen that the maximum shear at any point O due to a uniform load will occur when the load extends from the right end of the beam to the point O. The minimum shear will occur when the load extends from the left end of the beam to the point O.

212. Position of Loads for a Maximum Shear in Any Panel of a Truss with Parallel or Inclined Chords. (a) *Concentrated Moving Loads.* Let Fig. 144, a represent a truss loaded with concentrated loads. It is required to determine the position of the loads for a maximum shear in any panel, say b′c′. Let ΣP_1 represent the total load on the left of this panel, ΣP_2 that on the panel, and ΣP_3 the total load on the right of the panel b′c′. Let m = number of panels on the left of the panel b′c′, and n = total number of panels in the truss.

The influence diagram (Fig. 144, b) is constructed, as follows: For a load unity moving from a′ to b′, the shear in the panel b′c′ increases from zero when the load is at a′ to $-\frac{m}{n}$ when the load is at b′; and ab is the influence line for the load to the left of the panel b′c′. For a load unity moving from d′ to c′, the shear increases from zero to $+\frac{n-m-1}{n}$; and dc

is the influence line for the loads to the right of the panel b'c'. For a load unity on the panel b'c', the shear varies from $-\frac{m}{n}$ for the load at b' to $+\frac{n-m-1}{n}$ for the load at c'; and bc is the influence line for the load on this panel.* The load in the panel b'c' is carried to the joints b' and c' by the stringers, the amount of load transferred to each varying inversely as the distance from the load to the joint.

The influence diagram for the entire span is abcd, and it is seen that the lines ab and bc are parallel, and that the perpendicular distance between them is unity.

Referring to the influence diagram (Fig. 144, b), it is seen that for a maximum positive shear in b'c' due to a moving load,

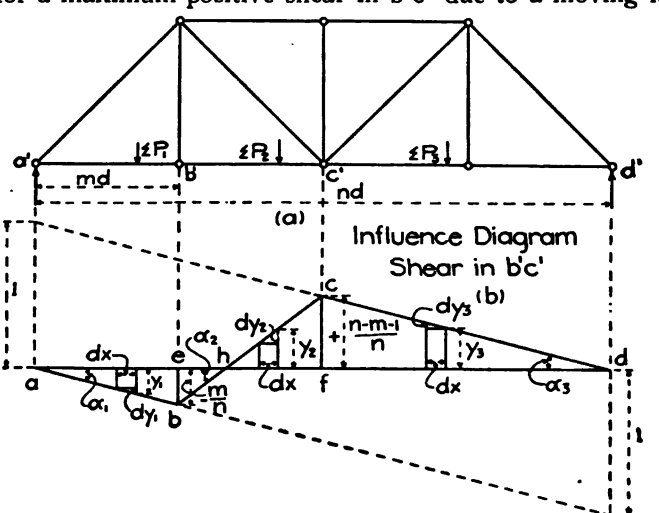


FIG. 144. SHEAR IN A TRUSS.

the load should be placed at c'; and for a maximum negative shear, the load should be placed at b'.

The maximum positive shear in the panel b'c' is

$$S = \Sigma P_3 y_3 + \Sigma P_2 y_2 - \Sigma P_1 y_1. \qquad (1)$$

Now move the loads a small distance dx to the left, the dis-

tribution of the loads remaining the same as before. The shear is then

$$S + dS = \Sigma P_3 (y_3 + dy_3) + \Sigma P_2 (y_2 - dy_2) - \Sigma P_1 (y_1 - dy_1) \quad (2).$$

Subtracting equation (1) from equation (2), we have

$$dS = \Sigma P_3 dy_3 - \Sigma P_2 dy_2 + \Sigma P_1 dy_1. \quad (3)$$

But $$dy_3 = dx \tan \alpha_3 = dx \frac{1}{nd},$$

$$dy_2 = dx \tan \alpha_2 = dx \frac{n-1}{nd},$$

and $$dy_1 = dx \tan \alpha_1 = dx \frac{1}{nd}.$$

Substituting these values of dy_3, dy_2, and dy_1 in equation (3), and dividing through by dx, we have

$$\frac{dS}{dx} = \Sigma P_3 \frac{1}{nd} - \Sigma P_2 \frac{n-1}{nd} + \Sigma P_1 \frac{1}{nd}. \quad (4)$$

Placing $\frac{ds}{dx} = o$ for a maximum, and multiplying through by nd, we have

$$\Sigma P_3 - \Sigma P_2 (n-1) + \Sigma P_1 = o, \quad (5)$$

from which $$\Sigma P_2 = \frac{\Sigma P_1 + \Sigma P_2 + \Sigma P_3}{n}, \quad (6)$$

which is the criterion for the maximum shear in any panel. This criterion expressed in words is—*the maximum shear in any panel of a truss will occur when the load in the panel is equal to the total load on the truss divided by the number of panels.* This is equivalent to saying that the average load in the panel must be equal to the average load on the entire bridge.

This criterion requires that some wheel near the head of the train shall be at the panel point to the right of the panel in which the shear is required. Any particular wheel P placed at the right-hand panel point will give a maximum shear in that panel if the entire load on the bridge lays between $\Sigma P_2 n$ and

$(\Sigma P_2 + P)\, n$; where ΣP_2 is the load in the panel other than P, and n is the number of panels in the truss.

(b) *Uniform Load.* Referring to Fig. 144, b, it is seen that the maximum shear in the panel b′c′ due to a uniform load will occur when the load extends from the right end of the truss to the point h, i. e., to the point where the shear changes sign. The minimum shear will occur when the uniform load extends from the left end of the truss to the point h. By minimum shear is meant the greatest shear of an opposite kind.

213. Position of Loads for a Maximum Stress in any Web Member of a Truss with Inclined Chords. The criterion for maximum stresses in chord members is the same for trusses with parallel chords as for those with inclined chords. The criterion for maximum stresses in web members for trusses with inclined chords differs from that for those with parallel chords; since in the former case, the inclined chord members take a part of the shear.

Let g′c′ (Fig. 145, a) be any web member of a truss with inclined chords; let p–p be a section cutting g′h′, g′c′, and b′c′; and let O be the point of intersection of g′h′ and b′c′, which is the center of moments for determining the stress in g′c′. Also let ΣP be the entire moving load on the truss when there is a maximum stress in g′c′; let ΣP_1 be the summation of the moving loads in the panel b′c′; and let ΣP_2 be the summation of the moving loads to the right of c′. It is required to draw the influence diagram for the moment at O, and to determine the position of the moving loads for a maximum stress in g′c′.

The stress in g′c′ is equal to the moment of the external forces to the left of the section p–p about the point O divided by the moment arm r; and the stress in this member is a maximum when the moment at O is a maximum.

The moment at O for a load unity at $c' = -\frac{L - s - d}{L}\, k$,

and for a load unity at $b' = +\frac{L - s}{L} \cdot k - k - s$. The mo-

ment passes through zero when the load is at some point between b′ and c′. The influence diagram for the moment at O may therefore be constructed by laying off ch (Fig. 145, b) $= \frac{L-s-d}{L}k$ (downward from c), $bg = \frac{L-s}{L}k - k - s$ (upward from b), and drawing ag, gh, and hf.

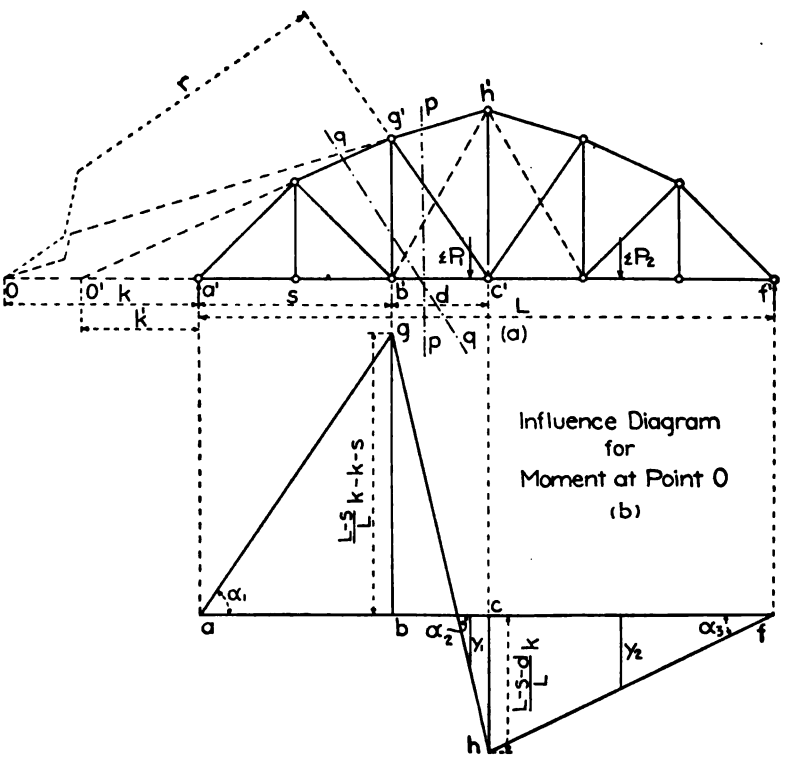


FIG. 145. STRESS IN WEB MEMBER—TRUSS WITH INCLINED CHORDS.

The maximum stress in g′c′ occurs when some of the wheels near the head of the train are in the panel b′c′. It will be assumed in this case that there are no loads to the left of b′; as this is the usual condition.

The criterion for a maximum stress in g′c′ may be deter-

mined in the following manner: Let the loads be moved a small distance dx toward the left end of the truss from the position shown in Fig. 145, a. The increase in the moment at O will be

$$dM = \Sigma P_2 dx \tan \alpha_3 - \Sigma P_1 dx \tan \alpha_2.$$

For a maximum,

$$\frac{dM}{dx} = \Sigma P_2 \tan \alpha_3 - \Sigma P_1 \tan \alpha_2 = 0. \qquad (1)$$

But $\tan \alpha_3 = \frac{ch}{L-s-d} = -\frac{k}{L}$, since $ch = -\frac{L-s-d}{L}k$;

and $\tan \alpha_2 = \frac{ch + gb}{d}$

$$= \frac{-\left(\frac{L-s-d}{L}k\right) + \left(\frac{L-s}{L}k - k - s\right)}{d}$$

$$= \frac{k}{L} - \frac{k+s}{d}.$$

Substituting these values of $\tan \alpha_3$ and $\tan \alpha_2$ in equation (1), and putting ΣP instead of $\Sigma P_1 + \Sigma P_2$, we have

$$\frac{\Sigma P}{L} - \frac{\Sigma P_1\left(1 + \frac{s}{k}\right)}{d} = 0,$$

or

$$\frac{\Sigma P}{L} = \frac{\Sigma P_1\left(1 + \frac{s}{k}\right)}{d}, \qquad (2)$$

which is the criterion for a maximum moment at O and therefore for a maximum stress in the member g'c'.

By comparing the above criterion with that given for a maximum shear in § 212, it is seen that they are very similar, the only difference being that in this case the load in the panel is to be increased by the $\frac{s}{k}$ th part of itself before dividing by the panel length.

To get the maximum stress in the member g'b' (Fig. 145, a), the same panel b'c' is partially loaded and k is replaced by k', the other quantities remaining the same as for g'c'.

214. Position of Loads for a Maximum Floorbeam Reaction. It is required to find the maximum load on the floorbeam at O (Fig. 146, a) due to loads on the panels MO and ON The loads are carried to M, O, and N by the floor stringers.

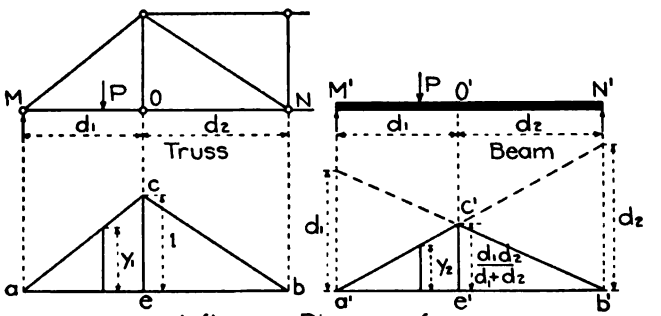


FIG. 146. FLOORBEAM REACTION.

Fig. 146, a shows the influence diagram for the floorbeam load at O. The influence line for the load in the panel MO is ac; and that for the load in the panel ON is cb. The ordinate ce is equal to unity.

Let M'N' (Fig. 146, b) be a beam whose length $d_1 + d_2$ is equal to the length of the two panels MN of the truss, and let a'b'c' be the influence diagram for the bending moment at O', whose distance from the left end of the beam equals d_1. The ordinate under O' equals $\frac{d_1 d_2}{d_1 + d_2}$.

By comparing the diagrams in Fig. 146, a and Fig. 146, b, it is seen that they differ only in the value of the ordinates ce and c'e'. It is also seen that the maximum floorbeam reaction occurs for the same position of the loads as does the maximum bending moment. The ratio of any two corresponding ordinates

y_1 and y_2 is as ce is to $c'e' = 1 \div \frac{d_1 d_2}{d_1 + d_2} = \frac{d_1 + d_2}{d_1 d_2}$.

Therefore, the maximum floorbeam reaction may be obtained by finding the maximum bending moment at a distance d_1 from the end of a beam whose length is equal to the sum of the two panel lengths $d_1 + d_2$, and multiplying this moment by $\frac{d_1 + d_2}{d_1 d_2}$.

If the two panel lengths are equal, the maximum moment at the center of the beam should be multiplied by $\frac{2}{d}$ where d is equal to the panel length.

CHAPTER XXIII.

MAXIMUM MOMENTS, SHEARS, AND STRESSES DUE TO ENGINE AND TRAIN LOADS.

This chapter will treat of the determination of maximum moments, shears, and stresses due to engine and train loads. Graphic methods of applying the criteria derived in Chapter XXII will also be shown. The dead load moments, shears, and stresses will not be considered; as they may be found by the methods already explained.

The chapter will be divided into two articles, as follows: Art. 1, Maximum Moments, Shears, and Stresses in Any Particular Girder or Truss; and Art. 2, Maximum Moments, Shears, and Stresses in Girders and Trusses of Various Types and Spans.

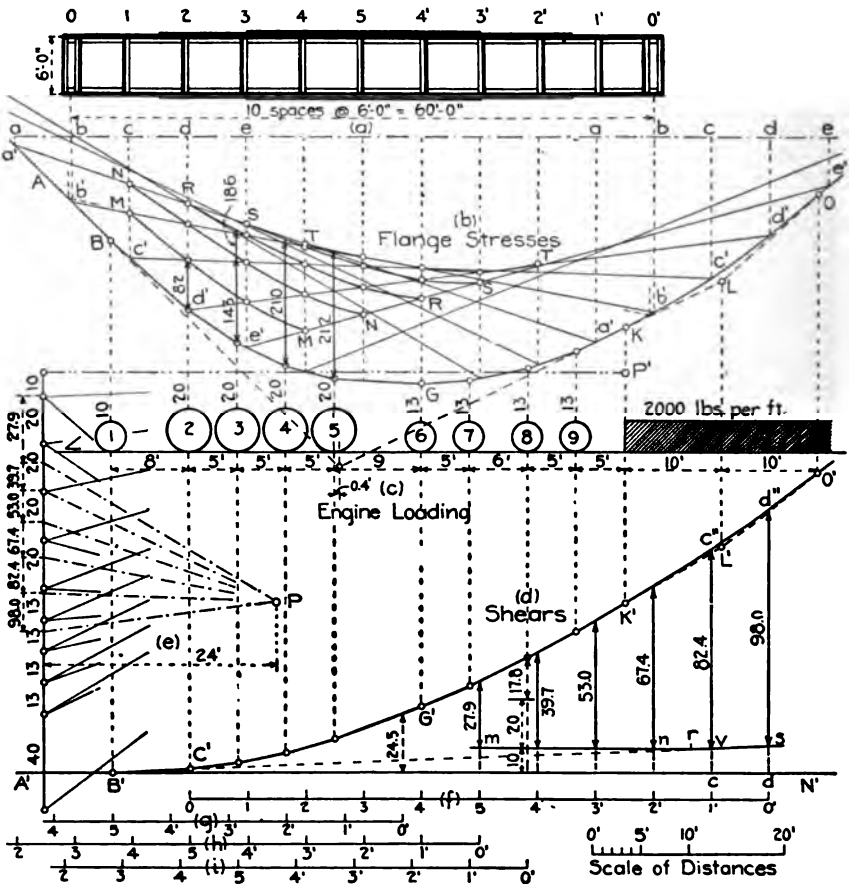
ART. 1. MAXIMUM MOMENTS, SHEARS, AND STRESSES IN ANY PARTICULAR GIRDER OR TRUSS.

When it is required to determine the maximum moments, shears, and stresses in any particular span, the methods which will now be given will be found convenient. These methods will be explained by the solution of two problems, involving (1) the plate girder, and (2) the Pratt truss.

215. (1) Maximum Flange Stresses and Shears in a Plate Girder. Let Fig. 147, a represent one girder of a single-track deck plate girder-bridge whose span, center to center of end bearings, is 60 ft., and whose effective depth is 6 ft. It is required to find the maximum flange stresses and shears at the tenth points

along the girder due to the engine and train loading shown in Fig. 147, c, the loading being that for one girder.

Flange Stresses. The diagram for flange stresses is shown in Fig. 147, b. To construct this diagram, lay off the engine dia-



FLANGE STRESSES AND SHEARS
IN A
PLATE GIRDER UNDER ENGINE AND TRAIN LOAD
Span = 60'-0"; Depth (eff.) = 6'-0"; Loading as shown.
All loads and stresses are in thousands of pounds.

FIG. 147. MAXIMUM FLANGE STRESSES AND SHEARS IN A PLATE GIRDER.

gram (Fig. 147, c) to any convenient scale, and mark the beginning, middle point, and end of the uniform load. Since the span of the girder is 60 ft., it will be necessary to consider only about 20 ft. of uniform load. Lay off the load line (Fig. 147, e) for the given engine and 20 ft. of train load, and with a pole distance equal to some multiple of the effective depth of the girder (in this case four times the depth), construct the funicular polygon ABGKO (Fig. 147, b). The portion of the funicular polygon KO for the uniform train load is an arc of a parabola, tangent to KL at K and to LO at O, the method used in constructing the parabola being that shown in Fig. 148, b. Prolong the line AB as far as necessary to complete the construction. Now divide the span of the girder into ten equal parts, and drop verticals from these points of division to meet the funicular polygon. The line b′b′ (Fig. 147, b), which connects the points of intersection of verticals 60 feet apart with the funicular polygon, is the closing string of the funicular polygon for the loads that come upon the girder when it is in the position shown in Fig. 147, a. For this position of the girder, wheel 2 is at section 2. The ordinates under the several points of division, intercepted between the closing string b′b′ and the funicular polygon, represent to scale the flange stresses at the several points along the girder for this position of the load. These ordinates represent actual flange stresses, and the scale used in measuring them is four times that used for the loads in the load line; since the pole distance was taken equal to four times the depth of the truss (see § 184). Now consider the girder to be shifted one division, or 6 feet, toward the right from the position shown in Fig. 147, a. (This has the same effect as moving the loads 6 feet to the left.) The closing string of the funicular polygon for this new position of the loads and truss is c′c′. The ordinate directly above wheel 2, intercepted between the closing string c′c′ and the funicular polygon, represents the flange stress at section 1 of the girder; and the other ordinates represent the flange stresses at various points along the girder. Likewise, d′d′ and e′e′ are closing strings for the girder moved two and three divisions to the right, respectively, from its original position; and the flange stresses

at different points along the girder are represented by the ordinates between these closing strings and the funicular polygon. Also, a′a′ is the closing string for the funicular polygon when the girder is moved one division to the left from the position shown in Fig. 147, a. From an inspection of the diagram shown in Fig. 147, b, it is seen that it is unnecessary to draw any other closing strings for this problem. Now if the curve MM is drawn through the points distant each one division horizontally from b′, c′, d′, and e′, respectively, it is seen that the ordinates between this curve and the funicular polygon will represent the successive flange stresses at section 1 as the girder is moved to the right, i. e., when the load moves along the girder toward the left. Likewise, the curve NN, drawn through the points distant each two divisions horizontally from a′, b′, c′, d′, and e′, respectively, will represent the successive flange stresses at section 2 as the load moves along the girder. Also, the curves RR, SS, and TT represent the successive flange stresses at sections 3, 4, and 5, respectively, as the load moves along the girder. By scaling the ordinates at the different points, it is seen that the maximum flange stress at section 1 occurs when wheel 2 is at section 1, and is 82 000 lbs. Likewise, it is seen that the maximum flange stress at section 2 occurs when wheel 3 is at section 2, and is 143 000 lbs.; that the maximum flange stress at section 3 occurs when wheel 3 is at section 3, and is 186 000 lbs.; that the maximum flange stress at section 4 occurs when wheel 4 is at section 4, and is 210 000 lbs.; and that the maximum flange stress at section 5 occurs when wheel 5 is at section 5, and is 212 000 lbs. It should be noted that the maximum moment and flange stress at any point always occurs when some wheel is at the point, i. e., the maximum ordinate is always at some vertex of the funicular polygon. If the ordinate is not at one of the division points through which the curves MM, NN, etc., were drawn, its length may be more accurately determined by drawing the closing line for the required position of the loads. It is seen that the construction shown in Fig. 147, b not only gives the maximum stresses, but also the wheels which cause these stresses. Referring to § 159, it is seen that the maximum moment, and therefore the maximum

flange stress in the girder, occurs at some wheel near the center of the span when that wheel is as far to one side of the center as the center of gravity of all the loads is to the other side of the center. The position of the center of gravity of all the wheels on the girder at this time is given by producing the extreme strings AB and KL (Fig. 147, b) until they intersect. It is seen that the center of gravity of all the wheels is 0.4 ft. to the right of wheel 5; therefore the maximum flange stress in the girder occurs when wheel 5 is 0.2 ft. to the left of the center. A part of the closing string for this position of the wheels is shown by the dotted line near b′ (Fig. 147, b); and the maximum flange stress in the girder is 213 000 lbs.

Shears. The maximum live load shears at the tenth points may be obtained from the diagram shown in Fig. 147, d. In constructing this diagram, use the same load line as for flange stresses, but take the pole at P′. The pole P′ is located on a horizontal line through the top of the load line with a pole distance equal to the span of the girder, 60 feet. Draw the funicular polygon A′B′C′G′K′O′ (Fig. 147, d) for the given loading, the portion K′O′ being the arc of a parabola tangent to K′L′ at K′ and to L′O′ at O′. With the first driver, wheel 2, at the left end of the girder, lay off the span to the right, and divide it into tenths, as shown in Fig. 147, f. The ordinate dd″ above the right end of the girder, intercepted between the horizontal line A′B′N′ and the funicular polygon A′B′C′G′K′O′, then represents the reaction at the left end of the span (see § 187); provided none of the loads used in the construction of the funicular polygon are off of the girder. Now when wheel 2 is at the left end of the girder, wheel 1 is off of the span, and should not be considered in drawing the funicular polygon, i. e., the horizontal line A′B′N′ should be replaced by the line B′C′rs. Since there are no loads on the girder to the left of wheel 2, the maximum shear at the left end of the span is equal to the left reaction, which is represented by the ordinate sd″ = 98 000 lbs. When wheel 2 is at section 1, which is 6 feet from the left end of the girder, wheel 1 is still off of the girder, and the maximum shear at section 1 is represented by vc″ = 82 400 lbs. The scale used in measuring these ordi-

nates is the same as that used for the loads. Now if sections 2, 3, and 4 (Fig. 147, f) are successively placed under wheel 2, then the intercepts above the right end of the girder will represent the left reactions. The shears at these points are equal to the reactions minus the weight of the pilot wheel 1; and are represented by the intercepts between the funicular polygon and the line mr, the vertical distance between A′B′N′ and mr representing the weight of wheel 1. The shears at sections 2, 3, and 4 are 67 400, 53 000, and 39 700 lbs., respectively. When wheel 2 is at section 5, the center of the girder (see Fig. 147, h), the shear at section 5 is 27 900 lbs.; when wheel 1 is at section 5 (Fig. 147, g), the shear at section 5 is 24 500 pounds; and when wheel 3 is at section 5 (Fig. 147, i), the shear at that section is 17 800 lbs. For maximum shears, it is unnecessary to consider any points to the right of the center of the girder. It is seen that wheel 2 gives the maximum shears at all points from the left end to the center of the span. It may be easily determined by trial, from the diagram shown in Fig. 147, d, which wheel—wheel 1 or wheel 2—gives the maximum shear at any point along the girder, or it may be determined directly by the criterion given in § 211.

Another method of obtaining the maximum shears will now be given, in which the shears are determined from the diagram for flange stresses. When wheel 2 is at the left end of the girder, wheel 1 is off the span, the closing string of the funicular polygon is d′d′ (Fig. 147, b), and the ray, drawn through P parallel to d′d′ to intersect the load line, will determine the two reactions (Fig. 147, e), the left reaction being 98 000 lbs., which is the shear at the left end of the girder. Likewise, when wheel 2 is at section 1, wheel 1 is still off the girder, the closing string of the funicular polygon is c′c′; and the ray, drawn through P parallel to c′c′, will determine the two reactions, the left reaction being 82 400 lbs., which is the shear at section 2 of the girder, Likewise, when wheel 2 is at section 2, wheel 1 is on the girder, the closing string is b′b′, the left reaction is 77 400 lbs., and the shear is $77\,400 - 10\,000 = 67\,400$ lbs. In a similar manner, the shears at sections 3, 4, and 5 of the girder are found to be 53 000, 39 700, and 27 900 lbs., respectively. In this problem, wheel 2 is

at one of the sections. If the wheel causing maximum shears is not at one of the sections, then another set of verticals and closing lines should be drawn to determine the maximum shears. In some cases, this method gives the simpler solution; while in others, the first method described is more efficient.

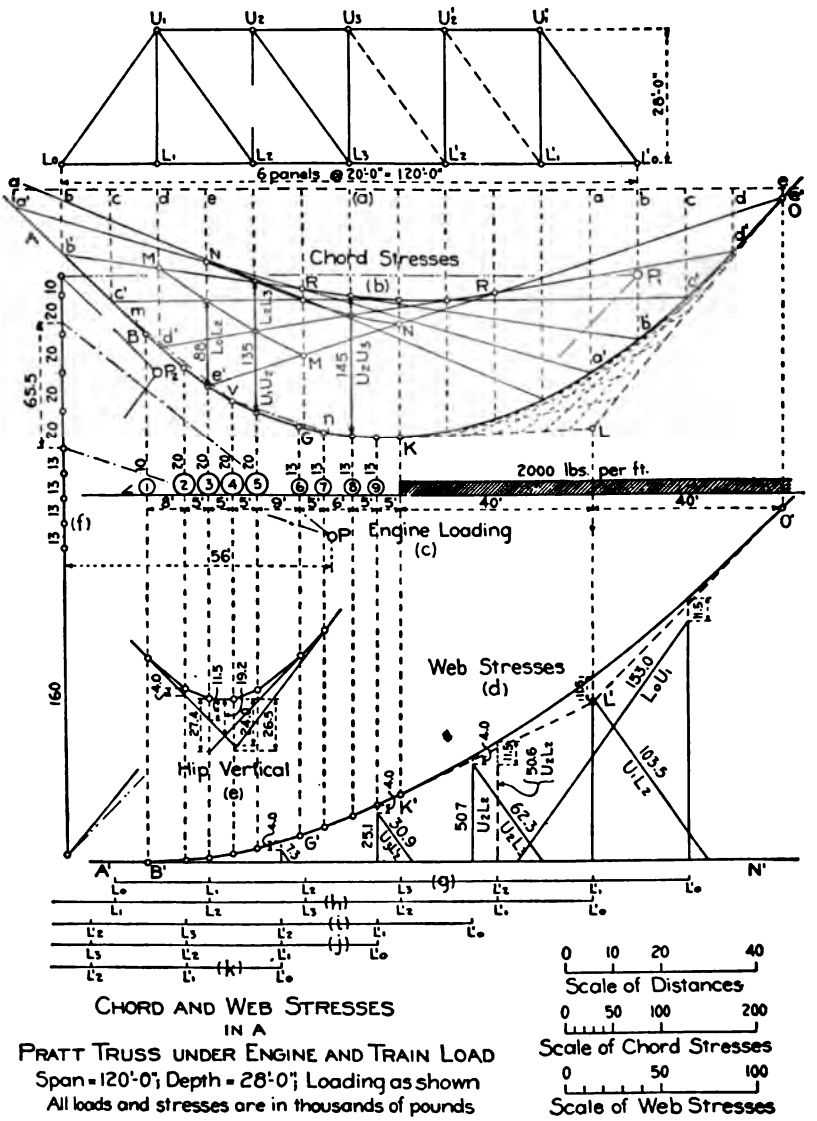
In the problem given, the loading consisted of one engine and tender followed by a uniform train load. If the loading consists of two engines and tenders followed by a uniform train load, the maximum shears will be somewhat larger. This is evident from the fact that the straight line AB (Fig. 147, b) would then be replaced by a broken line, thus increasing the length of the ordinates between the closing lines and the funicular polygon.

The process explained is more efficient if the equidistant verticals, which represent the divisions of the girder, are drawn upon a separate sheet of tracing cloth or transparent paper; as the position of the girder may then be readily shifted.

The method explained in this section affords an efficient solution for any particular span and loading; and if a large scale is used and care is exercised in making the constructions very good results may be obtained.

216. (2) **Maximum Chord and Web Stresses in a Pratt Truss.** Let Fig. 148, a, represent one truss of a single-track through Pratt truss-bridge, whose span is 120 ft., and whose depth is 28 ft. It is required to find the maximum chord and web stresses due to the engine and train loading shown in Fig. 148, c, the loading shown being that for one truss.

Chord Stresses. In this problem, the chords are parallel, and the upper chord panel points are directly over those of the lower chord, which simplifies the solution somewhat. The constructions and methods used for this problem are similar to those used for the plate girder (see Fig. 147, and § 215); and only the points in which the methods differ for the two cases need be explained. The diagram for chord stresses is shown in Fig. 148, b. By an inspection of the truss and loading, it was seen that about 80 feet of uniform train load would be sufficient. The load line is shown in Fig. 148, f; and since the pole distance was taken equal to twice the depth of the truss, the ordinates in the



CHORD AND WEB STRESSES
IN A
PRATT TRUSS UNDER ENGINE AND TRAIN LOAD
Span = 120'-0"; Depth = 28'-0"; Loading as shown
All loads and stresses are in thousands of pounds

FIG. 148. MAXIMUM CHORD AND WEB STRESSES IN A PRATT TRUSS.

chord stress diagram should be measured by a scale equal to twice that used for the loads. The chord stresses, together with the ordinates which represent them, are shown in Fig. 148, b. From an inspection of this diagram, it is seen that the maximum stress in L_0L_2 is 88 000 lbs. when wheel 3 is at L_1; that the maximum stresses in U_1U_2 and L_2L_3 are each 135 000 lbs. when wheel 5 is at L_2; and that the maximum stress in U_2U_3 is 145 000 lbs. when wheel 8 is at L_3. It is also seen that wheel 7 at L_3 gives almost as large a stress in U_2U_3 as does wheel 8.

If a truss with an odd number of panels had been given, then the maximum stress in the center panel of the lower chord would have occurred when the loads are so placed that there would be zero shear in the center panel.

If the live load had consisted of two engines and tenders followed by a uniform train load, then the stresses caused by the loads of the second engine should also have been considered.

Web Stresses. The diagram for all the web stresses, except the hip vertical, is shown in Fig. 148, d, and for the hip vertical, in Fig. 148, e. For the diagram shown in Fig. 148, d, the pole is taken at P_1 with a pole distance equal to the span of the truss. When wheel 3 is placed at L_1 (Fig. 148, g), the left reaction, which is represented by the intercept above L_0' (Fig. 148, g), = 136 000 lbs.; the shear in the panel L_0L_1 = 136 000 — 11 500 (the portion of the loads in the panel L_0L_1 that is carried to L_0 by the stringer) = 124 500 lbs.; and the stress in L_0U_1, which is obtained by drawing a line parallel to the member L_0U_1, = 153 000 lbs. (Fig. 148, d). This is the maximum stress for L_0U_1; as may be shown by successively placing wheels 2 and 4 at L_1, and finding the stresses in this member caused by these positions of the loads. Likewise, the maximum stress in U_1L_2 is obtained when wheel 3 is at L_2 (Fig. 148, h), and is 103 500 lbs.; the maximum stresses in U_2L_2 and U_2L_3 are both obtained when wheel 2 is at L_2 (Fig. 148, i), and are 50 700 and 62 300 lbs., respectively; the maximum live load stresses in U_3L_3 and U_3L_2' are both obtained when wheel 2 is placed at L_2' (Fig. 148, j), and are 25 100 and 30 900 lbs., respectively (if there is no dead load shear in the panel); and the maximum live load stresses in $U_2'L_2'$

and $U_2'L_1'$ are both obtained when wheel 2 is at L_1' (Fig. 148, k), and are 6000 and 7300 lbs., respectively (if there is no dead load shear in the panel $L_2'L_1'$). When wheel 2 is at any panel point, the part of the load carried to the panel point ahead by the stringer is 4000 lbs. The constructions when wheel 2 and wheel 3 are successively at L_3 are shown in Fig. 148, d, wheel 2 at L_3 giving the larger stress.

The diagram for determining the maximum stress in the hip vertical is shown in Fig. 148, e. The stress in this member is equal to that portion of the total load in the two panels L_0L_1 and L_1L_2 which is carried to L_1 by the stringers. It is seen that this is a maximum when the heavy wheels are near L_1. To determine the exact position of these wheels for a maximum stress in U_1L_1, and also the maximum stress in the member, place the center of gravity of the first six wheels (those in the panels L_0L_1 and L_1L_2) at L_1; and draw the funicular polygon shown in Fig. 148, e, with a pole distance equal to one panel length. If the pole distance is taken equal to a panel length, then the intercepts in the funicular polygon will represent the reactions at L_0 and L_2 caused by the loads in the panels L_0L_1 and L_1L_2, respectively. This is true because in the funicular polygon intercept $\times$ pole distance (one panel length) = moment; and the reactions = intercept $\times$ pole distance (one panel length) $\div$ moment arm (one panel length) = intercept. When the center of gravity of the six wheels is placed at L_1, the reactions at L_0 and L_2 are each equal to 19 200 lbs. (Fig. 148, e), and the stress in U_1L_1 = 103 000 (the total weight of the six wheels) — 38 400 = 64 600 lbs. Now place wheel 3 at L_1. For this position of the loads, the reactions at L_0 and L_2 are 11 500 and 27 400 lbs., respectively; and the stress in U_1L_1 = 103 000 — 11 500 — 27 400 = 64 100 lbs. Next place wheel 4 at L_1. For this position of the load, an additional wheel—wheel 7—is brought upon the length L_0L_2; the reactions at L_0 and L_2 are 24 000 and 26 500 lbs., respectively; and the stress in U_1L_1 = 116 000 (the total weight of the seven wheels) — 24 000 — 26 500 = 65 500 lbs. It is thus seen that the maximum tension in U_1L_1 = 65 500 lbs. when wheel 4 is at L_1.

Referring to Fig. 148, e, it is seen that the ordinates under

wheels 2 and 3 represent the loads that were deducted from the reactions in Fig. 148, d to obtain the shears.

The stress in the hip vertical may be obtained in a different manner, as follows: Place wheel 4 at L_1, and draw a vertical through wheel 4 to intersect the funicular polygon at v (Fig. 148, b). Also, mark the points m and n on the funicular polygon, distant horizontally one panel length from v. The lines mv and nv are the closing strings of the funicular polygons for the loads in the panels L_0L_1 and L_1L_2, respectively. Now draw rays through P parallel to these closing strings, and these rays will cut off on the load line the total load transferred to L_1, which is the stress in U_1L_1.

Art. 2. Maximum Moments, Shears, and Stresses in Girders and Trusses of Various Types and Spans.

When it is required to determine the moments and shears in girders and trusses of different spans, subjected to the same loading, the work may be greatly facilitated by constructing a load line and a moment diagram for the load on one rail. If the diagrams are drawn to a large scale, very good results may be obtained by their use. The construction of the diagrams will be explained, and then their application to the determination of moments and shears in girders and trusses will be shown.

217. Load Line and Moment Diagram. The load line and moment diagram for Cooper's E 40 loading are shown in Fig. 149. The diagrams shown in this figure were originally drawn upon cross-section paper, divided into one-tenth inch squares, using a scale of four times that shown in Fig. 149. The engine diagram was laid off to a scale of ten feet to the inch; the loads, to a scale of 50 000 pounds to the inch; and the moments, to a scale of 2 500 000 foot-pounds to the inch. The engine and uniform train load diagram is shown in the lower part of the figure. The engine wheels are numbered, the load on each wheel shown, and the spacing of the wheels given.

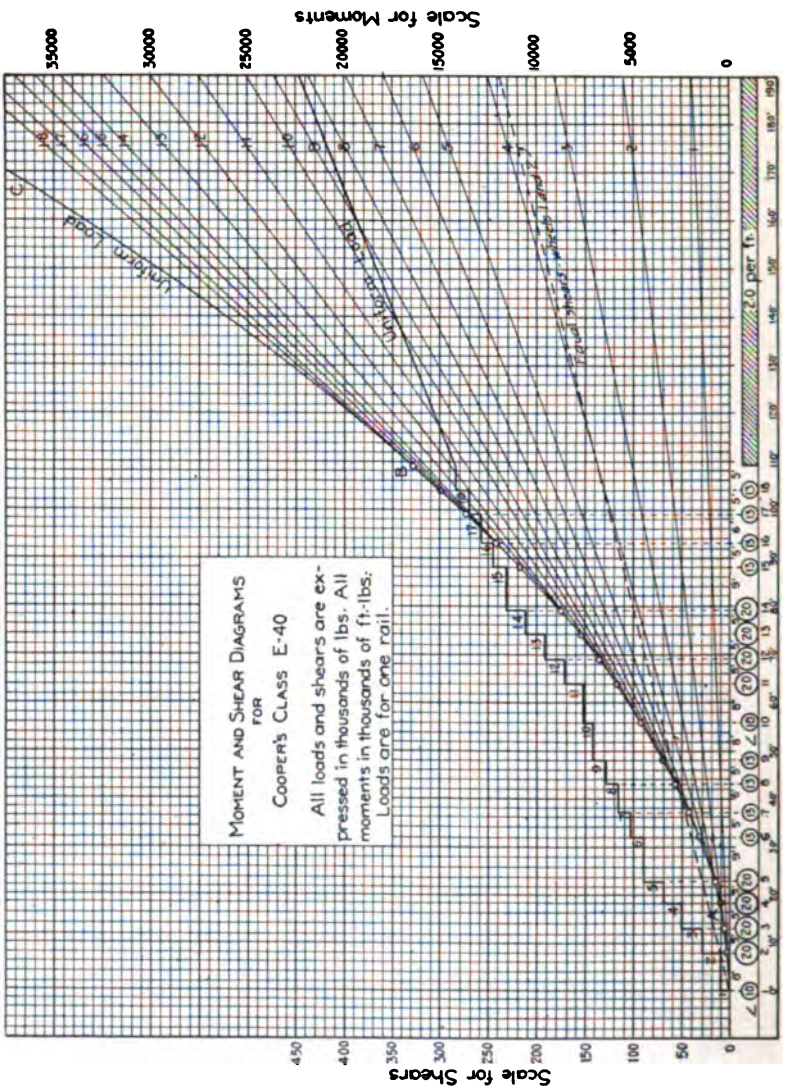


FIG. 149. MOMENT AND SHEAR DIAGRAMS—COOPER'S E40.

The load line, which is the heavy stepped line 1–2–3–4–5, etc., is a diagram whose ordinates measured from the line o–o repre-

sent the summation of the loads to the left. Each step in the load line represents to scale the load directly under it. The portion of the diagram above the uniform load is a straight line having a uniform slope of 2000 pounds per foot.

The moment lines, which are numbered 1, 2, 3, 4, 5, etc., near the right edge of the figure, are constructed as follows: Starting at 0, the right end of the horizontal reference line 0–0, lay off successively on a vertical line the moment of each wheel load about that point, beginning with wheel 1. The moment line 1 is then drawn from a point in the reference line, directly over wheel 1, to the first point at the right of the diagram; moment line 2 is drawn from a point on moment line 1, directly over wheel 2, to the second point at the right of the diagram; moment line 3, from a point on moment line 2, directly over wheel 3, to the third point, etc. The moment line BC is a parabolic curve; and may be easily constructed by computing the moments of portions of the uniform load between B and several points to the right about these points, and laying off these moments above the moment line 18. It is seen that the broken and curved line ABC is an equilibrium polygon for the given loads. The ordinate at any point, measured between the reference line 0–0 and any moment line, represents the sum of the moments about this point of all the loads up to and including the wheel load corresponding to the moment line under consideration. To illustrate, the sum of the moments of the wheel loads 1, 2, 3, and 4 about wheel 18 is represented by the ordinate over wheel 18, measured between the reference line 0–0 and moment line 4. Also, the moment at any point, measured between any two moment lines—as between 2 and 6—represents the sum of the moments of wheel loads 3 to 6, inclusive, about that point. The line of equal shears for wheels 1 and 2, shown in Fig. 149, has an upward slope of $\frac{P_1}{b}$; where P_1 is the first wheel load, and b is the distance between wheels 1 and 2. The use of this line will be explained in § 220 (1).

218. Application of Diagrams in Fig. 149 to Determining Maximum Moments in Plate Girders or at Joints of the

Loaded Chord of a Truss with Parallel or Inclined Chords. The diagrams shown in Fig. 149 may be used for finding the position of the wheels for a maximum moment, and also the value of the moment. To illustrate the use of these diagrams, let it be required to determine the position of the wheels for a maximum moment at L_2 (Fig. 150), this point being either a panel point of

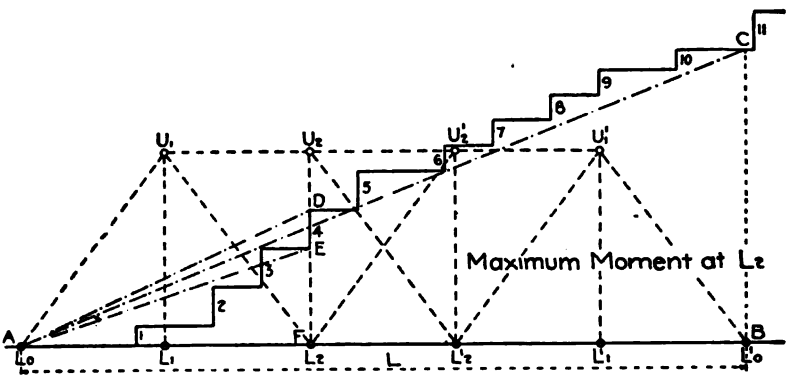


FIG. 150. MAXIMUM MOMENT AT LOADED CHORD JOINT.

the truss whose span is AB, or any point along a girder of the same span. To avoid confusion, a portion of the load line shown in Fig. 149 is reproduced to a larger scale in Fig. 150. Since the girder, or truss, must be shifted, the points along the girder, or the panel points of the truss, should be marked on the edge of a separate slip of paper or upon a piece of tracing cloth; and it will be assumed that the points marked on the span AB (Fig. 150) are on a separate slip of paper. The criterion for a maximum moment at L_2 is — the average load ΣP_1 to the left of L_2 must be equal to the average load ΣP on the entire span (see § 208). Try wheel 4 at L_2, i. e., shift the truss until L_2 is under this wheel (Fig. 150). The total load on the span is represented by the ordinate $BC = \Sigma P$; while the average load is $\frac{\Sigma P}{L}$, which is represented by the slope of the line AC. The load ΣP_1 to the left of L_2 is represented either by the ordinate FE or the ordinate FD, depending upon whether wheel 4 is at L_2 or just to the left

of L_2; and the average load $\frac{\Sigma P_1}{AF}$ to the left of L_2 is represented either by the slope of the line AE or the line AD. Since the slope of the line AC is less than that of AD and greater than that of AE, it is seen that wheel 4 at L_2 gives a maximum moment at this point. If the line AC lays above the line AD, the loads must be moved to the left; and if the line AC is below the line AE, then the loads must be moved to the right. It is thus seen that if the line whose slope is $\frac{\Sigma P}{L}$ cuts the vertical line representing the load at the point, this position of the load gives a maximum moment. Instead of actually drawing the line AC, its position is usually determined by stretching a thread. In the illustration just given, none of the wheels were off the bridge to the left, and the line AC starts from the zero shear line at L_0. If some of the loads had passed the left end of the bridge L_0, then AC should start in the load line vertically over L_0.

The position of the wheels having been determined, the moment itself may be easily found from the moment diagram (Fig. 149), as follows: With wheel 4 at L_2, read the ordinate at the right end of the span L_0' between the reference line o–o and the moment line 10, which is the moment of all the loads on the span about L_0'. The moment at L_2 is obtained from the moment at L_0' by multiplying it by $\frac{AF}{L}$ and subtracting the moment of the loads to the left of L_2, this latter moment being given by reading the ordinate at L_2 between the reference line o–o and moment line 4.

Since the line ABC (Fig. 149) formed by the segments of the moment lines is a funicular polygon for the given loads, the moment at L_2 is also equal to the ordinate at that point intercepted between the funicular polygon and the closing line. The extremities of this closing line are on the verticals which pass through the ends of the bridge.

In finding the maximum moment in a girder—which occurs near its center—it is necessary to locate the center of gravity of

all the loads on the girder (see § 159 (a)); and the center of gravity of any number of loads may be found from Fig. 149 by producing the extreme strings of the funicular polygon until they intersect.

219. Application of Diagrams in Fig. 149 to Determining Maximum Moments at Panel Points in the Unloaded Chord of a Truss with Parallel or Inclined Chords. The use of the diagrams in Fig. 149 for finding the maximum moment at any unloaded chord panel point will be shown by the following problem: It is required to determine the position of the engine load for a maximum moment at the panel point U_3 of the truss shown

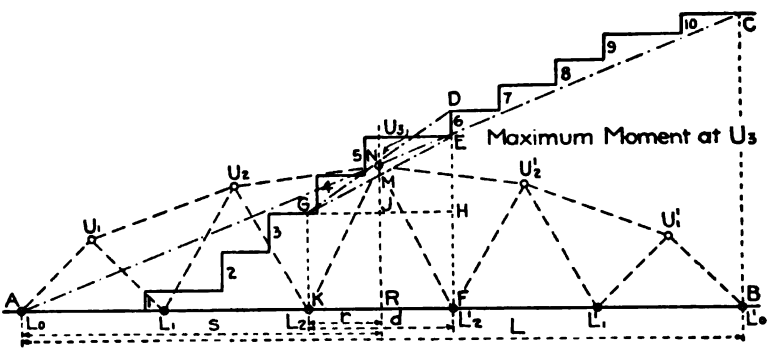


FIG. 151. MAXIMUM MOMENT AT UNLOADED CHORD JOINT.

in Fig. 151, together with value of the maximum moment. Let the heavy stepped line 1–2–3–4, etc. (Fig. 151), be a portion of the load line, and let L be the span of the truss. It has been shown (§ 209) that the criterion for a maximum moment at U_3 is

$$\frac{\Sigma P}{L} = \frac{\Sigma P_2 \frac{r}{d} + \Sigma P_1}{s}$$; where ΣP is the total load on the span,

ΣP_1 is the load to the left of L_2, ΣP_2 is the load in the panel L_2L_2', d is the panel length, r is the horizontal distance from U_3 to L_2, and s is the horizontal distance from U_3 to L_0. Try wheel 6 at L_2', which is the position of the load in Fig. 151. Now the total load ΣP on the span is represented by BC, and the aver-

age load $\frac{\Sigma P}{L}$, by the slope of the line AC. The load ΣP_1 to the left of L_2 is represented by GK, and that in the panel L_2L_2', by HD or HE (depending upon whether wheel 6 is just to the left or to the right of L_2'). The average load in the panel L_2L_2' is represented by the slope of the line GD, or the line GE. The term $\Sigma P_2 \frac{r}{d}$ (see above criterion) is represented by JN, or JM; the term $\Sigma P_2 \frac{r}{d} + \Sigma P_1$, by RN, or RM; and the term $\frac{\Sigma P_2 \frac{r}{d} + \Sigma P_1}{s}$, by the slope of the line AN, or the line AM (not drawn in the figure). Since the line AC cuts the vertical through U_3 between the points N and M, it is seen that the wheel 6 at L_2' satisfies the criterion for a maximum moment at U_3. If it had been impossible to satisfy the criterion by placing some wheel at L_2', then the criterion should have been tested by placing a wheel at L_2. In using the diagrams shown in Fig. 149, the upper and lower chord panel points should be marked on a separate slip of paper. The slip should then be shifted until the wheel which gives a maximum moment is at the panel point about which the moment is required. The auxiliary lines shown in Fig. 151 need not be drawn; as the positions of the lines GD and GE may be determined by stretching a thread, the points N and M being marked, and the thread then moved to the position AC.

The position of the engine load (wheel 6 at L_2') having been determined, the left reaction may be found as in § 218. The determination of the moment at the unloaded chord joint U_3 differs from that at a loaded chord joint, in that only the portion of the load in the panel L_2L_2', transferred to L_2' by the stringers, should be considered. The moment at U_3 is equal to that of the left reaction minus the moment of the portion of the loads in L_2L_2' transferred to L_2 together with the moment of all other loads to the left of U_3.

The moment at U_3 may also be readily found, as follows: With wheel 6 at L_2', find the moments at L_2 and L_2', as in § 218. Then if M_L and M_R represent the moments at L_2 and L_2' respectively, the moment at $U_3 = M_L + (M_R - M_L)\frac{r}{d}$.

220. Application of Diagrams in Fig. 149 to Determining Maximum Shears. Two cases will be considered, viz: (1) maximum shears in beams and girders; and (2) maximum shears in trusses.

(1) *Maximum Shears in Beams and Girders.* It has been shown (§ 211) that wheel 1 at any point will give a maximum shear when $\frac{P_1}{b} = \frac{\Sigma P_2}{L}$, and that wheel 2 will give a maximum shear when $\frac{P_1}{b} = \frac{\Sigma P_1}{L}$; where P_1 is the first wheel load, ΣP_1 is the total load on the span when P_1 is at the point, ΣP_2 is the total load when P_2 is at the point, b is the distance between wheels 1 and 2, and L is the span of the beam or girder.

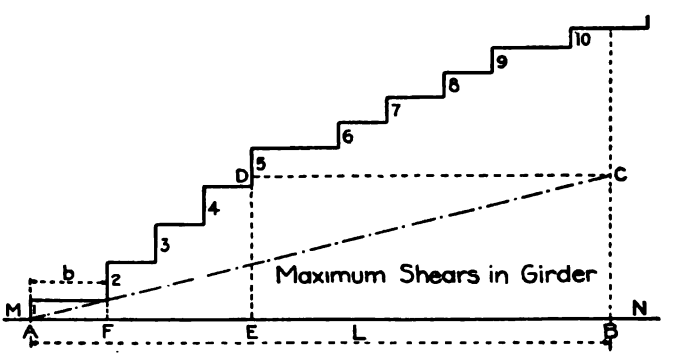


FIG. 152. MAXIMUM SHEARS IN A GIRDER.

Let 1–2–3–4, etc. (Fig. 152) be a portion of the load line, and let AB = L be the length of a beam or girder. It is required to determine the segment of the beam in which wheel 1 gives the maximum shear, and also the segment in which wheel 2 gives the maximum shear. Place wheel 1 at the left end of the beam; and

from A, draw the line AC having a slope of $\frac{P_1}{b}$ to intersect a vertical through the right end of the beam. Now the ordinate BC represents the load which divided by L equals $\frac{P_1}{b}$. It therefore represents the load ΣP_1, or ΣP_2. Draw the line CD parallel to AB to intersect the load line at D; and also draw the vertical line DE. It is then seen that the average load on the entire span equals $\frac{P_1}{b}$, when wheel 5 is at the right end of the beam. Now lay off the span AB on a separate slip of paper, and mark the points E′ and F′ on this slip to correspond with the points E and F (Fig. 152). Place the point B (on the slip) at E, i. e., directly under wheel 5, and mark the positions of wheels 1 and 2, calling these points 1′ and 2′. Then any point in the segment 2′B (between wheel 2 and the right end of the span) has its maximum shear when wheel 1 is at the point; and any point to the left of 1′ (wheel 1) has its maximum shear when wheel 2 is at the point. Between 1′ and 2′ (the distance between wheels 1 and 2), both positions should be tried.

The line marked equal shears, wheels 1 and 2 (Fig. 149), corresponds to the line AC (Fig. 152). Since Fig. 149 has cross-section lines, it is unnecessary to draw any additional lines.

(2) *Maximum Shears in Trusses.* It has been shown (§ 212) that the criterion for a maximum shear in any panel of a truss is—the load in the panel must be equal to the total load on the span divided by the number of panels. If ΣP is the total load on the span, and ΣP_1 is the load in the panel other than the wheel load at the panel point to the right, then any load P at this panel point will give a maximum shear in the panel if $\frac{\Sigma P}{L}$ lays between $\frac{\Sigma P_1}{d}$ and $\frac{\Sigma P_1 + P}{d}$.

Let L (Fig. 153) be the span of any truss, and 1–2–3–4, etc., a portion of the load line. It is required to determine which

wheel load placed at the panel point to the right will give a maximum shear in each panel of the truss. Place wheel 1 at the first panel point L_1, as shown in Fig. 153. Draw the lines AC, AD,

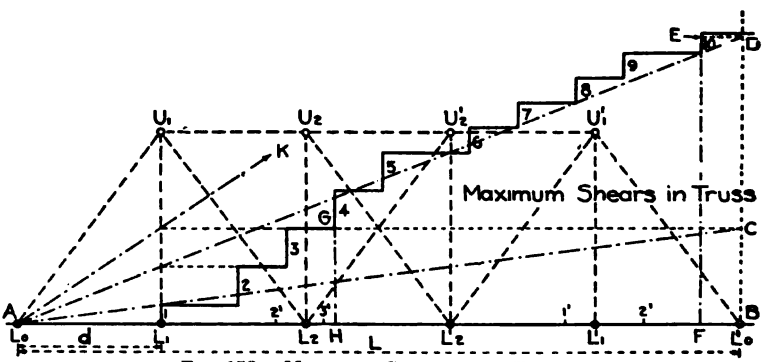


FIG. 153. MAXIMUM SHEARS IN A TRUSS.

and AK with ordinates at L_1 representing the wheel loads P_1, P_2, and P_3, respectively. These lines will have slopes of $\frac{P_1}{d}$, $\frac{P_1 + P_2}{d}$, and $\frac{P_1 + P_2 + P_3}{d}$, respectively. Wheel 1 placed at any panel point will give a maximum shear in the panel to the left when $\frac{P_1}{d} = \frac{\Sigma P}{L}$. Now $\frac{\Sigma P}{L}$ is represented by BC, the ordinate over the right end of the truss L_0'; and $\frac{P_1}{d} = \frac{\Sigma P}{L}$ as soon as wheel 3 enters the span and until wheel 4 enters the span at L_0'. Therefore wheel 1 will give a maximum shear in the panels to the left of each panel point passed by it in moving the loads to the left until wheel 4 enters the span at L_0', i. e., in the panel $L_2'L_1'$. Likewise, wheel 2 will give a maximum shear in any panel when $\frac{\Sigma P}{L}$ lays between $\frac{P_1}{d}$ and $\frac{P_1 + P_2}{d}$; i. e., for values of ΣP between BC and BD. Now $\frac{P_1 + P_2}{d} = \frac{\Sigma P}{L}$ when wheel 10 is at L_0'. Therefore wheel 2 will give a maximum shear in the panels to

the left of each panel point passed by it in moving the loads from the position with wheel 4 at L_0' to the position with wheel 10 at L_0', i. e., in the panels $L_2'L_1'$, L_2L_2', and L_1L_2. Likewise, wheel 3 will give a maximum shear in the panels to the left of each panel point passed by it in moving the loads from the position with wheel 10 at L_0' to the left end of the truss, i. e., in the panels L_0L_1 and L_1L_2. This is determined from the slopes of the lines AD and AK. From the above discussion, it is seen that both wheel 1 and wheel 2 satisfy the criterion for a maximum shear in the panel $L_2'L_1'$; and that both wheel 2 and wheel 3 satisfy the criterion for a maximum shear in the panel L_1L_2. Since the diagram in Fig. 149 has cross-section lines, the auxiliary lines shown in Fig. 153 need not be drawn, the actual process being as follows: Lay off the span AB = L on the edge of a separate slip of paper, and mark the panel points L_0, L_1, L_2, L_2', L_1', and L_0'. Place the edge of the slip on the reference line o–o (Fig. 149) with the right end L_0 of the truss under wheel 4, and mark the positions of wheel 1 and wheel 2 on the slip. Now move the slip to the right until L_0' is directly under wheel 10, and mark the positions of wheel 2 and wheel 3. The marking of the slip will now correspond to that of the line AB (Fig. 153). It is now seen that wheel 1 will satisfy the criterion for a maximum shear in the segment $L_0'1'$; that wheel 2 will satisfy the criterion for a maximum shear in the segment $2'2'$; and that wheel 3 will satisfy the criterion, in the segment $3'L_0$. It is also seen that the segment in which wheel 1 satisfies the criterion for a maximum shear is overlapped by that in which wheel 2 satisfies the criterion, by the distance $1'2'$, which is the distance between wheels 1 and 2. It is further seen that the segment in which wheel 2 satisfies the criterion for a maximum shear is overlapped by that in which wheel 3 satisfies the criterion, by the distance between wheels 2 and 3.

The position of the wheels having been determined, the shear itself may be easily found from the moment lines (Fig. 149). To determine the shear in any panel, say in L_2L_2' (Fig. 153), place the right panel point L_2' under wheel 2 (this position of

the load being the one which satisfies the criterion for a maximum shear); and read the ordinate to the funicular polygon ABC at the right end of the truss. The moment represented by this ordinate divided by the span is equal to the left reaction. Since the loads can only come upon the truss at the panel points, being transferred by the stringers, the shear in the panel L_2L_2' is equal to the left reaction minus the portion of the load in this panel which is carried to the left panel point L_2. The load which is carried to this panel point is equal to the moment of the load to the left of L_2' about L_2' divided by the panel length d; and is represented by the ordinate to the funicular polygon above the panel point L_2'.

221. Application of Diagrams in Fig. 149 to Determining Maximum Web Stresses in Trusses with Inclined Chords. The following problem will show the application of the diagrams in Fig. 149 to determining the maximum stress in any web mem-

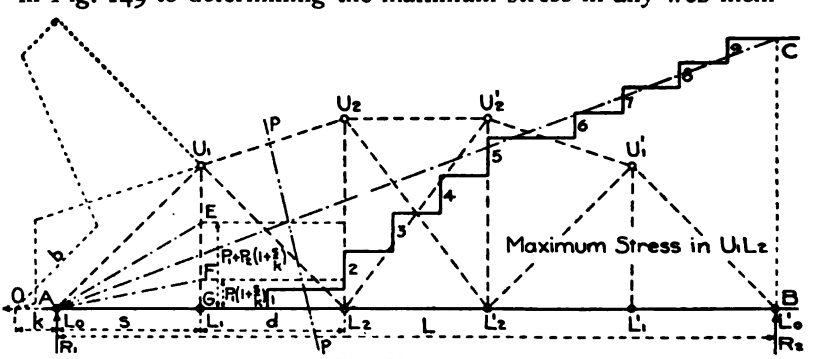


FIG. 154. MAXIMUM STRESS IN WEB MEMBER—TRUSS WITH INCLINED CHORDS.

ber of a truss with inclined chords. It is required to find the maximum live load stress in the member U_1L_2 of the truss shown in Fig. 154. A portion of the load line shown in Fig. 149 is reproduced to a larger scale in Fig. 154. It has been shown (§ 213) that the criterion for a maximum stress in the member

U_1L_2 is $\frac{\Sigma P}{L} = \frac{\Sigma P_1\left(1+\frac{s}{k}\right)}{d}$; where ΣP is the total load on the

span, ΣP_1 is the load in the panel L_1L_2, s is the distance from L_1 to the left end of the truss, and k is the distance from the left end of the truss to the point of intersection O of the members U_1U_2 and L_1L_2, this point being the center of moments for determining the stress in U_1L_2. It is seen that this criterion is similar to that for a maximum shear in the panel, indicating that some wheel near the head of the train placed at L_2 will give a maximum stress in U_1L_2. Try wheel 2 at L_2 (Fig. 154). The load ΣP_1 in the panel L_1L_2 is either P_1 or $P_1 + P_2$, depending upon whether wheel 2 is to the right or to the left of L_2. The term $\Sigma P_1\left(1 + \frac{s}{k}\right)$ of the criterion is represented by GF, or by GE; and the term $\frac{\Sigma P_1\left(1 + \frac{s}{k}\right)}{d}$, by the slope of the line AF, or the line AE. The total load on the span is represented by BC, and the average load, by the slope of the line AC. Since AC lays between AF and AE, it is seen that wheel 2 satisfies the criterion for a maximum stress in U_1L_2.

To determine the stress in the member U_1L_2 place wheel 2 at L_2, and read the ordinate to the funicular polygon (Fig. 149) at the right end of the truss. The moment represented by this ordinate divided by the span L is equal to the left reaction R_1. If G_1 represents the portion of the load to the left of the section p–p (i. e., the portion of the load carried to L_1 by the stringer), then the stress in $U_1L_2 = \frac{R_1k - G_1(k+s)}{b}$. The part of the load in L_1L_2 which is carried to L_1 may be found by reading the ordinate to the funicular polygon at L_2, and dividing by the panel length.

222. Determination of Maximum Stresses in a Truss with Subordinate Bracing. *Petit Truss.* A truss with subordinate bracing has been defined as one which has points of support for the floor system between the main panel points. The Baltimore trusses shown in Fig. 114, d and Fig. 114, e are examples of such

a truss, in which the chords are parallel; while the Petit trusses shown in Fig. 114, j and Fig. 155 are examples of this type, in which the chords are not parallel. The effect of the subordinate bracing upon the stresses in the main members of the truss will be shown by the following problem: It is required to determine the maximum stresses in the members in the panel L_4L_6 of the truss shown in Fig. 155. The members L_6M, U_2M, and L_5M are tension members; while L_4M is in compression when the counter U_3M is not acting, and in tension when the counter is acting. It is seen that U_3M does not act when the main members have their maximum stresses.

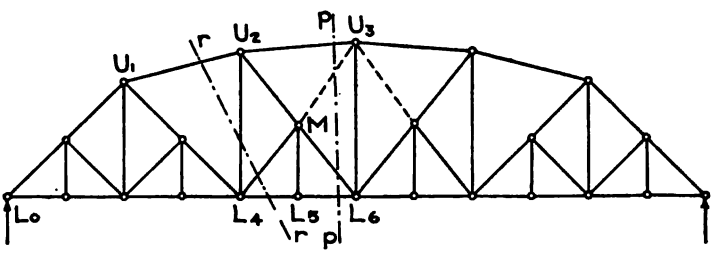


FIG. 155. MAXIMUM STRESSES—TRUSS WITH SUBORDINATE BRACING.

To determine the maximum stress in U_2U_3, cut the members U_2U_3, L_6M, and L_5L_6 by the section p–p, and take the center of moments at L_6. Since the counter U_3M is not acting, the position of the loads for a maximum moment at L_6, together with the value of this moment, may be obtained by the methods shown in § 218. The stress in U_2U_3 is equal to this moment divided by the perpendicular distance from L_6 to U_2U_3. It is thus seen that the stress in this chord member is the same as for a truss with the subordinate bracing omitted.

To find the maximum stress in the lower chord member L_4L_6, the same section should be taken with the center of moment at U_2. The position of the wheel loads for a maximum moment at U_2 may be determined from the criterion deduced in § 210. By the use of the diagrams in Fig. 149, it is easy to apply this criterion, and to determine which wheel placed at L_5 will give a maximum moment at U_2. The moment at this panel point is equal to the

reaction at L_0 multiplied by the distance L_0L_4 minus the moment of the loads to the left of L_4 about L_4 plus the moment of the portion of the loads in the panels L_4L_5 and L_5L_6 which is carried to L_5 by the stringers. It is necessary to consider the joint load at L_5; since it is to the left of the section p–p.

The maximum stress in L_5M is obtained when there is a maximum joint load at L_5, and is equal to that load. The stress in this member may be obtained as shown in Fig. 148, e, and explained in § 216.

The position of the wheel loads for a maximum stress in U_2M and the stress itself are the same as if the subordinate members L_5M and L_4M were omitted. The stress in U_2M may therefore be determined as shown in § 221, the subordinate members being omitted, and the panel length taken as L_4L_6. It is seen that this is true, since the small triangular truss L_4ML_6 merely acts as a trussed stringer to transfer the loads to L_4 and L_6.

The stress in L_6M is influenced by the subordinate members if there are any loads between L_4 and L_6. The stress in this member may be determined by taking the center of moments at the point of intersection of U_2U_3 and L_4L_6.

The maximum stress in U_2L_4 may be determined as in § 221, considering the members L_6M and L_4M removed. The section r–r should be cut, and the center of moments taken at the point of intersection of U_1U_2 and L_4L_6.

The stress in L_4M when the counter is not acting, i. e., when L_4M acts as a compression member, may be readily found by graphic resolution. Since U_2M and L_6M are collinear, the resolved components in L_4M and L_5M perpendicular to U_2L_6 must be equal.

The maximum tensile stress in L_4M, which occurs when the counter U_3M is acting, may be found in the following manner: Consider the numbers L_4M and U_3M replaced by a straight member L_4U_3. Now the maximum stress in this member may be found by the same method used for that in U_2M, i. e., by considering the members L_5M and U_2M removed. The member L_6M is not considered; as it does not act for this loading. After finding the stress in L_4U_3, the maximum stress in L_4M may be deter-

mined, as follows: Lay off on a line parallel to the member L_4U_3 a length L_4U_3, representing to scale the stress in that member. Through U_3, draw a line parallel to the member L_6M; and through L_4, draw lines parallel respectively to L_4M and U_3M to intersect the line parallel to L_6M. This construction gives the maximum stress in the member L_4M.

The maximum stress in the counter U_3M may be found by taking the section p–p, remembering that the member L_6M is not acting when the truss is loaded for a maximum stress in the counter. The load should be brought upon the truss at L_0, and the left segment loaded for a maximum in the member. The hanger L_5M should be considered; as the loads carried by it increase the stress in U_3M.

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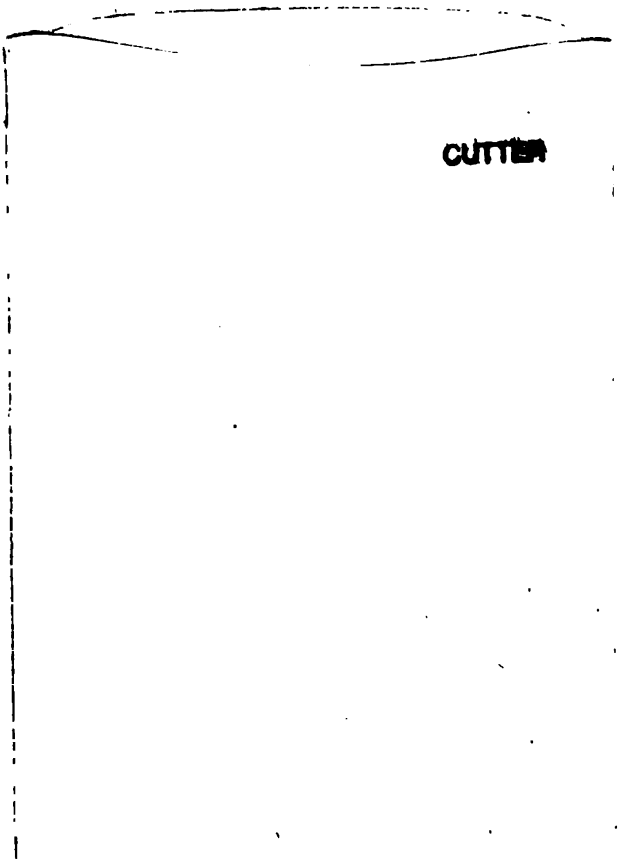
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